Effect of RNS Dynamic Range on Grayscale Images Filtering

Nikolay I. Chervyakov, Pavel A. Lyakhov, Diana I. Kalita, Kirill S. Shulzhenko
Department of Applied Mathematics and Mathematical Modeling,
North-Caucasus Federal University
Stavropol, Russia
k-fmf-primath@stavsu.ru, {ljahov, diana.kalita, kirill-joker}@mail.ru

Abstract — **In this paper we present the investigation results of RNS dynamic range effect on grayscale images filtering. We show how error occurs during filtering using RNS with insufficient dynamic range. Modelling of filtration with sharpening and edge detection filters demonstrates that the edge detection filters are less sensitive to insufficient RNS dynamic range. Quantitative evaluation of the simulation results with PSNR and SSIM showed applicability limits of different RNS for grayscale image filtering.**

I. Introduction

Digital image processing techniques play a significant role in medicine, space and scientific research, industry and information systems today [1]. Grayscale images, on one hand, are widely used in chromatics and polygraphy, and on the other hand, is one of the simplest and most illustrative tools for developing new methods of image processing, which later can be easily generalized for other color modes. Modern applications of digital image processing should be energy-efficient and have a sufficiently high speed performance computing for widespread use in energy-dependent mobile devices [2, 3]. A perspective way to achieve high-speed performance and low power consumption of digital filters is the use of Residue Number System (RNS) arithmetic instead of traditional Weighted Number System (WNS) [4-6]. For example, in paper [7] RNS-based image processor which implements discrete wavelet transform was proposed. Hardware implementation of that design showed increase in performance by 7% and reduction in hardware costs by 3,5% compared with WNS. In paper [8] authors demonstrated that using RNS for sharpening filter hardware implementation allows to increase in performance by 49% and decrease the energy consumption by 31% compared with WNS.

Important issue arising during the use of RNS is choosing the right moduli set that provides sufficient dynamic range [9]. In paper [10] authors use a special moduli set $\{2^{n-1}+1, 2^n-1, 2^n\}$ with $n=3$ for image processing and conclude sufficiency the dynamic range obtained in this case. In paper [8] a moduli set $\{2^{n-1}-1, 2^n-1, 2^n\}$ with $n=4$ is proposed to use. In this paper we will show that the use of these moduli sets can lead to incorrect results, distorting the image. Also, we will offer a rule for RNS moduli set selecting that will provide the correct processing of grayscale images.

Moduli set $\{2^{n-1}-1, 2^n-1, 2^n\}$ is one of the most attractive tools for RNS-based realization of digital signal processing. This moduli set consists of the most convenient modules for hardware implementation of modular operations since performing such operations by modulo 2^n and by modulo 2^n-1 easier than by modulo 2^n+1 [11]. Given these considerations, we will use moduli set $\{2^{n-1}-1, 2^n-1, 2^n\}$ for demonstrating correct image processing using RNS.

II. Introduction to RNS

Numbers in RNS represented in the basis of relatively prime numbers, are called modules $\beta=\{m_1,...,m_k\}$, $GCD(m_i, m_j)=1$, for $i \neq j$. Any integer number $0 \leq X < M = \prod_{i=1}^{k} m_i$ can be uniquely represented in RNS as a tuple $\{x_1, x_2, ..., x_k\}$, where $x_i = |X|_{m_i} = X \bmod m_i$ [11].

The dynamic range of RNS is usually divided into two approximately equal parts, so that about half of the range is represented by positive numbers, and the rest of the range by negative numbers. Thus, any integer number satisfying one of the following two relations can be represented in the RNS:

$$-\frac{M-1}{2} \leq X \leq \frac{M-1}{2}, \text{ if } M \text{ is odd;} \quad (1)$$

$$-\frac{M}{2} \leq X \leq \frac{M}{2}, \text{ if } M \text{ is even.} \quad (2)$$

Operations of addition, subtraction, and multiplication in RNS are defined by the formulas showing the carry-free parallel nature of RNS:

$$A \pm B = \left(|a_1 \pm b_1|_{m_1}, ..., |a_n \pm b_n|_{m_n}\right), \quad (3)$$

$$A \times B = \left(|a_1 \times b_1|_{m_1}, ..., |a_n \times b_n|_{m_n}\right). \quad (4)$$

Reverse conversion of number X from residues $\{x_1, x_2, ..., x_k\}$ is based on Chinese Remainder Theorem

$$X = \left| \sum_{i=0}^{n} \left| \left| M_i^{-1} \right|_{m_i} x_i \right|_{m_i} M_i \right|_M, \quad (5)$$

978-1-5090-4231-9/16/$31.00 ©2016 IEEE

where $M_i = \frac{M}{m_i}$ and $\left|M_i^{-1}\right|_{m_i}$ means a multiplicative inverse of M_i modulo m_i.

Thus, there are three main advantages of RNS [12].

1. There is no carry propagation between RNS arithmetic units. Large numbers are represented in the form of small residues, which leads to faster data processing.

2. When using the RNS, large numbers are encoded into a set of small residues, which reduces the complexity of the arithmetic units and simplifies the computing system.

3. RNS is non-positional system with independent arithmetic units, therefore, an error in one channel does not apply to others. Thus the processes of error detection and error correction are simplified.

However, such operations as sign detection, comparison, division and some others are time-consuming and expensive in the RNS [13]. Despite these shortcomings, modular arithmetic can be effectively implemented in applications where the main calculations accounted for the multiplication combined with addition and subtraction and digital image processing relates to such category of applications.

III. RNS IN IMAGE PROCESSING

As mentioned above, the RNS arithmetic is a perspective instrument for improving technical characteristics of the digital image processing systems. We will consider grayscale images below. In this format, the image is a rectangular array of integer values (pixels). The number of gray levels is integer power of 2, that is, the pixel represents the brightness or darkness. Thus, a larger number encoding pixel indicates brighter color. In 8-bit image processing applications, grayscale image pixels are encoded by 8-bit numbers and located in the range $[0,255]$ where 0 represents black color and 255 represents white color. If a negative number is obtained as a result of image processing then it is replaced by 0 (black color). In case of getting the number greater than 255, it is replaced by 255 (white color). Process of image filtering performs the convolution operation between image pixels and filter mask, which is a square matrix of coefficients.

When using the RNS for the digital image processing particular attention should be paid to choosing the right moduli set that will be given enough dynamic range. In the works [10] and [8] the moduli sets $\{5,7,8\}$ and $\{7,15,16\}$ were proposed and authors told about adequacy of their dynamic range for digital image processing. In the following example we will show that it is not always correct.

Example 1. Let us assume that the part of the grayscale image has the following values of the pixels:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 255 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Suppose that sharpening filter is used [14]:

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}.$$

In the WNS we obtain the following value of the pixel after the filtering:

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 255 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 1275,$$

where symbol $\times$ means sum of the products of the corresponding elements of the matrices.

The number $1275 > 255$ and therefore is considered as white color.

When filtering using RNS with moduli $\{7,15,16\}$ we have:

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 255 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$$
$$= \begin{bmatrix} \{0,0,0\} & \{6,14,15\} & \{0,0,0\} \\ \{6,14,15\} & \{5,5,5\} & \{6,14,15\} \\ \{0,0,0\} & \{6,14,15\} & \{0,0,0\} \end{bmatrix} \times$$
$$\times \begin{bmatrix} \{0,0,0\} & \{0,0,0\} & \{0,0,0\} \\ \{0,0,0\} & \{3,0,15\} & \{0,0,0\} \\ \{0,0,0\} & \{0,0,0\} & \{0,0,0\} \end{bmatrix} = \{1,0,11\} = 1275.$$

Dynamic range for RNS with moduli $\{7,15,16\}$ is equal to $M = 7 \cdot 15 \cdot 16 = 1680$. The number 1275 is in the second half of the range $840 \le 1275 < 1680$. Hence the final result after reverse conversion is $1275 - 1680 = -405$. This negative number gives the black color of pixel, which shows an incorrect result of filtering using moduli set $\{7,15,16\}$.

This example shows the importance of RNS moduli set selection, which provides sufficient dynamic range. Insufficient range may lead to incorrect results, therefore should use the criteria for overflow detection. The following formula can be used to determine the range of RNS:

$$M > 2 \cdot \left(2^N - 1\right) \cdot \max\{\pi, \nu\}, \qquad (6)$$

where $\pi = \sum_{a_{i,j} > 0} a_{ij}$ is the sum of the positive filter mask coefficients, $\nu = \left| \sum_{a_{ij} < 0} a_{ij} \right|$ is the absolute value of sum of the negative filter mask coefficients and N is the number of bits to represent each pixel of the image. Coefficient 2 in (6) shows the necessity of representation of positive and negative numbers in RNS according to the formulas (1) – (2).

For the above example, we have $\pi = 5$, $\nu = 4$ and $N = 8$. Thus the minimum RNS with moduli set $\left\{2^{n-1} - 1, 2^n - 1, 2^n\right\}$ provides the sufficient dynamic range achieved with $n = 5$ i.e.

$\{15,31,32\}$. The following example shows the correct digital image processing for the conditions from Example 1.

Example 2. Use of RNS with moduli $\{15,31,32\}$ for data processing from Example 1 gives the following result:

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 255 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} \{0,0,0\} & \{14,30,31\} & \{0,0,0\} \\ \{14,30,31\} & \{5,5,5\} & \{14,30,31\} \\ \{0,0,0\} & \{14,30,31\} & \{0,0,0\} \end{bmatrix} \times$$

$$\times \begin{bmatrix} \{0,0,0\} & \{0,0,0\} & \{0,0,0\} \\ \{0,0,0\} & \{0,7,31\} & \{0,0,0\} \\ \{0,0,0\} & \{0,0,0\} & \{0,0,0\} \end{bmatrix} = \{0,4,27\} = 1275$$

The number 1275 is in the first half of the RNS range $0 \le 1275 \le 7440$, that is a positive number which is equal to the result, obtained by using WNS.

IV. Modeling of Grayscale Image Processing Using RNS

In this section we will describe the results of grayscale image processing obtained by using different filters and different number systems. We used three RNS with moduli $\{5,7,8\}$, $\{7,15,16\}$ and $\{15,31,32\}$, as well as traditional WNS for modeling. The following masks are used as filters.

Edge detection filters:

$$1) \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}; \; 2) \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}.$$

Sharpening filters:

$$3) \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}; \; 4) \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}.$$

Further in the text these filters will be denoted as filter-1, filter-2, filter-3 and filter-4 respectively. All calculations were performed using MATLAB.

TABLE I.
THE SIMULATION RESULTS OF IMAGE PROCESSING USING RNS WITH DIFFERENT MODULI SETS

filter	moduli set					
	$\{5,7,8\}$		$\{7,15,16\}$		$\{15,31,32\}$	
	PSNR, dB	*SSIM*	*PSNR, dB*	*SSIM*	*PSNR, dB*	*SSIM*
filter-1	28.5883	0.9745	∞	1	∞	1
filter-2	27.6565	0.9658	∞	1	∞	1
filter-3	6.8032	0.2221	∞	1	∞	1
filter-4	6.2265	0.1257	45.1322	0.9999	∞	1

The results of applying filter-1 for Lena image are presented in Figure 1 (all resulting images were shifted by 128 to enhance their visibility). Figure 1a shows the original grayscale image. Figure 1b shows the result of image filtering in the WNS. Figures 1c-1e show the results of image filtering using RNS with moduli $\{5,7,8\}$, $\{7,15,16\}$ and $\{15,31,32\}$ respectively. Images 1c-1e are not very different from the image 1b visually, but in reality images 1d and 1e are identical to image 1b, whereas the image 1c is different from 1b.

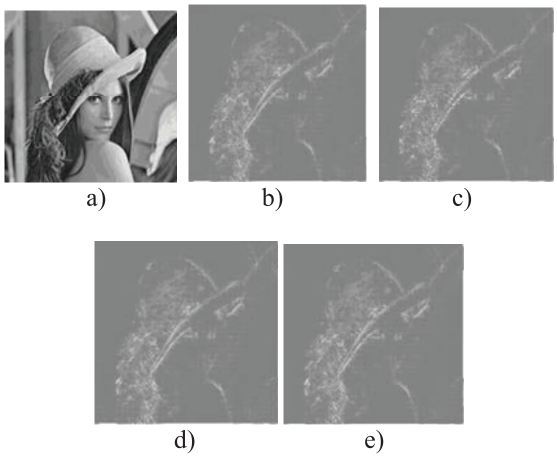

Fig. 1. The results of Lena image processing using filter-1: a) the original image, b) image processed in the WNS, c) image processed using RNS $\{5,7,8\}$, d) image processed using RNS $\{7,15,16\}$, e) image processed using RNS $\{15,31,32\}$.

Fig. 2. The results of Lena image processing using filter-3: a) the original image, b) image processed in the WNS, c) image processed using RNS $\{5,7,8\}$, d) image processed using RNS $\{7,15,16\}$, e) image processed using RNS $\{15,31,32\}$.

The results of applying filter-3 for Lena image are presented in Figure 2. Figure 2a shows the original grayscale image. Figure 2b shows the result of image filtering in the WNS. Figures 2c-2e show the results of image filtering using RNS with moduli $\{5,7,8\}$, $\{7,15,16\}$ and $\{15,31,32\}$ respectively. Figure 3c shows a very poor quality of an image processing using RNS $\{5,7,8\}$. This fact is explained by too small dynamic range of that RNS, which leads to serious distortions of the image pixels. Results of processing using RNS $\{7,15,16\}$ and $\{15,31,32\}$ are indistinguishable from the image processed in the WNS visually.

When comparing the results of filtering using different RNS it can be concluded that the moduli sets $\{5,7,8\}$ and $\{7,15,16\}$, may produce a distorted image processing results. This fact is due to insufficiency of dynamic range of investigated RNS. The worst result was showed by RNS with moduli $\{5,7,8\}$, since even visual quality of the image processing using sharpening filters is unacceptable in practice.

For the quantitative determination of image processing quality using different RNS moduli sets we used two numerical characteristics.

1. PSNR, or Peak Signal to Noise Ratio between two images [15]. This characteristic is calculated by the formula:

$$PSNR = 10\log_{10}\left(\frac{R^2}{MSE}\right), \tag{7}$$

where $MSE = \frac{\sum_{M,N}[I_1(m,n) - I_2(m,n)]^2}{M \cdot N}$ is mean square error of comparing the image quality; R is the maximum amplitude of the input image. Since the value of $PSNR$ has a logarithmic nature, unit of its measure is decibel (dB). The larger $PSNR$ value indicates the better image quality and for identical images $PSNR = \infty$. For example, in 8-bits grayscale image compression it is assumed that PSNR exceeding 40 dB corresponds to near lossless case when the distortions are hardly distinguished by a human being (high quality). PSNR values below 30 dB indicate significant visual distortions (low quality). For our case of filtering using RNS with different moduli sets we calculate the $PSNR$ value between the image obtained by using traditional WNS and an image obtained by using RNS.

2. SSIM, of Structural SIMilarity index between two images which is defined on the basis of full comparison of the original and the resulting images [16]. This characteristic is calculated by the formula:

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)}, \tag{8}$$

where μ_x is the mean value x, μ_y is the mean value y, σ_x^2 is dispersion x, σ_y^2 is dispersion y, σ_{xy} is covariance x and y, $c_1 = (k_1 L)^2$, $c_2 = (k_2 L)^2$ are two variables, L is the pixels dynamic range, $k_1 = 0{,}01$ and $k_2 = 0{,}03$ are constants. The value of $SSIM$ is between 0 and 1 (for identical equal images is equal 1). For our case of filtering using RNS with different moduli sets we calculate the $SSIM$ value between the image obtained by using WNS and an image obtained by using RNS.

Table 1 shows the results of quality of the processed images using four filters in the RNS with different moduli sets. Analyzing the results, it can be concluded that the moduli set $\{15,31,32\}$ shows absolutely accurate result in all cases (the same as the result of processing in the WNS). Moduli set $\{5,7,8\}$ provides bad result ($PSNR < 30\,\text{dB}$) in all cases. Thus, the use of moduli set $\{5,7,8\}$ does not guarantee the correct work of sharpening filters, and may limit applies in the image edge detection. Moduli set $\{7,15,16\}$ showed no errors in image filtering with filter-1, filter-2 and filter-3. Image quality deteriorated when filter-4 was used, but was at an acceptable level for practical use ($PSNR > 40\,\text{dB}$). Perhaps, for some applications, RNS with little insufficient dynamic range can be used. However, this question needs further investigations.

V. CONCLUSIONS

This paper analyzes the use of different RNS moduli sets for digital filtration of grayscale images by different filters. It is shown that the incorrect result in image filtering using RNS arises due to lack of dynamic range. Formula (6) for determining a sufficient dynamic range of RNS should be used for correct implementation of image processing. It was confirmed visually by Lena image filtration and confirmed quantitatively by calculating $PSNR$ and $SSIM$ for obtained images.

An interesting direction for further research of image filtering using RNS is the search for effective implementation for digital filters with more complicated structure. For example, masks of smoothing filters contain fractional values. However, the division operation is a difficult for implementing in RNS.

ACKNOWLEDGEMENTS

This work was written in the assignment of basic part of State task No. 2563.

This work was supported by the Russian Federation President Grant MK-5980.2016.9.

REFERENCES

[1] Li Tan and Jean Jiang, *Digital Signal Processing*, Second Edition: Fundamentals and Applications, Academic Press, 2013, 876 p.

[2] I.F. Akyildiz, W. Su, Y. Sankarasubramaniam and E. Cayirci, "Wireless sensor networks: A survey," *Computer Networks*, vol. 38, no. 4, 2002, pp. 393-422, doi: 10.1016/S1389-1286(01)00302-4.

[3] O. Younis and S. Fahmy, "HEED: A hybrid, energy-efficient, distributed clustering approach for ad hoc sensor networks," *IEEE Transactions on Mobile Computing*, vol. 3, no. 4, 2004, pp. 366-379, doi: 10.1109/TMC.2004.41.

[4] N.I. Chervyakov, P.A. Lyakhov and M.G. Babenko, "Digital filtering of images in a residue number system using finite-field wavelets," *Automatic Control and Computer Sciences*, vol. 48, no. 3, 2014, pp. 180-189, doi: 10.3103/S0146411614030031.

[5] G.L. Bernocchi, G.C. Cardarilli, A. Del Re, A. Nannarelli and M. Re, "Low-power adaptive filter based on RNS components," *IEEE*

International Symposium on Circuits and Systems, ISCAS 2007, pp. 3211-3214, doi: 10.1109/ISCAS.2007.378155.

[6] Jienan Chen and Jianhao Hu, "Energy-Efficient Digital Signal Processing via Voltage-Overscaling-Based Residue Number System," *IEEE Transactions on Very Large Scale Integration (VLSI) Systems*, vol. 21, no. 7, 2013, pp. 1322-1332, doi: 10.1109/TVLSI.2012.2205953.

[7] Azadeh Safari, C.V. Niras and Yinan Kong, "Power-performance enhancement of two-dimensional RNS-based DWT image processor using static voltage scaling," *Integration, the VLSI journal*, vol. 53, March 2016, 2016, pp. 145-156, doi: 10.1016/j.vlsi.2015.12.006.

[8] D. Younes and P. Steffan, "Efficient image processing application using Residue Number System," *Proceedings of the 20th International Conference on Mixed Design of Integrated Circuits and Systems*, June 2013. pp. 468-472.

[9] P.V.A. Mohan, *Residue Number Systems: Algorithms and Architectures*, Springer US, 2002, 254 p.

[10] D.K. Taleshmekaeil and A. Mousavi, "The use of residue number system for improving the digital image processing," *IEEE 10th International Conference on Signal Processing (ICSP)*, October 2010, pp. 775–780, doi: 10.1109/ICOSP.2010.5655920

[11] A. Omondi and B. Premkumar, *Residue Number Systems: Theory and Implementation*, Imperial College Press, 2007, 296 p.

[12] A.S. Molahosseini, S. Sorouri and A.A.E. Zarandi, "Research challenges in next-generation residue number system architectures," *Proc. IEEE 7th International Conference on Computer Science & Education (ICCSE)*, July 2012, pp. 1658-1661, doi: 10.1109/ICCSE.2012.6295382.

[13] B. Parhami, *Computer Arithmetic: Algorithms and Hardware Designs*, 2nd edition. Oxford University Press, New York, 2010, 641 P.

[14] R.C. Gonzalez and R.E. Woods, *Digital Image Processing*, 3rd edition, Pearson, 2007, 976 p.

[15] Q. Huynh-Thu and M. Ghanbari, "Scope of validity of PSNR in image/video quality assessment," *Electronics Letters*, vol. 44, no. 13, 2008, pp. 800–801, doi: 10.1049/el:20080522.

[16] Z. Wang, "Image quality assessment: from error visibility to structural similarity," *IEEE Transactions on image processing*, vol. 13, no. 4, 2004. pp. 600-612, doi: 10.1109/TIP.2003.819861.