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DFT-Based Frequency Estimation of Multiple Sinusoids

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ABSTRACT An accurate frequency estimation method of multi-component sinusoidal signal in additive white Gaussian noise (AWGN) is proposed. The algorithm is implemented in the frequency domain and based on discrete Fourier transform (DFT). The maximum DFT spectral line and two Discrete-Time Fourier Transform (DTFT) samples located on the same side of the maximum DFT spectral line are used to estimate the frequency of a sinusoidal component. And this algorithm is utilized in both the coarse estimation and the fine estimation. Simulation results show that compared with the competing estimators, the presented method is closer to the Cramer-Rao lower bound (CRLB). And it is almost independent of the frequency displacement. The numerical complexity of the presented method is similar with the competing DFT-based algorithms.

INDEX TERMS DFT, frequency estimation, multiple sinusoids, CRLB.

I. INTRODUCTION

In the area of digital signal processing, accurate and effective estimation of multi-component sinusoidal frequencies in the background of AWGN is a classic and important topic. Multi-component sinusoidal frequency estimation has been widely used in signal processing, system identification, power system, communications, measurement and radar systems [1]–[4]. For example, the frequency modulated continuous wave (FMCW) radar system transmits a continuous frequency modulated millimeter wave through the antenna, receives the reflected signal of the target, and estimates the frequency of multiple sinusoidal sums to realize the ranging and velocity measurement [5].

Frequency estimator of single-tone sinusoid is the basis of estimating the frequencies of multiple sinusoids. Many researchers have proposed their estimators of single-tone sinusoid in the background of AWGN. These estimators can be categorized into time domain estimators [6]–[11] and frequency domain estimators [12]–[24]. Time domain estimators include autocorrelation methods [6], [7], maximum likelyhood methods [8], [9] and least square methods [10], [11]. The three essential factors of evaluating frequency estimators are the estimation accuracy, the estimation range and the computation amount. Time domain estimators are usually accurate, but they need large amount of computation. As a result, they are inappropriate to deal with problems in timely manner in some application areas. The frequency domain methods are mainly based on fast Fourier transform (FFT), which have relatively low computational complexity and are appropriate for real-time operations. Therefore, they have been widely used. The three DFT spectral lines interpolation estimator (TDSL estimator) [12] is a general method which uses the largest DFT sample and two DTFT samples arbitrarily located in the DFT main lobe. To reduce the influence of spectrum leakage of interference signal on the desired frequency, the TDSL estimator with rectangular window is extended to the case of maximum sidelobe decay window [16]. In [17], three DFT samples near the maximum DFT sample are calculated, and parabolic interpolation of the DTFT peak is utilized to estimate the frequency. In [23], the Aboutanios and Mulgrew algorithm [14] is generalized, and the frequency estimate is obtained iteratively by interpolation on the shifted DFT coefficients.

Frequency estimators of multiple sinusoids have been proposed by researchers. MUSIC estimator [25] and ESPRIT estimator [26] are subspace-based parametric methods. Although they can reach the CRLB when the signal-to-noise ratio (SNR) is not low, they have relatively higher computational complexity [27]. In [27], the coarse-to-fine

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HAQSE (CFH) algorithm is extended to the case of multi-component complex sinusoids. In [28], the detection and estimation of multiple sinusoidal frequencies are implemented by the Djukanovic algorithm in the frequency domain [28]. In both the algorithms of [27] and [28], the first step is to detect the strongest spectrum of the signal by FFT for the multi-component complex sinusoidal signal. The second step is to eliminate the strongest component from the total signal. And then iterative algorithms are adopted until there are not peaks of sinusoids in the remaining signal spectrum. In [29], an efficient interpolation strategy is used to estimate each component of the multiple sinusoids in turn by combining the iterative leakage subtraction scheme.

In this paper, an algorithm for frequency estimation of multi-component sinusoidal signal in the background of AWGN is proposed. A three spectral line interpolation estimator (TSLI estimator) is proposed to estimate the frequency of a single component. The TSLI estimator utilizes the maximum DFT sample and two DTFT samples located on the same side of the maximum DFT sample to perform the frequency estimation. The presented estimator for multiple sinusoids is carried out in the frequency domain and based on the TSLI estimator. Firstly, TSLI estimator is used to estimate the frequency of the strongest component. Then the strongest component is removed from the total signal. At last, iterative methods are used until the last component is estimated. In this way, the coarse frequency estimates are obtained. The fine frequency estimation is carried out which is also based on the TSLI algorithm. As indicated by the simulation results, compared with the competing estimators, the presented estimator is closer to the CRLB. It is independent of the frequency displacement except when the frequency displacement is very small. The numerical complexity of the presented method is similar with the competing DFT-based algorithms.

The rest of the paper is organized as follows. Section II describes the proposed frequency estimation method. In Section III, the proposed estimator is contrasted with the existing estimators and the CRLB. Finally, Section IV concludes this paper.

II. PRESENTED ALGORITHM

In this section, we present a frequency estimator of multi-component sinusoidal signal in the background of AWGN. A three spectral line interpolation (TSLI) estimator is proposed to estimate the frequency of a single sinusoidal component. The presented estimator of multi-component sinusoidal signal is based on the TSLI estimator and includes two main steps: coarse estimation and fine estimation.

The multiple sinusoidal signal is

$$x(n) = s_K(n) + w(n), \quad n = 0, 1, \ldots, N-1 \tag{1}$$

$$s_K(n) = \sum_{k=1}^{K} A_k \exp\left[j(2\pi f_k n/f_s + \phi_k)\right], \quad n=0, 1, \ldots, N-1 \tag{2}$$

where $s_K(n)$ is the noise-free multiple sinusoids and $w(n)$ is AWGN with zero mean and variance σ^2. N is the signal length, and the total number of sinusoidal components is represented by K. A_k, f_k and ϕ_k denote the frequency, amplitude and initial phase of the k-th sinusoidal component, respectively. f_s is the sampling frequency. The frequency of each component can be expressed as

$$f_k = (m_k + \delta_k) \cdot \Delta f \tag{3}$$

where m_k is the index number of the maximum FFT spectral line corresponding to the k-th component, $\delta_k \in (-0.5, 0.5]$ is the fractional frequency deviation from f_k. $\Delta f = f_s/N$ is the FFT frequency resolution.

A. THREE SPECTRAL LINE INTERPOLATION METHOD

A single-tone sinusoid frequency estimator based on three DFT samples interpolation is proposed. Firstly, FFT is performed on the sampled sinusoidal signal, and the rough frequency estimation is carried out by searching the position of the discrete spectral line with the maximum amplitude. Then, the precise frequency estimate is acquired by interpolating the maximum FFT sample and two DTFT samples on the same side of the maximum FFT sample.

The discrete Fourier transform of a single-tone sinusoid in a noiseless case is as follows:

$$\begin{aligned} S[m+k] &= \sum_{n=0}^{N-1} A e^{j(2\pi f n/f_s + \phi)} e^{-j2\pi nk/N} \\ &= e^{j\phi} e^{j\pi \frac{N-1}{N}(\delta - k)} \frac{A \sin[\pi(\delta - k)]}{\sin[\pi(\delta - k)/N]}, \\ & k = 0, 1, \ldots\ldots, N-1 \end{aligned} \tag{4}$$

The rough frequency estimation is carried out by searching the discrete frequency index of the maximum DFT spectral line which is denoted as m.

Hereinafter, $S[m+k]$ is uniformly denoted as S_k for simplicity. The spectral line with the largest amplitude is denoted as S_0, and its expression can be written as

$$S_0 = e^{j[\phi + \pi\delta(1-1/N)]} \cdot \frac{A \sin(\pi\delta)}{\sin(\pi\delta/N)} \tag{5}$$

Then the precise frequency estimate is obtained by interpolating the maximum FFT spectral line and two DTFT samples located on the same side of the maximum spectral line. At the location $f = (m+q)\Delta f$, the DTFT sample value of the sampled sinusoid is

$$S_q = e^{j[\phi - \pi(q-\delta)(1-1/N)]} \cdot \frac{A \sin(\pi(q-\delta))}{\sin(\pi(q-\delta)/N)} \tag{6}$$

According to (5) and (6), the absolute values of S_0, $S_{0.1}$ and $S_{0.2}$ are expressed as follows:

$$|S_0| = \frac{A \sin(\pi\delta)}{\sin(\pi\delta/N)} \tag{7}$$

$$|S_{0.1}| = \frac{A \sin(\pi(0.1-\delta))}{\sin(\pi(0.1-\delta)/N)} \tag{8}$$

$$|S_{0.2}| = \frac{A \sin(\pi(0.2-\delta))}{\sin(\pi(0.2-\delta)/N)} \tag{9}$$

From (7)-(9), we can get

$$\frac{|S_{0.1}|}{|S_0|} = \frac{\sin(0.1\pi)\cot(\pi\delta) - \cos(0.1\pi)}{\sin(0.1\pi/N)\cot(\pi\delta/N) - \cos(0.1\pi/N)} \tag{10}$$

$$\frac{|S_{0.2}|}{|S_0|} = \frac{\sin(0.2\pi)\cot(\pi\delta) - \cos(0.2\pi)}{\sin(0.2\pi/N)\cot(\pi\delta/N) - \cos(0.2\pi/N)} \tag{11}$$

After some algebraic operations, we have

$$\begin{aligned} &|S_{0.1}|\sin(0.1\pi/N)\cot(\pi\delta/N) - |S_{0.1}|\cos(0.1\pi/N) \\ &\quad = |S_0|\sin(0.1\pi)\cot(\pi\delta) - |S_0|\cos(0.1\pi) \end{aligned} \tag{12}$$

$$\begin{aligned} &|S_{0.2}|\sin(0.2\pi/N)\cot(\pi\delta/N) - |S_{0.2}|\cos(0.2\pi/N) \\ &\quad = |S_0|\sin(0.2\pi)\cot(\pi\delta) - |S_0|\cos(0.2\pi) \end{aligned} \tag{13}$$

From (12) and (13), we have, (14), as shown at the bottom of the page.

In the above derivation process, we utilize the two DTFT samples $S_{0.1}$ and $S_{0.2}$ which are on the right of the maximum spectral line S_0. We can also use the two DTFT samples $S_{-0.1}$ and $S_{-0.2}$ which are on the left of S_0. After similar derivation process, we have, (15), as shown at the bottom of the page.

The steps of the presented TSLI estimator are shown in Table 1. To improve the estimation performance, the algorithm in [12] ($i = 1$) is utilized to obtain a preliminary estimate $\hat{\delta}_1$ of the fractional frequency deviation. And with this preliminary estimate $\hat{\delta}_1$, we can decide which estimation formula ((14) or (15)) should be used to get the final estimate of the single-component sinusoid.

TABLE 1. **The steps of TSLI algorithm.**

Step	Description
1	Conduct N-point FFT of $x(n)$, and search m
2	According to the estimator in [12] ($i=1$), $\hat{\delta}_1$ is obtained using three spectral lines X_0, X_1 and X_{-1}
3	Calculate the difference between $\lvert\hat{\delta}_1\rvert$ and 0.1, obtaining $d=\lvert\hat{\delta}_1\rvert-0.1$
4	If $\hat{\delta}_1>0$, compute X_d, $X_{d+0.1}$ and $X_{d+0.2}$ via $X_P=\sum_{n=0}^{N-1}x(n)e^{-j2\pi n\frac{m+P}{N}}$, $P=d,d+0.1,d+0.2$ and generate $\hat{\delta}_2$ via (14). And obtain $\hat{f}=(m+d+\hat{\delta}_2)\cdot\Delta f$
5	If $\hat{\delta}_1\le 0$, compute X_{-d}, $X_{-d-0.1}$ and $X_{-d-0.2}$ via $X_P=\sum_{n=0}^{N-1}x(n)e^{-j2\pi n\frac{m+P}{N}}$, $P=-d,-d-0.1,-d-0.2$ and generate $\hat{\delta}_2$ via (15). And obtain $\hat{f}=(m-d+\hat{\delta}_2)\cdot\Delta f$

B. COARSE FREQUENCY ESTIMATION

The proposed estimation method for multiple sinusoids starts from the strongest component. The coarse frequency estimate of the strongest component is obtained by TSLI algorithm and denoted as f_k^c (c stands for the coarse estimate).

Then the strongest component is removed from the total signal $x(n)$ by the methods below:

(1) Firstly, $x(n)$ is moved through negative frequency shift to low frequency band, and the result is

$$x^F(n) = x(n)e^{-j2\pi n f_k^c/f_s} \tag{16}$$

(2) The amplitude of the strongest component is estimate as follows

$$\hat{A}_k = \overline{x^F(n)} \tag{17}$$

where $\overline{x^F(n)}$ expresses the mean value of $x^F(n)$.

(3) Eliminate the strongest component from $x(n)$ as follows

$$x^*(n) = x(n) - \hat{A}_k e^{j2\pi n f_k^c/f_s} \tag{18}$$

After the first step, the strongest components will be converted to the low frequency band. In the third step, most energy of the strongest component is eliminated and $x^*(n)$ is obtained.

By repeating the above three steps, the proposed method can coarsely estimate all frequencies and amplitudes until each frequency in the total signal is estimated. When the frequency of the previous component is estimated and eliminated, the bias in the estimation gradually decreases. Therefore, the estimation performance of the last component is better than that of the other components. In order to reduce the bias effect, we suggest that all parameters of all the K components should be coarsely estimated by the coarse estimation method.

C. FINE FREQUENCY ESTIMATION

Inspired by [27] and [28], the fine frequency estimation is performed in this part. By eliminating all the coarsely estimated single-component sinusoids except the k-th com-ponent from the total signal, the fine estimate of the k-th component can be acquired as

$$\hat{x}_k(n) = x(n) - \sum_{\substack{m=1 \\ m\neq k}}^{K} \hat{A}_m e^{j2\pi n\hat{f}_m/f_s} \tag{19}$$

$$\hat{\delta} = \frac{N}{\pi}\tan^{-1}\left\{\frac{|S_{0.2}|\sin(\frac{\pi}{5N})\sin(\frac{\pi}{10}) - |S_{0.1}|\sin(\frac{\pi}{10N})\sin(\frac{\pi}{5})}{|S_0|\sin(\frac{\pi}{10}) + |S_{0.2}|\cos(\frac{\pi}{5N})\sin(\frac{\pi}{10}) - |S_{0.1}|\cos(\frac{\pi}{10N})\sin(\frac{\pi}{5})}\right\} \tag{14}$$

$$\hat{\delta} = \frac{N}{\pi}\tan^{-1}\left\{\frac{|S_{-0.2}|\sin(\frac{\pi}{5N})\sin(\frac{\pi}{10}) - |S_{-0.1}|\sin(\frac{\pi}{10N})\sin(\frac{\pi}{5})}{-|S_0|\sin(\frac{\pi}{10}) - |S_{-0.2}|\cos(\frac{\pi}{5N})\sin(\frac{\pi}{10}) + |S_{-0.1}|\cos(\frac{\pi}{10N})\sin(\frac{\pi}{5})}\right\} \tag{15}$$

where $\hat{A}_m$ is the amplitude of each component in the coarse estimation. Then, the fine frequency estimate of the k-th component is obtained by applying the TSLI algorithm with the following formula

$$\hat{f}_k^f = TSLI\left\{\hat{x}_k(n)\right\} \tag{20}$$

In order to accurately estimate the frequency values of all the components, (19)-(20) are performed on all K components.

In the fine estimation process, all the other single-tone components except the k-th component are removed. Therefore, the influence of the bias effect caused by other components can be minimized. And a better frequency estimate will be produced. Table 2 describes the steps of the proposed frequency estimator for multiple sinusoids. Steps 1-6 describe the coarse frequency estimation stage. And steps 7-11 describe the fine estimation.

TABLE 2. **The steps of the presented algorithm.**

Step	Description
1	Set $x^*(n) \leftarrow x(n)$
2	For k=1 to K do
3	$\hat{f}_k^c = TSLI\left\{x^*(n)\right\}$
4	$\hat{A}_k = \overline{x^*(n)e^{-j2\pi n f_k^c/f_s}}$
5	Set $x^*(n) \leftarrow \left(x^*(n) - \hat{A}_k e^{j2\pi n f_k^c/f_s}\right)$
6	End
7	Set $\hat{f}_k^f \leftarrow \hat{f}_k^c,\ k=1,2,\ldots,K$
8	For k=1 to K do
9	$\hat{x}_k(n) = x(n) - \sum_{\substack{m=1\\ m\neq k}}^{K} \hat{A}_m e^{j2\pi n \hat{f}_m^f/f_s}$
10	$\hat{f}_k^f = TSLI\left\{\hat{x}_k(n)\right\}$
11	End

III. SIMULATION RESULTS

In this section, the performance of the presented estimator is evaluated by computer simulations and compared with the existing estimators and the CRLB. In all the experiments of this section, the initial phase of each sinusoid is random-ly selected in the range of (-π, π), and the sampling frequen-cy $f_s = N$. The signal length is $N = 256$. The SNR of the k-th sinusoidal component is computed by the following formula

$$SNR_k = \frac{A_k^2}{\sigma^2} \tag{21}$$

In addition, the CRLB of frequency estimation for multi-component complex sinusoidal signal is expressed as [30]

$$\text{var}\left(\hat{f}_k\right) \geq \frac{3f_s^2}{2\pi^2 SNR_k N\left(N^2-1\right)} \tag{22}$$

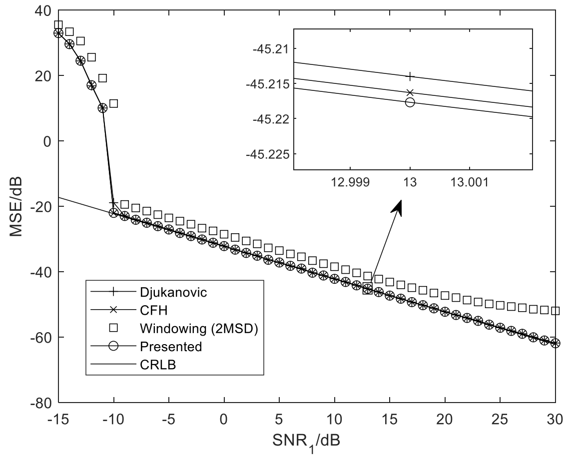

FIGURE 1. **MSE of bi-component sinusoidal signal with respect to *SNR* (*N* = 256, MSE of f_1).**

We consider the case of bi-component sinusoid at first. The frequencies are selected as $f_1 = 50$Hz and $f_2 = 55.7$Hz. The amplitudes are $A_1 = 1$ and $A_2 = 0.9$. Fig.1 and Fig.2 show the mean square error (MSE) of f_1 and f_2 respectively. The presented algorithm is compared with CFH algorithm [27], Djukanovic algorithm [28] and TDSL windowing algorithm [16]. The *SNR* varies from −15dB to 30dB. For different values of *SNR*, 20,000 runs are considered. In Fig.1, the enlarged figure demonstrates that the presented algorithm is closer to the CRLB than CFH algorithm and Djukanovic algorithm. And the MSE of TDSL windowing algorithm is larger than that of the other three algorithms. That is due to the fact that though TDSL windowing algorithm reduces the impact of spectrum leakage from other components with time-domain windowing, it can't eliminate the effect of spectrum leakage completely. The accuracy of the presented estimator is the highest among all the algorithms. And the TDSL windowing algorithm is characterized by a deviation from the CRLB starting at $SNR = 15$dB. In Fig.2, similar conclusions can be drawn.

Then we consider a bi-component complex sinusoid with frequencies f_1 and $f_2 = f_1 + \Delta fd$, where Δfd is the frequency displacement between the two signal components. Fig.3 shows the MSE of different algorithms versus Δfd which is uniformly distributed from 1Hz to 64Hz with a step of 1Hz. The frequency f_1 is randomly selected from 0 to 128Hz. In addition, the amplitudes are $A_1 = 1$ and $A_2 = 0.9$, and $N = 256$, $SNR_1 = 15$dB. For each value of Δfd, 20,000 runs are considered.

As illustrated in Fig.3, the MSE of the TDSL windowing method is about 4dB higher than that of the other algorithms. And it can be observed from the enlarged figure that the presented estimator is closer to CRLB than CFH method [27] and Djukanovic method [28]. Therefore, the accuracy of the presented estimator is the highest among all the algorithms. When $\Delta fd \leq 2$Hz, the MSE of all algorithms are very large. This is because when f_1 and f_2 are very close, the main

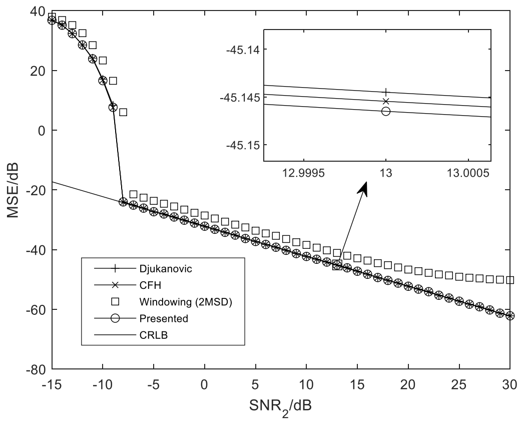

FIGURE 2. MSE of bi-component sinusoidal signal with respect to *SNR* (*N* = 256, MSE of f_2).

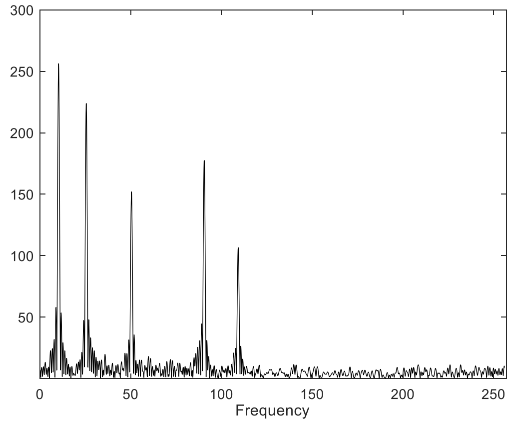

FIGURE 4. Spectrum of five-component signal embedded in AWGN (*N* = 256, SNR_1 = 10*dB*).

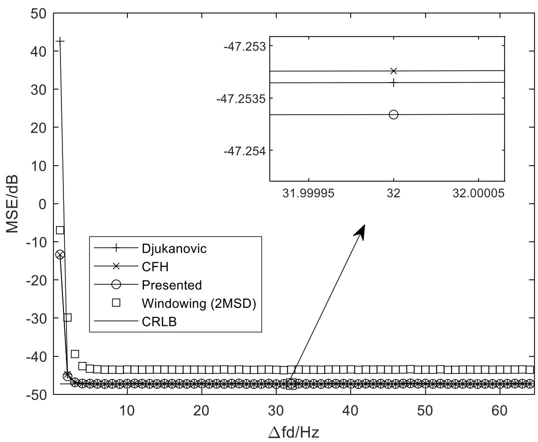

FIGURE 3. MSE of bi-component sinusoidal signal with respect to the frequency displacement Δ*fd* (*N* = 256, MSE of f_1).

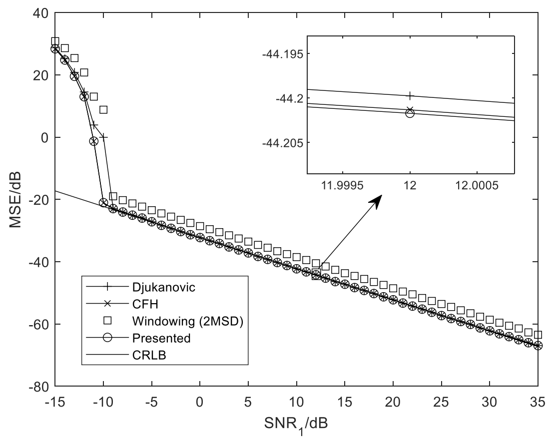

FIGURE 5. MSE of five-component sinusoidal signal with respect to *SNR* (*N* = 256, MSE of f_1).

lobes of the two components overlap, and all the estimators based on DFT interpolation cannot estimate the frequency accurately. When $\Delta fd > 3$Hz, all the curves tend to be stable. Therefore, the performance of the presented estimator is independent of Δfd except when Δfd is very small.

Next, we consider a five-component signal. The frequencies are selected as $f_1 = 10.12$Hz, $f_2 = 25.33$Hz, $f_3 = 50.27$Hz, $f_4 = 90.38$Hz and $f_5 = 109.09$Hz, and the corresponding amplitudes are $A_1 = 1$, $A_2 = 0.85$, $A_3 = 0.58$, $A_4 = 0.69$ and $A_5 = 0.43$. The number of samples $N = 256$. Fig.4 shows the spectrum of this five-component signal in AWGN and $SNR_1 = 10$dB. Fig.5 and Fig.6 show the MSE of the presented estimator compared with CFH algorithm [27], Djukanovic algorithm [28] and TDSL windowing algorithm [16] calculated at variable *SNR*. For different values of *SNR*, 20,000 runs are considered.

In Fig.5, it can be observed from the enlarged figure that the presented estimator is closer to the CRLB than CFH algorithm and Djukanovic algorithm. And it is obvious that the MSE of TDSL windowing algorithm is larger than that of the other three algorithms. That is due to the fact that though TDSL windowing algorithm reduces the impact of spectrum leakage from other components with time-domain windowing, it can't eliminate the effect of spectrum leakage completely. Therefore, the accuracy of the presented estimator is the highest among all the algorithms. In Fig.6, similar conclusions can be drawn.

The numerical complexity of different algorithms is shown in Table 3. When performing complexity analysis for all algorithms, we ignore all complexity items with $O(1)$ and complexity items which are independent of N. For K-component signal, the presented algorithm needs K TSLI operations to coarsely estimate the K frequencies. TSLI algorithm requires $O\left(N\log_2 N\right)$ complex operations. In coarse estimation, the presented estimator needs KN-point FFT calculation, K FFT maximum search, K TSLI algorithm calculation and K sinusoidal removal from the considered signal (formula (18)). Therefore, the coarse estimation needs $O\left(KN\log_2 N\right)$ complex operations. Similarly, the fine estimation needs $O\left(KN\log_2 N\right)$ complex operations. From the above discussion, we can conclude that the total amount of calculation complexity is $O\left(KN\log_2 N\right)$. This is the same as

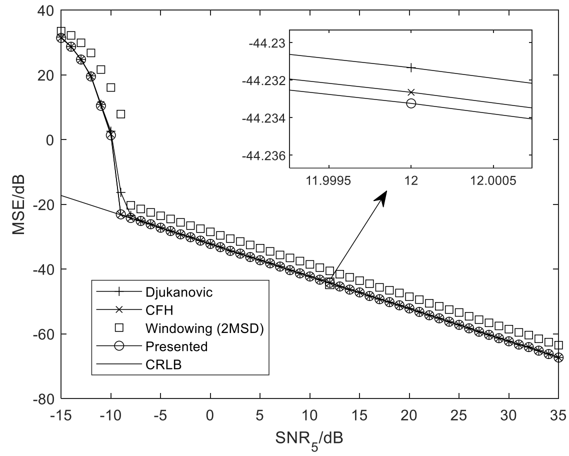

FIGURE 6. MSE of five-component sinusoidal signal with respect to *SNR* ($N = 256$, MSE of f_5).

TABLE 3. Numerical complexity of different algorithms.

Algorithm	Computational complexity
Presented algorithm	$O(KN\log_2 N)$
Djukanovic algorithm [27]	$O(KN\log_2 N)$
CFH algorithm [28]	$O(KN\log_2 N)$
Windowing algorithm (2MSD) [16]	$O(N\log_2 N)$

CFH algorithm [27] and Djukanovic algorithm [28]. Therefore, compared with the other methods, the presented estimator has relatively similar computation cost, but achieves more accurate frequency estimation. Though the computational complexity of TDSL windowing algorithm [16] is $O(N\log_2 N)$ which is the lowest among all the methods, the MSE of TDSL windowing method is the largest according to the simulation results of Fig.1, Fig.2, Fig.3, Fig.5 and Fig.6.

IV. CONCLUSION

Frequency estimation of multi-component sinusoidal signal has a wide range of applications. In this paper, an estimator of multiple sinusoids based on DFT is proposed. Firstly, the spectrum of the strongest component is estimated by FFT. Then, the strongest component is removed from the total signal and the iterative algorithm is used until the coarse estimate of the last component is obtained. Finally, all the other components except for the component to be finely estimated are subtracted from the total signal, and the fine estimation is performed by the three DFT spectral line interpolation algorithm. Simulation experiments are conducted and the results show that the presented estimator reaches CRLB under the condition of variable *SNR*. Its performance is better than that of CFH algorithm, Djukano-vic algorithm and TDSL windowing algorithm. And the proposed algorithm is almost independent of the frequency displacement. The complexity is similar to Djukanovic algorithm and CFH algorithm. Therefore, the presented estimator is suitable for estimating the frequencies of multiple sinusoids and can be used in practical applications.

REFERENCES

[1] L. Xu and F. Ding, "Iterative parameter estimation for signal models based on measured data," *Circuits Syst. Signal Process.*, vol. 37, no. 7, pp. 3046–3069, Jul. 2018.

[2] L. Xu, "The parameter estimation algorithms based on the dynamical response measurement data," *Adv. Mech. Eng.*, vol. 9, no. 11, pp. 1–12, Nov. 2017.

[3] S. Montagner, N. Benvenuto, and S. Tomasin, "Taming the complexity of mm-wave massive MIMO systems: Efficient channel estimation and beamforming," in *Proc. IEEE Int. Conf. Commun. Workshop (ICCW)*, Jun. 2015, pp. 1251–1256.

[4] H. Wen, J. Zhang, Z. Meng, S. Guo, F. Li, and Y. Yang, "Harmonic estimation using symmetrical interpolation FFT based on triangular self-convolution window," *IEEE Trans. Ind. Informat.*, vol. 11, no. 1, pp. 16–26, Feb. 2015.

[5] Y. Kim, H. Ji, J. Lee, Y. Nam, and B. Ng, "Full dimension MIMO (FD-MIMO): The next evolution of MIMO in LTE systems," *IEEE Wireless Commun.*, vol. 21, no. 2, pp. 26–33, Apr. 2014.

[6] Y. Cao, G. Wei, and F.-J. Chen, "An exact analysis of modified covariance frequency estimation algorithm based on correlation of single-tone," *Signal Process.*, vol. 92, no. 11, pp. 2785–2790, Nov. 2012.

[7] Y.-Q. Tu and Y.-L. Shen, "Phase correction autocorrelation-based frequency estimation method for sinusoidal signal," *Signal Process.*, vol. 130, pp. 183–189, Jan. 2017.

[8] A. J. S. Dutra, J. F. L. Oliveira, T. M. Prego, S. L. Netto, and E. A. B. Silva, "High-precision frequency estimation of real sinusoids with reduced computational complexity using a model-based matched-spectrum approach," *Digit. Signal Process.*, vol. 34, pp. 67–73, Nov. 2014.

[9] P. Stoica, R. L. Moses, B. Friedlander, and T. Soderstrom, "Maximum likelihood estimation of the parameters of multiple sinusoids from noisy measurements," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, no. 3, pp. 378–392, Mar. 1989.

[10] C. Qian, L. Huang, H. C. So, N. D. Sidiropoulos, and J. Xie, "Unitary Puma algorithm for estimating the frequency of a complex sinusoid," *IEEE Trans. Signal Process.*, vol. 63, no. 20, pp. 5358–5368, Oct. 2015.

[11] S. Nandi, "Estimation of parameters of two-dimensional sinusoidal signal in heavy-tailed errors," *J. Stat. Planning Inference*, vol. 142, no. 10, pp. 2799–2808, Oct. 2012.

[12] L. Fan and G. Qi, "Frequency estimator of sinusoid based on interpolation of three DFT spectral lines," *Signal Process.*, vol. 144, pp. 52–60, Mar. 2018.

[13] L. Fan, G. Qi, and D. Zhang, "Sinusoid frequency estimation based on DFT and the argument of the sample autocorrelation function," in *Proc. 34th Chin. Control Conf. (CCC)*, Jul. 2015, pp. 4895–4898.

[14] E. Aboutanios and B. Mulgrew, "Iterative frequency estimation by interpolation on Fourier coefficients," *IEEE Trans. Signal Process.*, vol. 53, no. 4, pp. 1237–1242, Apr. 2005.

[15] C. Candan, "Analysis and further improvement of fine resolution frequency estimation method from three DFT samples," *IEEE Signal Process. Lett.*, vol. 20, no. 9, pp. 913–916, Sep. 2013.

[16] L. Fan, G. Qi, J. Liu, J. Jin, L. Liu, and J. Xing, "Frequency estimator of sinusoid by interpolated DFT method based on maximum sidelobe decay Windows," *Signal Process.*, vol. 186, Sep. 2021, Art. no. 108125.

[17] S. Djukanovi, T. Popovi, and A. Mitrovi, "Precise sinusoid frequency estimation based on parabolic interpolation," in *Proc. 24th Telecommus. Forum (TELFOR)*, Nov. 2016, pp. 1–4.

[18] L. Fan, G. Qi, and W. He, "Accurate estimation method of sinusoidal frequency based on FFT," in *Proc. 35th Chin. Control Conf. (CCC)*, Jul. 2016, pp. 5164–5167.

[19] C. Candan and U. Çelebi, "Frequency estimation of a single real-valued sinusoid: An invariant function approach," *Signal Process.*, vol. 185, Aug. 2021, Art. no. 108098.

[20] D. Belega and D. Petri, "Frequency estimation by two-or three-point interpolated Fourier algorithms based on cosine windows," *Signal Process.*, vol. 117, pp. 115–125, Dec. 2015.

[21] J. Liu, L. Fan, J. Jin, X. Wang, and J. Xing, "An accurate and efficient frequency estimation algorithm by using FFT and DTFT," in *Proc. 39th Chin. Control Conf. (CCC)*, Jul. 2020, pp. 2913–2917.

[22] Y. Chen, A. C. Ko, W. Tam, C. Kok, and H. So, "Non-iterative and accurate frequency estimation of a single cisoid using two DFT coefficients," *Digit. Signal Process.*, vol. 98, Mar. 2020, Art. no. 102652.

[23] A. Serbes, "Fast and efficient sinusoidal frequency estimation by using the DFT coefficients," *IEEE Trans. Commun.*, vol. 67, no. 3, pp. 2333–2342, Mar. 2019.

[24] U. Orguner and C. Candan, "A fine-resolution frequency estimator using an arbitrary number of DFT coefficients," *Signal Process.*, vol. 105, no. 10, pp. 17–21, Dec. 2014.
[25] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. AP-34, no. 3, pp. 276–280, Mar. 1986.
[26] R. Roy and T. Kailath, "Esprit-estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, no. 7, pp. 984–995, Jul. 1989.
[27] A. Serbes and K. Qaraqe, "A fast method for estimating frequencies of multiple sinusoidals," *IEEE Signal Process. Lett.*, vol. 27, pp. 386–390, 2020.
[28] S. Djukanovi and V. Popovic-Bugarin, "Efficient and accurate detection and frequency estimation of multiple sinusoids," *IEEE Access*, vol. 7, pp. 1118–1125, 2018.
[29] S. Ye and E. Aboutanios, "Rapid accurate frequency estimation of multiple resolved exponentials in noise," *Signal Process.*, vol. 132, pp. 29–39, Mar. 2017.
[30] D. Rife and R. Boorstyn, "Single tone parameter estimation from discrete-time observations," *IEEE Trans. Inf. Theory*, vol. IT-20, no. 5, pp. 591–598, Sep. 1974.

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