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Sampled-Data Control for Asynchronously Switched Linear Systems Without MDT Constraints

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ABSTRACT In this paper, the sampled-data control problem is studied for asynchronously switched linear systems (SLSs) without minimum dwell time (MDT) constraints. The asynchronous phenomenon exists due to that the information of system mode can be acquired only at the sampling instant. First, a sufficient condition of global asymptotic stability (GAS) is presented for sampled-date switched control systems with a novel class of switching signals, which allows the switching number to be a non-affine function of time, and does not involve any point-wise bound on the switching number. Moreover, unlike the existing literature concerned with sampled-data control problem of switched systems, the MDT constraints are removed. We allow that no sampling happens between two adjacent switching instants, which makes the results more applicable to practice. Then, a sufficient condition checking the existence of sampled-data controllers is presented in terms of linear matrix inequalities (LMIs). Finally, it is shown by a boost converter circuit system that the designed sampled-data controller and switching signals can stabilize the system cooperatively.

INDEX TERMS Asynchronously switched linear systems, sampled-data control, asymptotic stability, multiple Lyapunov functions.

I. INTRODUCTION

Switched systems consist of several subsystems and a switching signal selecting the activated mode at any constant. Studies on the switched systems have become a hot spot direction due to their practical and theoretical values in past decades. The switching feature is very common in real-world systems, thereby a lot of systems of practice can be modeled as switched systems, such as chemical system [1], traffic system [2], teleoperation system [3] and network control system [4]. Up to now, rich research results have been achieved on switched systems, such as stability and performance analysis [5]–[9], controller design [4], [10]–[13], estimation and filtering [14]–[19].

One basic problem of switched systems is to design switching controllers and admissible switching signals to stabilize the system cooperatively. There already exist many results on this issue. For instance, in [11], state-feedback switching controllers and mode-dependent average dwell time switching signals have been designed to stabilize the closed-loop switched system, in [10], switching controllers and dwell time switching signals have been designed to ensure the positivity and stability of the closed-loop system cooperatively. In these literature, continuous state information must be acquired in real time to design the control input. However, the digital controllers are very common now as a result of the development of digital computation. Compared to the continuous state feedback input approach, the digital state feedback input approach is more convenient since it only requires the information of the states at each discrete sampling instants, and holds until next sampling instant comes. The discrete state information is more easy to get, which makes the sampled-data control approach more flexible. Therefore, it is of both theoretical and practical significance to study the sampled-data control problem. The discrete-time approach [20], the input delay approach [21] and the impulsive system approach [22] are three main approaches to study the stability issues of sampled-data control systems. Until now, there already exist lots of results on sampled-data control problems for LTI systems [20], [23], piecewise affine

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systems [24], linear impulsive systems [25], Markovian jump systems [26], Takagi-Sugeno fuzzy systems [27], [28] and so on.

Despite the systems mentioned above, sampled-data control problem of switched systems has also attracted many scholars? attentions. In some recent works [25], [29], [30], it?s assumed that the sampling and switching always happen at the same time. Under this assumption, the mode of the controller remains consistent with the one of the system. However, this cannot be guaranteed for switching systems with unpredictable switching signals. The information of switching signal can be updated only at the sampling instant, and holds until next sampling happens. If a switching happens between two adjacent sampling instant, the mode of the system will change but the mode of the controller will hold. This causes that the system and controller runs asynchronously after the switching happens. In [31], the sampled and quantized measurements of the states have been used to stabilize SLSs with MDT constraints. It?s assumed in [31] that the dwell time of the system will be no smaller than the maximum sampling interval, which can ensure that there exists less than one switching between adjacent two sampling instants, or it can be said that there exists no less than one sampling between two adjacent switching instants. In last several years, under the assumption in [31], some results on sampled-data control problem for switched system [32]–[40] have been derived. In most of aforementioned works, the designed switching signals are required to have both a MDT and an average dwell time (ADT) [41]. These restrictions will no doubt lower the practicability of the results. But if we remove the MDT constraints, more than one switching will happen between two adjacent sampling instants, in other words, it?s possible that no sampling happens between two adjacent switching instants. This may cause that the switching delay of the controller covers the dwell time of the subsystem, which increases the difficulty of analysis. *Can we solve out the sample-data control problem of switched systems without MDT constraints?* This problem motivates our research interests.

Inspired by [42]–[44], a novel class of switching signals and a set of sampled-data controllers are designed to guarantee the GAS of the closed-loop SLSs cooperatively. The main contributions are as follows:

1) Unlike the existing literature [31]–[40] concerned with switched systems' sampled-data control problem, we do not require the MDT of the switching signals to be larger than the maximum sampling interval, i.e., we allow that no sampling happens between two adjacent switching instants, which shows the results in this paper are more applicable to practice.

2) A novel class of constrained switching signals is designed for the closed-loop sampled-data control system. Some asymptotic properties such as the switching frequency, the fraction of activation of the subsystem with synchronous or asynchronous controller, and the admissible transitions density between each subsystems are used to characterize this class of switching signals. Unlike the traditional ADT switching signals, the novel ones contain no point-wise bounds on switching number, and allow it grows faster than an affine function with respect to time.

3) Sufficient conditions verifying the existence of controllers which can guarantee GAS have been given for the switched systems, and the controller gain matrices can be obtained by solving a set of LMIs.

Outlines: The paper is organized as follows: System descriptions and preliminaries are presented in Section II. Main results are given in Section III. A numerical example is given in Section IV, and the conclusion is given in Section V.

Notations: 'T': matrix transposition. '*': transposed elements in the symmetric positions. $\|\cdot\|$: Euclidean vector norm. $\mathbb{N}$ ($\mathbb{N}^+$): the set of non-negative (positive) integers. $\mathbb{R}$ ($\mathbb{R}^+$): the set of (positive) real numbers. $\mathbb{R}^n$ ($\mathbb{R}^{m\times n}$): the set of n-dimensional vectors (m×n-dimensional matrices) with real entries. I: identity matrix with appropriate dimension. $diag\{H_1, H_2\}$: block-diagonal matrix of H_1 and H_2. $P > 0$ ($P < 0$) denotes that matrix P is positive (negative) definite. Given matrices $A \in \mathbb{R}^{n\times n}$, we denote $\text{He}\{A\} = A + A^T$. A function $\kappa : [0, +\infty) \to [0, +\infty)$ is of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, unbounded, and $\kappa(0) = 0$.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the following SLSs with control inputs:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t),\ t \geq t_0 = 0, \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control input, switching signal $\sigma(t)$ is a piecewise constant function with respect to t, the set of subsystems is denoted as $\mathcal{I} = \{1, 2, \ldots, N\}$, where $N > 1$ denotes the number of subsystems. The switching sequence for system (1) is denoted as $\{t_0, t_1, t_2, \ldots, t_k, t_{k+1}, \ldots\}$, $k = 0, 1, 2, \ldots$, where t_k denotes the k^{th} switching instant. For a switching sequence $t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$, $\sigma(t)$ is continuous from right everywhere. N_σ^t denotes the number of switches during $[0, t)$.

To facilitate the research as well as to have a clearer switching model, we will give the system model on arbitrary dwell time interval $[t_k, t_{k+1})$, $k = 0, 1, 2, \ldots$. Yet the general, we assume that the nearest sampling instant before the k^{th} switching instant t_k is $t_{k,0}$, and there exist n_k sampling instants $t_{k,1}, t_{k,2}, \ldots, t_{k,n_k}$ on dwell time interval $[t_k, t_{k+1})$, $n_k \geq 0$. $t_{k,l}$ represents the l^{th} sampling instant on $[t_k, t_{k+1})$, $l = 1, 2, \ldots, n_k$. Specially, the case $n_k = 0$ means that there exists no sampling instant on $[t_k, t_{k+1})$. Moreover, we denote $h_{k,l} = t_{k,l+1} - t_{k,l}$, $l = 0, 1, \ldots, n_k - 1$ as the sampling interval between consecutive two sampling instants $t_{k,l}$ and $t_{k,l+1}$ (when $n_k = 0$, $h_{k,0}$ represents the sampling interval between sampling instant $t_{k,0}$ and the next sampling instant). The sampling interval satisfies that $0 < h_{k,l} \leq h$, where $h > 0$ is called the maximum sampling interval.

In order to let the sampled-data controller be more practicable, we assume that the mode sensor keeps working all the time, but the system signal is sent to the zero-order holder (ZOH) only at the sampling instant, and no delay exists in the transmission process. Thus the control inputs on $[t_k, t_{k+1})$ can be denoted as follows:

$$u(t) = K_{\sigma(\widetilde{t})} x(\widetilde{t}), \tag{2}$$

where $K_{\sigma(\widetilde{t})} \in \mathbb{R}^{n_u \times n_x}$ is a constant matrix, $\widetilde{t} = t - d(t)$, of witch $d(t) : \mathbb{R}^+ \to (0, h]$ is a time-varying function called the switching delay, and denoted as follows:

$$\begin{cases} n_k = 0 : d(t) = t - t_{k,0}, & t \in [t_k, t_{k+1}), \\ n_k \geq 1 : d(t) = \begin{cases} t - t_{k,0}, & t \in [t_k, t_{k,1}), \\ t - t_{k,l}, & t \in [t_{k,l}, t_{k,l+1}), \\ t - t_{k,n_k}, & t \in [t_{k,n_k}, t_{k+1}), \end{cases} \end{cases} \tag{3}$$

where $l = 1, 2, \ldots, n_k - 1$. By substituting control inputs $u(t)$ (2) into SLSs (1), we can get the closed-loop system model on arbitrary dwell time interval $[t_k, t_{k+1})$, $k = 0, 1, 2, \ldots$, which is shown as follows:

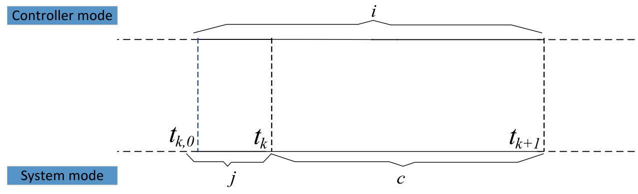
$$\dot{x}(t) = A_{\sigma(t)} x(t) + B_{\sigma(t)} K_{\sigma(\widetilde{t})} x(\widetilde{t}), \quad t \in [t_k, t_{k+1}). \tag{4}$$

From (4) we can see that a delay $d(t)$ exists between the switching signals of the controller and the system, i.e., asynchronous phenomenon may exist on the dwell time interval $[t_k, t_{k+1})$. We denote $T_k = t_{k+1} - t_k$ as the activation time of the $\sigma(t_k)^{th}$ subsystem between consecutive two switching instants t_k and t_{k+1}. Moreover, we denote T_k^s and T_k^a as the holding time of the $\sigma(t_k)^{th}$ subsystem in the synchronous and asynchronous period of T_k, respectively. Obviously, T_k^s and T_k^a satisfy that $T_k = T_k^s + T_k^a$.

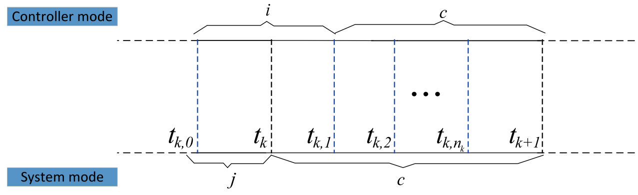
In practice, the switching instants are usually unpredictable, which means that it is impossible to make the sampling and switching instants remain totally consistent. Hence the following assumption is made:

Assumption 1: The switching will not happen at any sampling instant of system (4), *i.e.,* $t_k \neq t_{k,1}$, $k = 1, 2, \ldots$.

According to whether sampling happens on dwell time interval $[t_k, t_{k+1})$, $k = 0, 1, 2, \ldots$, the switching signals are divided into two situations, which are shown in Fig. 1. In Fig. 1 (a), no sampling happens on the dwell time interval, and the controller and system may keep running synchronously or asynchronously, which depends on whether $c = i$ or not. In Fig. 1 (b), the sampling happens more than once. The controller and system run asynchronously or synchronously on $[t_k, t_{k,1})$, then, the mode of the system is gotten at sampling instant $t_{k,1}$, and the mode of the controller is thus updated. Their modes are matched during the rest of the dwell time interval. From Fig. 1 we can know that the asynchronous period may occupy the whole dwell time interval, i.e., $T_k^a = T_k$, and is upper bounded by the maximum sampling interval h. Moreover, we remove the MDT constraints on switching signals, which means we allow that no sampling happens on the dwell time interval. This is different from the existing literature [31]–[40], which only consider the case that at least one sampling occurs on any dwell time interval. Obviously, the situation considered in this paper is more practical and comprehensive.



(a) $n_k = 0$



(b) $n_k > 0$

FIGURE 1. Two situations of the switching signal on arbitrary dwell time interval $[t_k, t_{k+1})$, $k \geq 0$ ($i, j, c \in \mathcal{I}, j \neq c$).

To solve the sampled-data control problems of system (4), in the sequel, we will define some important symbols, which will be used in the proof of the main results.

First, we choose a continuous monotone increasing function $\rho : [0, +\infty] \to [0, +\infty]$, which satisfies $\rho(0) = 0$ and $\lim_{t \to +\infty} \rho(t) = +\infty$, i.e., $\rho(t) \in \mathcal{K}_\infty$. $\rho(t)$ will be related to several important symbols of the switching signals. We let

$$\upsilon_\rho(t) := \frac{N_\sigma^t}{\rho(t)}, t > 0 \tag{5}$$

be the *ρ-frequency of switching* from 0 to t, and let

$$\hat{\upsilon}_\rho := \limsup_{t \to +\infty} \upsilon_\rho(t) \tag{6}$$

be the *asymptotic upper density* of $\upsilon_\rho(t)$. Then, we denote $E(\mathcal{I})$ as the set containing each possible switching pair: $i \to j$, $i, j \in \mathcal{I}$. Choose a switching pair $(i, j) \in E(\mathcal{I})$. We define *the transition frequency* from subsystem i to subsystem j on $[0, t]$ as

$$\omega_{ij}(t) := \frac{\#\{i \to j\}_t}{N_\sigma^t}, \tag{7}$$

where $\#\{i \to j\}_t$ represents the number of transitions from subsystem i to subsystem j on $[0, t]$. Moreover, we define the *asymptotic upper density* of $\omega_{ij}(t)$ as

$$\hat{\omega}_{ij} := \limsup_{t \to +\infty} \omega_{ij}(t). \tag{8}$$

Next, we denote the *ρ-fraction of activation* of the i^{th} system on synchronous period of $[0, t]$ as

$$\mathscr{T}_\rho^s(i, t) := \sum_{k:\sigma(t_k)=i} \frac{T_k^s}{\rho(t)}, \tag{9}$$

and let the *asymptotic lower densities* of $\mathscr{T}_\rho^s(i, t)$ be

$$\check{\mathscr{T}}_\rho^s(i) := \liminf_{t \to \infty} \mathscr{T}_\rho^s(i, t) \tag{10}$$

Similarly, we denote the *ρ-fraction of activation* of the i^{th} system on asynchronous period of $[0, t]$ as

$$\mathcal{T}^a_\rho(i,t) := \sum_{k:\sigma(t_k)=i} \frac{T^a_k}{\rho(t)}, \tag{11}$$

and let the *asymptotic upper densities* of $\mathcal{T}^a_\rho(i,t)$ be

$$\hat{\mathcal{T}}^a_\rho(i) := \limsup_{t\to\infty} \mathcal{T}^s_\rho(i,t). \tag{12}$$

Finally, a lemma which will be used in next section is introduced as follows:

Lemma 1 (([45])): The following inequality holds for any matrix $P > 0$, any scalars ε_1, ε_2, $\varepsilon_1 < \varepsilon_2$, and any vector-valued function $x(t) : [\varepsilon_1, \varepsilon_2] \to \mathbb{R}^n$:

$$-\int_{\varepsilon_1}^{\varepsilon_2} x^T(s)Px(s)ds \le -\frac{1}{\varepsilon_2-\varepsilon_1}\int_{\varepsilon_1}^{\varepsilon_2} x^T(s)dsP\int_{\varepsilon_1}^{\varepsilon_2} x(s)ds.$$

III. MAIN RESULTS

First, a sufficient condition checking the GAS of closed-system (4) with a class of constrained switching signals is presented as follows:

Theorem 1: Consider system (4)*. Given a set of scalars $\alpha_i > 0$, $\beta_i > 0$, $\mu_{i\to j} > 1$, $i, j \in \mathcal{I}$, $i \ne j$. If there exist $\mathcal{K}_\infty$ functions $\kappa_1(\|x\|)$ and $\kappa_2(\|x\|)$, and a set of continuously differentiable non-negative functions $V_{\sigma(t)\sigma(\tilde t)}(t)$: $\mathbb{R}^n \to \mathbb{R}^+$ such that $\forall i, j, c \in \mathcal{I}$, $i \ne j$, $c \ne j$,*

$$\kappa_1(\|x(t)\|) \le V_{jc}(t) \le \kappa_2(\|x(t)\|), \tag{13}$$
$$\dot V_{ii}(t) + \alpha_i V_{ii}(t) \le 0, \quad t \in [t_k + T^a_k, t_{k+1}), \tag{14}$$
$$\dot V_{ij}(t) - \beta_i V_{ij}(t) \le 0, \quad t \in [t_k, t_k + T^a_k), \tag{15}$$
$$V_{ii}(t_{k,1}) - V_{ij}(t^-_{k,1}) \le 0, \tag{16}$$
$$V_{ii}(t_{k,l}) - V_{ii}(t^-_{k,l}) \le 0, \quad l = 1, 2, \ldots, n_k, \tag{17}$$
$$V_{ci}(t_k) - \mu_{j\to c}V_{ji}(t^-_k) \le 0, \tag{18}$$
$$V_{ij}(t_k) - \mu_{j\to i}V_{jj}(t^-_k) \le 0. \tag{19}$$

Then, the GAS of system (4) *with any switching signal satisfying the following conditions can be guaranteed.*

$$\begin{cases} \check{\upsilon}_\rho := \liminf_{t\to+\infty} \upsilon_\rho(t) > 0, \\ \hat{\upsilon}_\rho \sum_{(i,j)\in E(\mathcal{I})} \tilde{\mu}_{i\to j}\hat{\omega}_{ij} - \sum_{i\in\mathcal{I}} \alpha_i \check{\mathcal{T}}^s_\rho(i) + \sum_{i\in\mathcal{I}} \beta_i \hat{\mathcal{T}}^a_\rho(i) < 0, \end{cases} \tag{20}$$

where $\tilde{\mu}_{i\to j} = \ln \mu_{i\to j}$.

Proof: Let $\phi(t) = \begin{cases} 1, & \textit{if } \sigma(t) = \sigma(\tilde t) \\ 0, & \textit{otherwise} \end{cases}$, $t \ge 0$. From (14) and (15) we have that $\forall t \in [t_k, t_{k+1})$,

$$\dot V_{\sigma(t)\sigma(\tilde t)}(t) + (\phi(t)\alpha_{\sigma(t)} - (1-\phi(t))\beta_{\sigma(t)})V_{\sigma(t)\sigma(\tilde t)}(t) \le 0. \tag{21}$$

Furthermore, according to Fig. 1, from (16)-(19) we have that $\forall k = 0, 1, 2, \ldots$ and $\forall l = 1, 2, \ldots, n_k$, the following conditions hold:

$$\begin{cases} V_{\sigma(t_k)\sigma(t^-_k)}(t_k) \le \mu_{\sigma(t^-_k)\to\sigma(t_k)} V_{\sigma(t^-_k)\sigma(\tilde t^-_k)}(t^-_k), \\ V_{\sigma(t_{k,l})\sigma(\tilde t_{k,l})}(t_{k,l}) \le V_{\sigma(t^-_{k,l})\sigma(\tilde t^-_{k,l})}(t^-_{k,l}). \end{cases} \tag{22}$$

Then, from (21) and (22) we can get

$$\begin{aligned} &V_{\sigma(t)\sigma(\tilde t)}(t) \\ &\quad\le e^{-\alpha_{\sigma(t)}T^s_{N^t_\sigma}(t_{N^t_\sigma},t)+\beta_{\sigma(t)}T^a_{N^t_\sigma}(t_{N^t_\sigma},t)} V_{\sigma(t_{N^t_\sigma})\sigma(\tilde t_{N^t_\sigma})}(t_{N^t_\sigma}) \\ &\quad\le e^{\beta_{\sigma(t)}(t-t_{N^t_\sigma})} V_{\sigma(t_{N^t_\sigma})\sigma(\tilde t_{N^t_\sigma})}(t_{N^t_\sigma}) \\ &\quad\le e^{\beta_{\sigma(t)}(t-t_{N^t_\sigma})} \mu_{\sigma(t^-_{N^t_\sigma})\to\sigma(t_{N^t_\sigma})} V_{\sigma(t^-_{N^t_\sigma})\sigma(\tilde t^-_{N^t_\sigma})}(t^-_{N^t_\sigma}) \\ &\quad\le e^{\beta_{\sigma(t)}(t-t_{N^t_\sigma})-\alpha_{\sigma(t_{N^t_\sigma-1})}T^s_{N^t_\sigma-1}+\beta_{\sigma(t_{N^t_\sigma-1})}T^a_{N^t_\sigma-1}} \\ &\qquad\times \mu_{\sigma(t^-_{N^t_\sigma})\to\sigma(t_{N^t_\sigma})}\mu_{\sigma(t^-_{N^t_\sigma-1})\to\sigma(t_{N^t_\sigma-1})} \\ &\qquad\times V_{\sigma(t^-_{N^t_\sigma-1})\sigma(\tilde t^-_{N^t_\sigma-1})}(t^-_{N^t_\sigma-1}) \\ &\qquad\ldots \\ &\quad\le e^{\beta_{\sigma(t)}(t-t_{N^t_\sigma})-\sum_{k=0}^{N^t_\sigma-1}\alpha_{\sigma(t_k)}T^s_k+\sum_{k=0}^{N^t_\sigma-1}\beta_{\sigma(t_k)}T^a_k} \\ &\qquad\times \prod_{k=0}^{N^t_\sigma-1} \mu_{\sigma(t_k)\to\sigma(t_{k+1})} V_{\sigma(0)\sigma(0)}(0). \end{aligned} \tag{23}$$

Note that

$$\begin{aligned} &\prod_{k=0}^{N^t_\sigma-1} \mu_{\sigma(t_k)\to\sigma(t_{k+1})} \\ &\quad= \exp\left(\sum_{k=0}^{N^t_\sigma-1} \tilde{\mu}_{\sigma(t_k)\to\sigma(t_{k+1})}\right) \\ &\quad= \exp\left(\sum_{i\in\mathcal{I}}\sum_{k=0}^{N^t_\sigma-1}\sum_{\substack{i\to j\\ j\in\mathcal{I}\\ i\ne j\\ \sigma(t_k)=i\\ \sigma(t_{k+1})=j}} \tilde{\mu}_{i\to j}\right) \\ &\quad= \exp\left(N^t_\sigma \sum_{(i,j)\in E(\mathcal{I})} \tilde{\mu}_{i\to j}\frac{\#\{i\to j\}_t}{N^t_\sigma}\right), \end{aligned} \tag{24}$$

and

$$\begin{aligned} &\exp\left\{-\sum_{k=0}^{N^t_\sigma-1}\alpha_{\sigma(t_k)}T^s_k + \sum_{k=0}^{N^t_\sigma-1}\beta_{\sigma(t_k)}T^a_k\right\} \\ &\quad= \exp\left\{-\sum_{i\in\mathcal{I}}\alpha_i\sum_{k:\sigma(t_k)=i}T^s_k + \sum_{i\in\mathcal{I}}\beta_i\sum_{k:\sigma(t_k)=i}T^a_k\right\}. \end{aligned} \tag{25}$$

By taking (24) and (25) into (23) we get that

$$V_{\sigma(t)\sigma(\tilde t)}(t) \le \exp(\zeta(t))V_{\sigma(0)\sigma(0)}(0), \tag{26}$$

where $\zeta(t)$ is denoted by

$$\zeta(t) := N_\sigma^t \sum_{(i,j)\in E(\mathcal{I})} \widetilde{\mu}_{i\to j}\frac{\#\{i\to j\}_t}{N_\sigma^t} - \sum_{i\in\mathcal{I}}\alpha_i \sum_{k:\sigma(t_k)=i} T_k^s + \sum_{i\in\mathcal{I}}\beta_i \sum_{k:\sigma(t_k)=i} T_k^a + \beta_{\sigma(t)}(t - t_{N_\sigma(t)}).$$

We continue to denote

$$f(t) := \sum_{(i,j)\in E(\mathcal{I})} \widetilde{\mu}_{i\to j}\frac{\#\{i\to j\}_t}{N_\sigma^t},$$

and

$$g(t) := -\sum_{i\in\mathcal{I}}\alpha_i \sum_{k:\sigma(t_k)=i} \frac{T_k^s}{\rho(t)} + \sum_{i\in\mathcal{I}}\beta_i \sum_{k:\sigma(t_k)=i} \frac{T_k^a}{\rho(t)} + \beta_{\sigma(t)}\frac{t - t_{N_\sigma^t}}{\rho(t)},$$

then $\zeta(t) := \rho(t)(\upsilon_\rho(t)f(t) + g(t))$. According to the properties of limsup, we can get that

$$\limsup_{t\to+\infty}(\upsilon_\rho(t)f(t) + g(t)) \leq \limsup_{t\to+\infty}\upsilon_\rho(t)\limsup_{t\to+\infty}f(t) + \limsup_{t\to+\infty}g(t).$$

From (7) we have that

$$\limsup_{t\to+\infty}f(t) \leq \sum_{(i,j)\in E(\mathcal{I})}\widetilde{\mu}_{i\to j}\limsup_{t\to+\infty}\omega_{ij}(t).$$

Moreover, (20) implies that $\beta_{\sigma(t)}(t - t_{N_\sigma^t})$ is $o(\rho(t))$ as $t\to+\infty$. Otherwise it results in $\check{\upsilon}_\rho = 0$, which is inconsistent with (20). Hence we can know that $\frac{t-t_{N_\sigma^t}}{\rho(t)} \to 0$ as $t\to 0$. Then, from (9) and (11) we obtain

$$\limsup_{t\to+\infty}g(t) \leq -\sum_{i\in\mathcal{I}}\alpha_i\liminf_{t\to+\infty}\mathscr{T}_\rho^s(i,t) + \sum_{i\in\mathcal{I}}\beta_i\liminf_{t\to+\infty}\mathscr{T}_\rho^a(i,t),$$

which with (10), (12) and (20) ensures that

$$\begin{aligned}\limsup_{t\to+\infty}(\upsilon_\rho(t)f(t) + g(t)) &\leq \limsup_{t\to+\infty}\upsilon_\rho(t)\sum_{(i,j)\in E(\mathcal{I})}\widetilde{\mu}_{i\to j}\limsup_{t\to+\infty}\omega_{ij}(t)\\ &\quad - \sum_{i\in\mathcal{I}}\alpha_i\liminf_{t\to+\infty}\mathscr{T}_\rho^s(i,t) + \sum_{i\in\mathcal{I}}\beta_i\limsup_{t\to+\infty}\mathscr{T}_\rho^a(i,t)\\ &< 0,\end{aligned}$$

which further implies that

$$\lim_{t\to+\infty}\exp(\rho(t)(\upsilon_\rho(t)f(t) + g(t))) = 0.$$

By combining (13) and (26) we get that $\|x(t)\| \leq \kappa_1^{-1}(\kappa_2(\|x(0)\|)\exp(\zeta(t)))$. Thus it can conclude that for any initial states $x(0)$, and any $\sigma(t)$, $\lim_{t\to+\infty}\|x(t)\| = 0$ always holds. The GAS of system (4) is established. The proof is completed. □

Remark 1: The function $\rho(t)$ appears in (5), (9) and (11) enlarges the range of admissible switching signals which can be asymptotically stabilize system (4). We can choose the order of $\rho(t)$ according to the order of N_σ^t. For instance, we can choose $\rho(t) = t^{1.5}$ for switching number satisfying $N_\sigma^t < \alpha t^{1.5} + \beta t + \gamma$, $\alpha > 0$, $\beta > 0$, $\gamma > 0$ on $[0, t]$, where N_σ^t can grow faster than an affine function with respect to t.

Remark 2: In [41], the definition of ADT is that let $N_\sigma(t, T)$ be the switching number on $[t, T]$, and N_0 be a chatter bound, if there exist two positive scalars N_0 and τ_a such that $N_\sigma(t, T) \leq N_0 + \frac{T-t}{\tau_a}$ holds $\forall 0 \leq t \leq T$, then we say $\sigma(t)$ has an ADT τ_a. The ADT condition requires the switching number to grow at most as fast as an affine function for any interval from t to T. Unlike this, the condition (20) contains no point-wise bounds on N_σ^t, only the asymptotic properties of the symbols defined in (5), (7), (9) and (11) need to be considered.

Remark 3: The conditions (16) and (17) ensure that the energy function remains non-increasing at any sampling instant. Hence we only need to consider the change extent of energy function at the switching instants, and the change rate of energy function during each dwell time interval.

Remark 4: It can be noted that the mode sensor has been assumed to keep working all the time, hence the switching signal curves of the system and the controller can be acquired. Then, with aid of the curves, the switching frequency, the fraction of activation of the subsystem with asynchronous or synchronous controllers can be calculated. Then, the GAS of the system can be checked according to whether (20) holds or not.

Based on the results of Theorem 1, sufficient conditions checking the existence of the sampled-data controllers are given as follows in terms of LMIs:

Theorem 2: Consider system (4). Given a set of scalars $\alpha_i > 0$, $\beta_i > 0$, $\mu_{i\to j} > 1$, $i, j \in \mathcal{I}$, $i \neq j$. If there exists a set of matrices Y_c, $Y_c \in \mathbb{R}^{n_u\times n_x}$, $H_c > 0$, $H_c \in \mathbb{R}^{n_x\times n_x}$, $\mathcal{P}_{ic} > 0$, $\mathcal{P}_{ic} \in \mathbb{R}^{n_x\times n_x}$, $\mathcal{Q}_{ic} > 0$, $\mathcal{Q}_{ic} \in \mathbb{R}^{n_x\times n_x}$, $c, i \in \mathcal{I}$ such that $\forall i, j, c \in \mathcal{I}$, $i \neq j$, $j \neq c$, the following conditions hold:

$$\Xi_{ii} \leq 0, \quad \Xi_{ii} + h[0\ I]^T\mathcal{Q}_{ii}[0\ I] \leq 0, \quad \begin{bmatrix}\Xi_{ii} & h\Pi_{ii}\\ * & -he^{-\alpha_i h}\mathcal{Q}_{ii}\end{bmatrix} \leq 0, \tag{27}$$

$$\Xi_{ij} \leq 0, \quad \Xi_{ij} + h[0\ I]^T\mathcal{Q}_{ij}[0\ I] \leq 0, \quad \begin{bmatrix}\Xi_{ii} & h\Pi_{ij}\\ * & -h\mathcal{Q}_{ij}\end{bmatrix} \leq 0, \tag{28}$$

$$\begin{bmatrix}\mathcal{P}_{ii} - 2H_i & H_j\\ * & -\mathcal{P}_{ij}\end{bmatrix} \leq 0, \tag{29}$$

$$\mathcal{P}_{ci} - \mu_{j\to c}\mathcal{P}_{ji} \leq 0,\ \mathcal{Q}_{ci} - \mu_{j\to c}e^{-\theta(\beta_j,\xi_c)h}\mathcal{Q}_{ji} \leq 0, \tag{30}$$

$$\mathcal{P}_{ij} - \mu_{j\to i}\mathcal{P}_{jj} \leq 0,\ \mathcal{Q}_{ij} - \mu_{j\to i}e^{-(\alpha_j+\beta_i)h}\mathcal{Q}_{jj} \leq 0, \tag{31}$$

where $\theta(\beta_j, \xi_c) = \max\{0, \xi_c - \beta_j\}$, $\xi_c = \begin{cases}-\alpha_c, & \textit{if } c = i\\ \beta_c, & \textit{otherwise}\end{cases}$, and

$$\Pi_{ii} = \begin{bmatrix}-Y_i^TB_i^T\\ -Y_i^TB_i^T\end{bmatrix}, \quad \Pi_{ij} = \begin{bmatrix}-Y_j^TB_i^T\\ -Y_j^TB_i^T\end{bmatrix},$$

$$\Xi_{ii} = \begin{bmatrix} He\{A_iH_i + B_iY_i\}+\alpha_i\mathcal{P}_{ii} & H_iA_i^T + Y_iB_i^T + \mathcal{P}_{ii} - H_i \\ * & -2H_i \end{bmatrix},$$

$$\Xi_{ij} = \begin{bmatrix} He\{A_iH_j + B_iY_j\}-\beta_i\mathcal{P}_{ij} & H_jA_i^T + Y_jB_i^T + \mathcal{P}_{ij} - H_j \\ * & -2H_j \end{bmatrix}.$$

Then, the GAS of system (4) with any switching signal satisfying (20) can be guaranteed. Furthermore, the controller gain matrices can be calculated by $K_c = Y_cH_c^{-1}$, $c \in \mathcal{I}$.

Proof: Let $\lambda_i(t) = \phi(t)\alpha_i - (1-\phi(t))\beta_i$, where $\phi(t) = \begin{cases} 1, & if\ \sigma(t) = \sigma(\tilde{t}) \\ 0, & otherwise \end{cases}$. We construct the following Lyapunov function:

$$\begin{aligned} &V_{\sigma(t)\sigma(\tilde{t})}(t) \\ &\quad = x^T(t)P_{\sigma(t)\sigma(\tilde{t})}x(t) \\ &\qquad + (h(t) - d(t))\int_{\tilde{t}}^{t} e^{\lambda_{\sigma(t)}(t)(s-t)}\dot{x}^T(s)Q_{\sigma(t)\sigma(\tilde{t})}\dot{x}(s)ds, \end{aligned} \tag{32}$$

where $t \geq 0$, $P_{ic} > 0$, $Q_{ic} > 0$, $i, c \in \mathcal{I}$, $h(t) : \mathbb{R}^+ \to (0, h]$ is a time-varying function defined as follows:

$$\begin{cases} n_k = 0 : h(t) = h_{k,0}, & t \in [t_k, t_{k+1}), \\ n_k \geq 1 : h(t) = \begin{cases} h_{k,0}, & t \in [t_k, t_{k,1}), \\ h_{k,l}, & t \in [t_{k,l}, t_{k,l+1}), \\ h_{k,n_k}, & t \in [t_{k,n_k}, t_{k+1}), \end{cases} \end{cases} \tag{33}$$

where $l = 1, 2, \ldots, n_k - 1$.

First, consider the synchronous case, i.e., $\sigma(t) = \sigma(\tilde{t})$. Yet the general, we assume that the i^{th} subsystem and controller are activated, i.e., $\sigma(t) = \sigma(\tilde{t}) = i$. Taking the derivative of (32) with respect to t gives that

$$\begin{aligned} &\dot{V}_{ii}(t) + \alpha_iV_{ii}(t) \\ &\quad = 2\dot{x}^T(t)P_{ii}x(t) + \alpha_ix^T(t)P_{ii}x(t) \\ &\qquad + (h(t)-d(t))\dot{x}^T(t)Q_{ii}\dot{x}(t) - \int_{\tilde{t}}^{t} e^{\alpha_i(s-t)}\dot{x}^T(s)Q_{ii}\dot{x}(s)ds. \end{aligned}$$

We denote $\mathcal{P}_{ic}$, $\mathcal{Q}_{ic}$, $\varphi_i(t)$ and $\psi_i(t)$ as follows:

$$\begin{cases} \mathcal{P}_{ic} = H_cP_{ic}H_c, \ \mathcal{Q}_{ic} = H_cQ_{ic}H_c, \\ \varphi_i(t) = [x^T(t)H_i^{-1}\ \dot{x}^T(t)H_i^{-1}]^T, \\ \psi_i(t) = [x^T(t)H_i^{-1}\ \dot{x}^T(t)H_i^{-1}\ e^T(t)H_i^{-1}]^T, \end{cases}$$

where $e(t) = \frac{1}{d(t)}\int_{\tilde{t}}^{t}\dot{x}(s)ds$. From Lemma 1 and (4) we can further get that

$$\begin{aligned} &\dot{V}_{ii}(t) + \alpha_iV_{ii}(t) \\ &\quad \leq 2\dot{x}^T(t)P_{ii}x(t) + \alpha_ix^T(t)P_{ii}x(t) \\ &\qquad + (h(t)-d(t))\dot{x}^T(t)Q_{ii}\dot{x}(t) - e^{-\alpha_ih}d(t)e^T(t)Q_{ii}e(t) \\ &\qquad + 2[x^T(t) + \dot{x}^T(t)]H_i^{-1}[A_ix(t) + B_iK_ix(t) \\ &\qquad - d(t)B_iK_ie(t) - \dot{x}(t)] \\ &\quad = 2\dot{x}^T(t)H_i^{-1}\mathcal{P}_{ii}H_i^{-1}x(t) + \alpha_ix^T(t)H_i^{-1}\mathcal{P}_{ii}H_i^{-1}x(t) \\ &\qquad + (h(t)-d(t))\dot{x}^T(t)H_i^{-1}\mathcal{Q}_{ii}H_i^{-1}\dot{x}(t) \\ &\qquad - e^{-\alpha_ih}d(t)e^T(t)H_i^{-1}\mathcal{Q}_{ii}H_i^{-1}e(t) \\ &\qquad + 2[x^T(t)H_i^{-1} + \dot{x}^T(t)H_i^{-1}][(A_iH_i + B_iY_i)H_i^{-1}x(t) \\ &\qquad - d(t)B_iY_iH_i^{-1}e(t) - H_iH_i^{-1}\dot{x}(t)] \\ &\quad = \varphi_i^T(t)\Xi_{ii}\varphi_i(t) + (h(t)-d(t))\varphi_i^T(t)[0\ I]^T\mathcal{Q}_{ii}[0\ I]\varphi_i(t) \\ &\qquad + d(t)\psi_i^T(t)\Psi_{ii}\psi_i(t) \\ &\quad = \tau(t)\varphi_i^T(t)(\Xi_{ii} + h(t)[0\ I]^T\mathcal{Q}_{ii}[0\ I])\varphi_i(t) \\ &\qquad + (1-\tau(t))\psi_i^T(t)(\Phi_{ii} + h(t)\Psi_{ii})\psi_i(t) \\ &\quad = \tau(t)\varphi_i^T(t)\Big[\chi(t)\Xi_{ii} + (1-\chi(t))(\Xi_{ii} \\ &\qquad + h[0\ I]^T\mathcal{Q}_{ii}[0\ I])\Big]\varphi_i(t) + (1-\tau(t))\psi_i^T(t) \\ &\qquad \times [\chi(t)\Phi_{ii} + (1-\chi(t))(\Phi_{ii} + h\Psi_{ii})]\,\psi_i(t), \end{aligned}$$

where $\Phi_{ii} = \begin{bmatrix} \Xi_{ii} & 0 \\ * & 0 \end{bmatrix}$, $\Psi_{ii} = \begin{bmatrix} 0 & \Pi_{ii} \\ * & -e^{-\alpha_ih}\mathcal{Q}_{ii} \end{bmatrix}$, $\tau(t) = \frac{h(t)-d(t)}{h(t)}$, $\chi(t) = \frac{h-h(t)}{h}$. Since $0 \leq \tau(t) \leq 1$, $0 \leq \chi(t) \leq 1$, from (27) we have that $V_{ii}(t) + \alpha_iV_{ii}(t) \leq 0$, $t \in [t_k + T_k^a, t_{k+1})$, i.e., (14) holds.

Then, consider the asynchronous case, i.e., $\sigma(t) \neq \sigma(\tilde{t})$. In general, we assume that the i^{th} subsystem and the j^{th} controller are activated, i.e., $\sigma(t) = i$, $\sigma(\tilde{t}) = j$, $i \neq j$. Taking the derivative of (32) with respect to t gives that

$$\begin{aligned} &\dot{V}_{ij}(t) - \beta_iV_{ij}(t) \\ &\quad = 2\dot{x}^T(t)P_{ij}x(t) - \beta_ix^T(t)P_{ij}x(t) \\ &\qquad + (h(t)-d(t))\dot{x}^T(t)Q_{ij}\dot{x}(t) \\ &\qquad - \int_{\tilde{t}}^{t} e^{-\beta_i(s-t)}\dot{x}^T(s)Q_{ij}\dot{x}(s)ds \\ &\quad \leq 2\dot{x}^T(t)P_{ij}x(t) - \beta_ix^T(t)P_{ij}x(t) \\ &\qquad + (h(t)-d(t))\dot{x}^T(t)Q_{ij}\dot{x}(t) - d(t)e^T(t)Q_{ij}e(t) \\ &\qquad + 2[x^T(t) + \dot{x}^T(t)]H_j^{-1}[A_ix(t) + B_iK_jx(t) \\ &\qquad - d(t)B_iK_je(t) - \dot{x}(t)] \\ &\quad = 2\dot{x}^T(t)H_j^{-1}\mathcal{P}_{ij}H_j^{-1}x(t) - \beta_ix^T(t)H_j^{-1}\mathcal{P}_{ij}H_j^{-1}x(t) \\ &\qquad + (h(t)-d(t))\dot{x}^T(t)H_j^{-1}\mathcal{Q}_{ij}H_j^{-1}\dot{x}(t) \\ &\qquad - d(t)e^T(t)H_j^{-1}\mathcal{Q}_{ij}H_j^{-1}e(t) \\ &\qquad + 2[x^T(t)H_j^{-1} + \dot{x}^T(t)H_j^{-1}][(A_iH_j + B_iY_j)H_j^{-1}x(t) \\ &\qquad - d(t)B_iY_jH_j^{-1}e(t) - H_jH_j^{-1}\dot{x}(t)] \\ &\quad = \varphi_j^T(t)\Xi_{ij}\varphi_j(t) + (h(t)-d(t))\varphi_j^T(t)[0\ I]^T\mathcal{Q}_{ij}[0\ I]\varphi_j(t) \\ &\qquad + d(t)\psi_j^T(t)\Psi_{ij}\psi_j(t) \\ &\quad = \tau(t)\varphi_j^T(t)(\Xi_{ij} + h(t)[0\ I]^T\mathcal{Q}_{ij}[0\ I])\varphi_j(t) \\ &\qquad + (1-\tau(t))\psi_j^T(t)(\Phi_{ij} + h(t)\Psi_{ij})\psi_j(t) \\ &\quad = \tau(t)\varphi_j^T(t)\Big[\chi(t)\Xi_{ij} + (1-\chi(t))(\Xi_{ij} \\ &\qquad + h[0\ I]^T\mathcal{Q}_{ij}[0\ I])\Big]\varphi_j(t) + (1-\tau(t))\psi_j^T(t) \\ &\qquad \times \left[\chi(t)\Phi_{ij} + (1-\chi(t))(\Phi_{ij} + h\Psi_{ij})\right]\psi_j(t), \end{aligned}$$

where $\Phi_{ij} = \begin{bmatrix} \Xi_{ij} & 0 \\ * & 0 \end{bmatrix}$, $\Psi_{ij} = \begin{bmatrix} 0 & \Pi_{ij} \\ * & -\mathcal{Q}_{ij} \end{bmatrix}$. Then, from (28) we can know that $\dot{V}_{ij}(t) - \beta_iV_{ij}(t) \leq 0$, $t \in [t_k, t_k + T_k^a)$, which implies that (15) holds.

Next, consider the switching instant t_k. First, consider the case that the controller and system are asynchronous at t_k^-. Multiplying both sides of (30) with H_i^{-1} gives that $P_{ci} - \mu_{j\to c}P_{ji} \leq 0$, $Q_{ci} - \mu_{j\to c}e^{-\theta(\beta_j,\xi_c)h}Q_{ji} \leq 0$. In general, we assume that the j^{th} subsystem and the i^{th} controller are activated at the instant before t_k, and the j^{th} subsystem switches to the c^{th} one at switching instant t_k, $i, j, c \in \mathcal{I}, j \neq i$, $j \neq c$. From (32) we have that

$$
\begin{aligned}
&V_{ci}(t_k)\\
&= (h(t_k) - d(t_k)) \int_{\tilde{t}_k}^{t_k} e^{-\xi_c(s-t_k)}\dot{x}^T(s)Q_{ci}\dot{x}(s)ds\\
&\quad + x^T(t_k)P_{ci}x(t_k)\\
&\leq \mu_{j\to c}(h(t_k) - d(t_k)) \int_{\tilde{t}_k}^{t_k} e^{-\xi_c(s-t_k)-\theta(\beta_j,\xi_c)h}\dot{x}^T(s)Q_{ji}\dot{x}(s)ds\\
&\quad + \mu_{j\to c}x^T(t_k)P_{ji}x(t_k)\\
&\leq \mu_{j\to c}V_{ji}(t_k^-),
\end{aligned}
$$

which implies that (18) holds. Then, consider the other case that the controller and the system are synchronous at t_k^-. Multiplying both sides of (31) with H_j^{-1} gives that $P_{ij} - \mu_{j\to i}P_{jj} \leq 0$, $Q_{ij} - \mu_{j\to i}e^{-(\alpha_j+\beta_i)h}Q_{jj} \leq 0$. In general, we assume that the j^{th} subsystem and controller are activated at t_k^-, and the j^{th} subsystem switches to the i^{th} one at switching instant t_k, $i, j \in \mathcal{I}, j \neq i$. From (32) we have that

$$
\begin{aligned}
&V_{ij}(t_k)\\
&= (h(t_k) - d(t_k)) \int_{\tilde{t}_k}^{t_k} e^{-\beta_i(s-t_k)}\dot{x}^T(s)Q_{ij}\dot{x}(s)ds\\
&\quad + x^T(t_k)P_{ij}x(t_k)\\
&\leq \mu_{j\to c}(h(t_k) - d(t_k)) \int_{\tilde{t}_k}^{t_k} e^{-\beta_i(s-t_k)-(\alpha_j+\beta_i)h}\dot{x}^T(s)Q_{jj}\dot{x}(s)ds\\
&\quad + \mu_{j\to i}x^T(t_k)P_{jj}x(t_k)\\
&\leq \mu_{j\to i}V_{jj}(t_k^-),
\end{aligned}
$$

which implies that (19) holds.

Finally, consider sampling instants $t_{k,l}$, $l = 1, 2, \ldots, n_k$ on $[t_k, t_{k+1})$, $k = 0, 1, 2, \ldots$. By multiplying both sides of (29) with $diag\{H_i^{-1}\ H_j^{-1}\}$ and using the schur complement lemma, we can get $P_{ii} \leq 2H_i^{-1} - H_i^{-1}P_{ij}^{-1}H_i^{-1} \leq P_{ij}$. First, consider the case that the controller and system are asynchronous at $t_{k,1}$. We assume that the i^{th} subsystem and the j^{th} controller are activated at $t_{k,1}$, $i, j \in \mathcal{I}$, $i \neq j$, then we can know that the j^{th} controller will switch to the i^{th} one at sampling instant t_k^1. From (32) we have that $V_{ii}(t_{k,1}) = x^T(t_{k,1})P_{ii}x(t_{k,1}) \leq x^T(t_{k,1})P_{ij}x(t_{k,1}) = V_{ij}(t_{k,1}^-)$, which implies that (16) holds. Then, consider the synchronous case. We assume that the i^{th} subsystem and controller are activated at $t_{k,1}$, then we can know that the mode of the controller and the system will not change at the sampling instant. From (32) we can get that $V_{ii}(t_{k,l}) = V_{ii}(t_{k,l}^-) = x^T(t_{k,l})P_{ii}x(t_{k,l})$, $l = 2, \ldots, n_k$, which implies that (17) holds. The proof is completed. □

Remark 5: For system (4), if we let the maximum sampling interval h be an infinitesimal, then the asynchronous phenomenon can be avoided, and the sample-data feedback control problem becomes a continuous-time synchronous feedback control problem, but the sampling cost will become very high. If we let h be very large, then less sampling points are needed, and the sampling cost is reduced, but the proportion of asynchronous time will also become larger, which may result in instability of the system. Hence we can let h be as large as possible as long as conditions (20) and (27)-(31) are satisfied.

Remark 6: The periodic sampled-data controller design results for system (4) with switching signals satisfying (20) can be derived from Theorem 2 directly. Moreover, the aperiodic sampled-data controller design results for switched linear systems without switching delay (i.e., the system signal is sent to the ZOH all the time) can also be derived from Theorem 2 directly. Hence they are omitted for brevity.

The objective of this paper is to find admissible switching signals and controller gains which can stabilize the system cooperatively. The following algorithm is given to show how the switching signals and controller gains are obtained via Theorems 1 and 2:

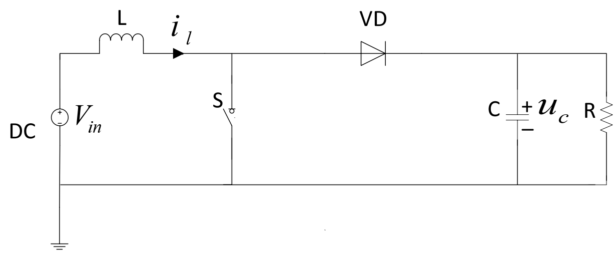
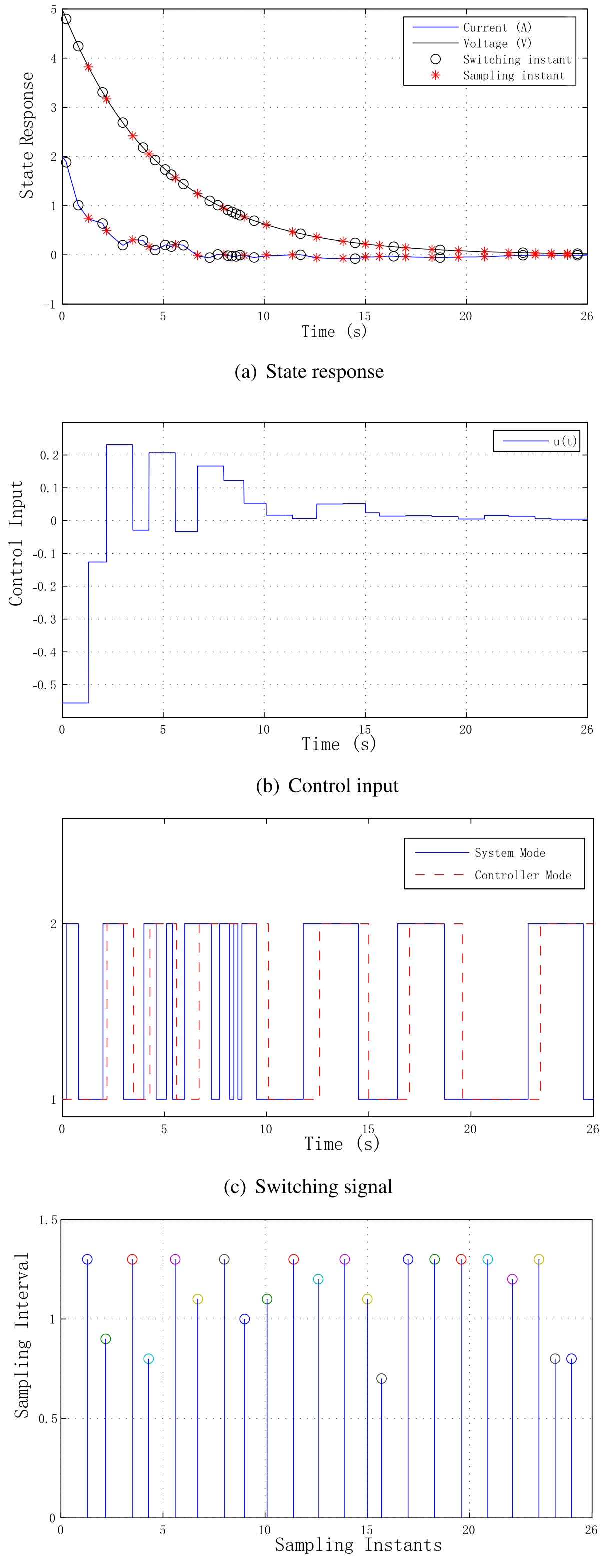


FIGURE 2. Boost converter circuit.

Algorithm 1 : Find Admissible Switching Signals and Controller Gains Which Can Stabilize the System Cooperatively

Step 1. Given a set of scalars $\alpha_i > 0$, $\beta_i > 0$, $\mu_{i\to j} > 1$, $i, j \in \mathcal{I}$, $i \neq j$.
Step 2. Check the LMIs in Theorem 2. If the conditions in Theorem 2 are feasible, record α_i, β_i, $\mu_{i\to j}$ and controller gains K_i, go to Step 3. Otherwise, change the values of α_i, β_i, $\mu_{i\to j}$, and return to Step 2.
Step 3. Define a set of signal parameters $\rho(t)$, $\upsilon_\rho(t)$, $\omega_{ij}(t)$, $\mathscr{T}_\rho^s(i, t)$ and $\mathscr{T}_\rho^a(i, t)$.
Step 4. Compute the values of $\hat{\upsilon}_\rho$, $\hat{\omega}_{ij}$, $\check{\mathscr{T}}_\rho^s(i, t)$ and $\hat{\mathscr{T}}_\rho^a(i, t)$ according to the signal parameters.
Step 5. If condition (20) holds, record $\rho(t)$, $\upsilon_\rho(t)$, $\omega_{ij}(t)$, $\mathscr{T}_\rho^s(i, t)$ and $\mathscr{T}_\rho^a(i, t)$, EXIT. Otherwise, change signal parameters $\upsilon_\rho(t)$, $\omega_{ij}(t)$, $\mathscr{T}_\rho^s(i, t)$ and $\mathscr{T}_\rho^a(i, t)$, and return to Step 4.

Remark 7: From Algorithm 1 we can see that the controller gains and switching signals are computed separately. The switching signals are designed after getting the controller gains K_i and the corresponding parameters α_i, β_i, $\mu_{i\to j}$. The switching condition (20) is related to these parameters. It is obvious that (20) can be satisfied more easily by employing



(a) State response

(b) Control input

(c) Switching signal

(d) Sampling signal

FIGURE 3. Simulation results for system (34).

larger α_i, $\mu_{i\to j}$ and smaller β_i. To obtain a larger switching signal set, we can let α_i and $\mu_{i\to j}$ be as large as possible, and let β_i be as small as possible.

IV. SIMULATION

The boost converter circuit shown in Fig. 2 can be modeled as a switched system [46]. V_{in} represents the source voltage, S represents the switching, while L, C and R represent the inductance, the capacitance and the load resistance, respectively. The circuit equation is shown as follows:

$$\begin{aligned} \dot{i}_l(t) &= -(1-S(t))\frac{1}{L}u_c(t) + S(t)\frac{1}{L}V_{in}, \\ \dot{u}_c(t) &= -\frac{1}{RC}u_c(t) + (1-S(t))\frac{1}{C}i_l(t), \end{aligned}$$

where $S(t)=1$ if the switching S is connecting, and $S(t)=0$ if S is disconnecting. Let $x^T(t) = [i_l(t)\ u_c(t)]$, $u(t) = V_{in}$, and choose $R = 60m\Omega$, $C = 80F$, $L = 5H$, then the circuit equation can be rewritten as follows:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \tag{34}$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 0 \\ 0 & -0.2083 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix} \\ A_2 &= \begin{bmatrix} 0 & -0.2000 \\ 0.0125 & -0.2083 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}. \end{aligned}$$

Next, a set of sampled-data controllers and an admissible switching signal will be designed to stabilize system (34) cooperatively. We solve out the controller gain matrices first. Denote $\alpha_1 = 0.2$, $\alpha_2 = 0.2$, $\beta_1 = 0.080$, $\beta_2 = 0.065$, $\mu_{1\to2} = 1.27$, $\mu_{2\to1} = 1.14$. With the help of the package YALMIP [47] and SDP solver SeDuMi [48], we find that feasible solutions for the conditions in Theorem 2 exist for $h \leq 1.32$. Let $h = 1.32$, we get the following controller gain matrices:

$$K_1 = [-1.9061\ 0.2065], \quad K_2 = [-1.8679\ 0.6569].$$

Then, we try to find an admissible switching signal which satisfies (20). Let $\rho(t)$ be identity function, and assume $\sigma(t)$ satisfies that

$$\begin{aligned} &N_\sigma^t = 0.6t + t^{0.575}, \ \omega_{12} = \omega_{21} = \frac{1}{2}, \\ &\mathcal{T}_\rho^s(1,t) = 0.303 + 0.225t^{-0.5} - 0.158t^{-0.7}, \\ &\mathcal{T}_\rho^s(2,t) = 0.378 - 0.225t^{-0.5} + 0.158t^{-0.7}, \\ &\mathcal{T}_\rho^a(1,t) = 0.180 + 0.12t^{-0.8} - 0.08t^{-0.5}, \\ &\mathcal{T}_\rho^a(2,t) = 0.139 - 0.12t^{-0.8} + 0.08t^{-0.5}. \end{aligned}$$

Then, according to (5)-(12) we have $\check{\upsilon}_\rho = 0.6$, $\check{\omega}_{12} = \hat{\omega}_{21} = \frac{1}{2}$, $\check{\mathcal{T}}_\rho^s(1) = 0.303$, $\check{\mathcal{T}}_\rho^s(2) = 0.378$, $\hat{\mathcal{T}}_\rho^a(1) = 0.180$, $\hat{\mathcal{T}}_\rho^a(2) = 0.139$. Substituting them into (20) gives that

$$\begin{cases} \check{\upsilon}_\rho = 0.6 > 0, \\ \hat{\upsilon}_\rho \displaystyle\sum_{(i,j)\in E(\mathcal{I})} \tilde{\mu}_{i\to j}\hat{\omega}_{ij} - \sum_{i\in\mathcal{I}} \alpha_i \check{\mathcal{T}}_\rho^s(i) + \sum_{i\in\mathcal{I}} \beta_i \hat{\mathcal{T}}_\rho^a(i) \\ = 0.1110 - 0.1128 = -0.0018 < 0. \end{cases}$$

This shows that condition (20) is satisfied.

Choose a period of time $t = 26$, the we have that $N_\sigma^t = 22$, $\mathcal{T}_\rho^s(1, t) = 0.3310$, $\mathcal{T}_\rho^s(2, t) = 0.3500$, $\mathcal{T}_\rho^a(1, t) = 0.1732$, $\mathcal{T}_\rho^a(2, t) = 0.1458$. A possible execution of switching signal $\sigma(t)$ is shown in Fig. 3 (c), and the sampling signal is shown in Fig. 3 (d). Then, the switching signal of the controller can be determined and is also shown Fig. 3 (c). Let initial states be $x^T(0) = [2\ 5]$, Fig. 3 (a) and (b) show the state trajectory and the control input of system (34). It can be seen that the trajectories of the states and control input converge to zero as time goes on. This shows the designed sampled-data controller and switching signals can globally asymptotically stabilize the system cooperatively. Furthermore, from Fig. 3 (a) it can be seen that the switching happens more than once on some sampling intervals, which implies that the results in the literature [31]–[37] on sampled-data control of SLSs are not applicable in this example. Actually, the dwell time can be arbitrarily small as long as condition (20) can be satisfied, which means that the MDT constraints have been removed.

V. CONCLUSION

This paper has investigated the sampled-data control problem of asynchronously SLSs without MDT constraints. A novel class of switching signals and a set of sampled-data controllers have been designed to stabilize the system cooperatively. Sufficient condition on the existence of sampled-data controllers has been given in terms of LMIs. In contrast to the existing literature which requires the MDT of the system to be no smaller than the maximum sampling interval, this novel class of switching signals covers the traditional ADT switching signals, and needs no restrictions on MDT. The derived results have a better practicality since the case that no sampling happens on a dwell time interval is allowed. Future work focuses on the sampled-date control problem of more complex switched systems without MDT constraints.

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