

A New Proposal For A Multi-Objective Technique Using SMPSO and Tabu Search

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Abstract— This paper presents a new multi-objective technique which consists of a hybrid between a particle swarm optimization approach (PSO) and tabu search (TS) technique. The main idea of the approach is to combine the high convergence rate of PSO with a local search technique based on Tabu Search. Besides, in our study, we proposed to apply local search to improve the capacity of exploitation of PSO. The mechanisms proposed are validated using fifteen different functions from specialized literature of multi-objective optimization. The obtained results show that using this kind of hybridization is justified as it is able to improve the quality of the solutions in the majority of cases.

Keywords— *Particle Swarm Optimization; SMPSO; Tabu Search; Multi-objective Optimization.*

I. STATE OF ART

Particle Swarm Optimization was proposed by Kennedy and Eberhart [1]. It uses a population of particles to find solutions through hyperdimensional search space. The change of the particles position is based on the socio-psychological tendency of particles to emulate the success of other individuals. Each particle has an associated velocity vector which drives the optimization process and reflects the socially exchanged information. In the last years some proposals for extending Particle Swarm Optimization algorithms to treat MOPs, have been published. In the literature, there is no way to ensure that an optimization method will give the best results for all the possible instances of a given problem. In practice, you can always default to one method over another. The hybridization can be regarded as an effective means of finding a compromise between the advantages and disadvantages of several optimization methods. In the literature, several approaches to PSO hybridizing with other techniques have been developed. The major characteristic of these hybridizations is the fact that they make use of local search techniques. For example [2,3] the authors suggest a

hybridization of a PSO algorithm with local search techniques such as scatter search and rough sets theory; the authors suggest a hybridization of a PSO algorithm (TRIBES) with local search techniques such as tabu search and simulated annealing [4,5]. Another paper present hybrid approaches for MOPSO using global search technique PESA II and local search technique tabu search [6]. The authors suggest a hybridization of a MOPSO algorithm with a local search method for solving multi-objective flexible job-shop problem [7]. In addition, we have another works that incorporated mechanism to improve the performance of PSO algorithm (TRIBES). For example this paper present two new operators: a mutation, which is applied to good particles and four processes of resets, which are applied to bad particles to improve exploration and/or exploitation of TRIBES [8]. The authors suggest two new mechanisms: inheritance and approximation, those mechanisms improve the performance of MO-TRIBES [9]. In this paper, the authors present a state of art of different operators used to enhance the performance of evolutionary multiobjective optimization [10]. Indeed they conduct comparative study of them. However, the theoretical convergence of the PSO has demonstrated that it doesn't converge to a global or local optimum [11]. It showed that the PSO is different from other metaheuristics: its convergence is ensured, but its solution is considered as neither a global solution nor a local solution; the convergence is ensured only to the best visited by the whole swarm.

This paper is organized as follows. Section 1 discusses the state of art of PSO, and briefly reviews the studied approaches of hybrid MOPSO algorithms. In section 2, we present our proposed approach. As for the comparative results, they will be described in section 3 and the conclusion will be stated in section 4.

II. OUR APPROACH

Hybrid approaches have many advantages. They allow different optimization methods to be used and their properties

to be combined in such a way that improves the overall performance obtained by each of them and ensures best results. Indeed, it suffers from the premature convergence which is caused by the rapid loss of diversity in the swarm. For this, we propose to hybridize the MOPSO with a local Search technique such as Tabu Search because it is highly efficient in the exploitation of the search space and it allows the escape of a local optimum using a tabu list. In order to improve the capacity of exploitation of MOPSO, we apply a local search technique: TS around to the particles which are situated in the least crowded zones. All this is meant to show that the hybridization procedure improves most of performances of MOPSO.

A. SMPSO

The authors presented in [12] Speed-constrained MOPSO (SMPSO). They have a mechanism to limit the speed of the OMOPSO algorithm that improves the research capacity of the algorithm. Indeed, following an experimental study of the OMOPSO behavior, the authors found that poor performance may be justified by the speed values of the majority of particles undergoing a kind of irregular behavior in some points of execution, alternating very high values to very low values. As a result, these particles move to their extreme values continuously, and therefore, they do not contribute to the research.

B. Tabu Search

Tabu Search (TS) is a local search metaheuristic introduced by Glover. It is used to solve complex problems and it helps to overcome local optima by the use of a tabu list and it is therefore an adaptive memory method. This method is characterized by high efficiency in computation time and a relatively simple operating mechanism. In addition, it depends on an aggressive search for the best choice of movement at each iteration to adapt and integrate problem specific knowledge. In fact, TS algorithm is combinatory and few works considered its adaptation for the continuous optimization. Among whom we can mention the approach of Chelouah and Siarry [13]. In that case this method is similar to the classic TS. The difference lies essentially in the generation of the neighbourhood.

C. Hybridizing SMPSO with TS

In this approach, we used SMPSO as a global search technique and we integrated Tabu Search to intensify the research around to the particles which are situated in the least crowded zones. Once the swarm is initialized, the leader's archive is initialized by the non-dominated swarm of particles. At each generation, for each particle, the speed and the position are calculated through the equations:

$$x_i(t) = x_i(t-1) + v_i(t) \quad (1)$$

$$v_i(t) = \gamma v_i(t-1) + c_1 r_1 (x_{pi} - x_i(t)) + c_2 r_2 (x_{gi} - x_i(t)) \quad (2)$$

$$\text{Where } \gamma = \frac{2}{2-\rho-\sqrt{\rho^2-4\rho}} \quad (3)$$

$$\rho = \begin{cases} c_1 + c_2 > 4 & \text{if } c_1 + c_2 \\ 0 & \text{if } c_1 + c_2 \leq 4 \end{cases} \quad (4)$$

Furthermore, a mechanism such that the speed of each accumulated variable j (in each of the particles) is defined by the following rate equation constriction:

$$v_{ij}(t) = \begin{cases} \text{delta}_j & \text{if } v_{ij}(t) > \text{delta}_j \\ -\text{delta}_j & \text{if } v_{ij}(t) \leq \text{delta}_j \\ v_{ij}(t) & \text{otherwise} \end{cases} \quad (5)$$

Where

$$\text{delta}_j = \frac{(\text{upper}_{\text{limit}_j} - \text{lower}_{\text{limit}_j})}{2} \quad (6)$$

In summary, the speed of particles is calculated according to equation (2), the rate obtained is then multiplied by the constriction (3), and the resulting value is limited to using equation (5).

```

Begin
  Swarm initialize
  Initialize the archive of
  leaders
  While (t < tmax)
    Calculate the speed for each
    particle eqs.2-6
    Calculate the position for each
    particle eq.1
    Perform the mutation
    TS(stopping criterion)
    Update leaders in the archive
    of leaders
    Update memory of particles
    t ++
  End While
  Returns the archive of leaders
End

```

Fig.1. TS-SMPSO pseudo-code.

Then we apply tabu search, and the neighbourhood is defined by using the concept of "ball". A ball B (x, r) is

centered on x (current solution) with radius r . To obtain a homogeneous exploration of the space, we consider a set of balls centered on the current solution x with radius $r_0, r_1, r_2, \dots, r_n$. Hence the space is partitioned into concentric crowns. The n neighbours of p_{best} are obtained by random selection of a point which does not belong to the tabu list inside each crown C_i , for i varying from 1 to n . The radius is Finally, we select the best neighbour x' even if it is worse than p_{best} and we insert it in the tabu list.

The generation of the neighborhood is based on Pareto dominance. The current solution is x . The next best configuration to replace x is the best solution in the neighborhood. Indeed, the best neighbor is one who is not dominated by any solution in the neighborhood. The best neighbor can be retained even if it is worse than the current solution.

III. EXPRESSIONS AND RESULTS

A. Test Functions

TABLE I. PROPERTIES OF THE TEST FUNCTIONS.

Test functions	Objective	Modality	Geometry
OKA2	f_1 f_2	Uni-modal Multi-modal	Concave
ZDT1	$f_{1:2}$	Uni-modal	Convex
ZDT2	$f_{1:2}$	Uni-modal	Concave
ZDT3	f_1 f_2	Uni-modal Multi-modal	Discontinuous
ZDT4	f_1 f_2	Uni-modal Multi-modal	Convex
ZDT6	f_1 f_2	Uni-modal Multi-modal	Concave
DTLZ1	$f_{1:3}$	Multi-modal	Linear
DTLZ2	$f_{1:3}$	Uni-modal	Concave
DTLZ3	$f_{1:3}$	Multi-modal	Concave
DTLZ4	$f_{1:3}$	Uni-modal	Concave
DTLZ5	$f_{1:3}$	Uni-modal	Undefined
DTLZ6	$f_{1:3}$	Uni-modal	Undefined
WFG1	$f_{1:2}$	Uni-modal	Convex
WFG8	$f_{1:2}$	Uni-modal	Concave
WFG9	$f_{1:2}$	Multi-modal	Concave

To validate the performances of our approaches, we compare their performance with SMPSO and PESA II on the following standard test problems (OKA, ZDT, WFG and DTLZ) [16]. These multi-objective functions have different difficulties such as convexity, concavity, and multimodality.... The detailed description of these functions was omitted due to space restrictions. However, all of them are unconstrained minimization and have between 3 and 30 decision variables. Indeed, we fix the maximal size of the archive to 100 for the

two-objective functions and to 150 for the three-objective ones. We also fixed the size of the neighborhood to 5 for the TS algorithm. In addition, the parameters adopted here are the same proposed in [12].

To realize our work, we used, as a stopping criterion for the TS-SMSPO and SMSPO algorithms, the number of iterations equal to 500 iterations (for the equivalent of the stopping criterion PESAII algorithm is a maximum number of evaluations of the objective function equal to 50,000).

All the algorithms have been implemented using jMetal Durillo and al., [17] a Java-based framework for developing metaheuristics for solving multi-objective optimization problems.

TABLE II. PARAMETERIZATION ($L = \text{INDIVIDUAL LENGTH}$).

Parameterization used in SMSPO [11]	
Swarm size	100 particles
Mutation	polynomial, $p_m = 1.0/L$
Parameterization used in PESAII [14], [15]	
Population size	100 individuals
Selection	PESA2Selection
Recombinaison	simulated binary, $p_c = 0.9$
Mutation	polynomial, $p_m = 1.0/L$
Parameterization used in TS-SMSPO	
Swarm size	100 particles
Mutation	polynomial, $p_m = 1.0/L$
Tabu list size	5

We analyze the quality of the results of Pareto fronts after 30 independent runs of each function. Since the treatment is conducted with stochastic algorithms and we want to provide the results with confidence, we have also included a testing phase which allows us to perform a multiple comparison of samples [18]. We have used the multi-comparative function provided by Matlab for that purpose. We always put down a confidence level of 95% (i.e., significance level of 5% or p-value below 0.05) in the statistical tests. Successful tests are marked with "+" symbols in the last column in tables of hypervolume quality indicator and epsilon quality indicator and the unsuccessful tests are marked with "-" symbols means that no statistical confidence was found ($p\text{-value} > 0.05$).

B. Metrics of Comparison

The following metrics are used to provide a qualitative assessment of the performance of the studied algorithms on the standard test problems. We have considered two quality indicators: additive unary epsilon indicator ($I_{\epsilon+}^1$), and hypervolume (HV) [19]. The first indicator measures the convergence of the resulting Pareto fronts, while the last one measures both convergence and diversity.

C. Epsilon indicator ($I_{\epsilon+}^1$)

The binary additive epsilon indicator, $I_{\epsilon+}^1(A,B)$, gives the minimum factor ϵ that can be added to each point of B such that the resulting transformed approximated set is weakly dominated by A . The unary additive epsilon indicator $I_{\epsilon+}^1(A)$ is calculated to compare A with a reference set of points. In that case, smaller values mean better results.

D. Hypervolume indicator (HV)

The hypervolume indicator measures the hypervolume of that portion of the objective space that is weakly dominated by an approximation set A , and is to be maximized. Here we consider the hypervolume difference to a reference set R ; where smaller values correspond to higher quality.

E. Results

The results of the statistical tests for the HV, presented in Tables 4 and 5, show that the TS-SMPSO algorithm shown present improvements from those of SMPSO and PESAI. For each function, we consider the best, the mean and the worst values over 30 independent runs and the values in bold show the best results found. We notice that: The fronts found for test functions OKA2, ZDT1, ZDT2, ZDT4, ZDT6, WFG1, WFG9, DTLZ2, DTLZ3 and DTLZ4 show that TS-SMPSO is the best in terms of non dominated solution distribution along the fronts of Pareto. Indeed, it gives smaller values, thus showing better quality of the hypervolume indicator. It can be explained by the good exploitation of the research space done by the tabu search on certain particles of the swarm. However, the fronts found for test functions ZDT1, ZDT3, DTLZ1 and DTLZ5 show that MOTS is better than the rest of algorithms. For the test function WFG8, SMPSO has provided the best results and performances compared to measures given by the HV indicator. A bad performance behavior is noticed for WFG1 and DTLZ6 given by SMPSO, but TS-SMPSO shows the good results of convergence and diversity. On the other hand, PESAI shows better results than the proposed reference fronts. The statistical results are significant, as it can be seen in the last column, where the most of "+" symbols are found. However, the results corresponding to problems ZDT4, WFG1, WFG9, DTLZ5 and DTLZ6 deserve additional comments. We have used the "-" symbol in Tables 4 and 5 to indicate those experiments in which the HV values have no statistical confidence. The $I_{\epsilon+}^1$ indicator values, included in Tables 6 and 7 show that TS-SMSPO provides good results for the following problems: OKA2, ZDT1, ZDT2, ZDT3, ZDT6, DTLZ1, DTLZ3, DTLZ5, DTLZ6, WFG8 and

WFG9, which shows that the convergence is assured thanks to the exploitation of the research space. Then the SMPSO and TS-SMPSO algorithm given the best results for the functions test WFG1, DTLZ2 and DTLZ4. For the problem ZDT4, we observed that PESAI provides a better result. Generally, for ZDT4 function the performance measures found by MOPSO in the literature are less compared to the results found by genetic algorithms. All the results have statistical significance, as it can be seen in the last column in tables 6 and 7, where only "+" symbols are found.

TABLE III. EXECUTION TIME (MEAN VALUES)

Test functions	TS-SMPSO	SMPSO	PESAI
OKA2	1258	1125	5998,25
ZDT1	1681,66	1687	10911,66
ZDT2	1516	1824	9859,8
ZDT3	1478,75	1367,4	10794,2
ZDT4	1324	1475,33	7614,8
ZDT6	3819,25	2848,25	8786,4
WFG1	2913,75	2875,75	9261
WFG8	1881,25	1852,4	7891,5
WFG9	1515,5	1344	10523,75
DTLZ1	2287	2375	19363,5
DTLZ2	4783,25	4796	28521
DTLZ3	2447	2155	17325,3
DTLZ4	1232	1052	31559,5
DTLZ5	6789,33	7014	20698
DTLZ6	1125,5	1220,7	24493

The time results are shown in table III. For each function, we present average measures over 30 independent runs. The unit time measurement used is the millisecond. We have concluded that most of the problems have the best obtained time execution by MOTS. However, for the functions ZDT1, ZDT2, ZDT4, DTLZ2 and DTLZ4 the best time obtained was by TS-SMPSO and the rest of functions by SMPSO. Therefore, the results of time execution show that SMPSO are more efficient than TS-SMPSO. Hence the cost of hybridization is not high TS-SMSPO which can be explained by TS algorithm is neither complex nor continuous in terms of memory and time but PESAI is complex as the genetic algorithm. The first conclusion is that all numeric results observed by tables 4, 5, 6 and 7 showed that the TS-SMPSO algorithm out performs generally, for most test functions, SMPSO and PESAI. In addition, the choice of neighborhoods informer is done in order to accelerate the swarm's

convergence towards specific search space zones. This can be considered as an intensification process.

TABLE IV. RESULTS FOR HV INDICATOR (TWO-OBJECTIVE FUNCTIONS).

Test functions		TS-SMPSO	SMPSO	PESAII	
OKA2	Best	3.71 e-2	9.3 e-2	9.31 e-2	+
	Mean	7.84 e-2	1.19 e-1	1.39 e-1	
	Worst	1.34 e-1	2.13 e-1	1.75 e-1	
ZDT1	Best	3.84 e-1	3.96 e-1	4.37 e-1	+
	Mean	4.18 e-1	4.32 e-1	5.76 e-1	
	Worst	6.59 e-1	4.45 e-1	7.44 e-1	
ZDT2	Best	2.9 e-1	3.22 e-1	3.20 e-1	+
	Mean	2.95e-1	3.28 e-1	3.24 e-1	
	Worst	3.28 e-1	4.01 e-1	3.25 e-1	
ZDT3	Best	5.10 e-1	6.61 e-1	7.02 e-1	+
	Mean	5.11 e-1	7.59 e-1	7.34 e-1	
	Worst	5.14 e-1	8.54 e-1	8.88 e-1	
ZDT4	Best	2.61 e-2	3.54 e-2	6.32 e-1	-
	Mean	3.18 e-2	4.58 e-2	6.48 e-1	
	Worst	3.77 e-2	5.2 e-2	7.18 e-1	
ZDT6	Best	3.43 e-1	3.65 e-1	3.92 e-1	+
	Mean	3.62 e-1	3.75 e-1	4.5 e-1	
	Worst	4. e-1	4.22 e-1	4.61 e-1	
WFG1	Best	9.95 e-2	1.46 e-1	3.48 e-2	-
	Mean	9.98 e-2	2.22 e-1	1.94e-1	
	Worst	1.06 e-1	2.26 e-1	2.86 e-1	
WFG8	Best	6.98 e-2	7.16 e-2	6.59 e-2	+
	Mean	8.22e-2	7.97e-2	8.03e-2	
	Worst	8.44 e-2	8.46 e-2	9.38 e-2	
WFG9	Best	4.9 e-2	4.93 e-2	5.39 e-2	-
	Mean	4.91 e-2	5.35 e-2	5.42e-2	
	Worst	2.23 e-1	5.62 e-2	7.22 e-2	

TABLE V. TABLE 4. RESULTS FOR HV INDICATOR (THREE-OBJECTIVE FUNCTIONS).

Test functions		TS-SMPSO	SMPSO	PESAII	
DTLZ1	Best	3.21 e-2	1.68 e-1	7.86 e-2	+
	Mean	2.99 e-1	3.85 e-1	2.41 e-1	
	Worst	4.05 e-1	5.17 e-1	3.31 e-1	
DTLZ2	Best	3.32e-1	3.25 e-1	2.3 e-2	+
	Mean	3.37 e-1	3.67 e-1	5.61 e-1	
	Worst	3.45 e-1	3.71 e-1	7.87 e-1	
DTLZ3	Best	1.65 e-2	1.37 e-2	1.35 e-2	+
	Mean	1.11 e-1	1.19 e-1	5.52 e-2	
	Worst	1.38 e-1	1.96e-1	6.84 e-2	
DTLZ4	Best	2.31 e-1	2.97 e-1	1.15 e-1	+
	Mean	2.35 e-1	3.22 e-1	2.89 e-1	
	Worst	2.79 e-1	3.67 e-1	3.05 e-1	
DTLZ5	Best	8.87 e-2	9.93 e-2	8.88 e-2	-
	Mean	8.96 e-2	1.01 e-1	9.14 e-2	
	Worst	9.09 e-2	1.21 e-1	9.98 e-2	
DTLZ6	Best	8.67 e-2	9.94 e-2	6.26 e-2	-
	Mean	9.49 e-2	9.56 e-1	7.948 e-2	
	Worst	9.95 e-2	9.78 e-2	9.98 e-2	

IV. CONCLUSION

We have introduced a novel SMPSO-based algorithm to work on multi-objective optimization problems using a TS hybridized SMSPO. In brief, the results of TS-SMSPO are more competitive than those of SMPSO and PESAII for most test functions. The hybrid approach aims at combining the high convergence rate of SMPSO with the good neighborhood exploration performed by the TS algorithm. The results show that the hybridization is a promising approach to multi-objective optimization. Future work would be using another mechanisms to hybridize PSO algorithm in order to enhance the convergence and the diversity such as the Island Models.

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