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Observer-Based Dynamic Event-Triggered Robust H_∞ Control of Networked Control Systems Under DoS Attacks

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ABSTRACT This paper designs an observer-based controller with a dynamic event-triggered strategy for networked control systems with external disturbance, system uncertainty, and unknown periodic Denial-of-Service (DoS) attacks. First, the system under DoS attacks is modeled as active subsystem and dormant subsystem by using the switching system method. The controller is built based on dynamic event-triggered sequence and observer with the hold-input method used as the control signal for DoS attacks active subsystem. An augmented system is constructed with the time delay method, and the H_∞ performance index of the closed-loop switched system is given. Then, using Lyapunov functional theory and some technical lemmas, the sufficient conditions for the stability of the system are given without the disturbance, and with disturbance, the sufficient conditions for the closed-loop system with H_∞ performance index are deduced. Finally, a numerical example is given to verify the effectiveness of the theory. Through the minimum optimization problem, the H_∞ performance parameter is optimized to increase the disturbance suppression degree of the system.

INDEX TERMS Dynamic event-triggered scheme, networked control systems, denial-of-service attacks, uncertainty, H_∞ performance.

I. INTRODUCTION

In recent years, due to the rapid development of sensor and communication and computing technology, networked control systems have captured more and more attention. Networked control systems have advantages in information sharing, low installation cost, easy maintenance, and high flexibility. Thus, they are widely applied in many fields, including industrial control, smart grid, petrochemical, automobile device [1].

As introducing a communication network into the traditional system destroys the system's closure, the open industrial network is vulnerable to network attacks [2]. The studies in [3]–[5] demonstrate that the common security threats of networked control systems are DoS attacks, which attempt to transmit a large amount of invalid data and occupy limited network resources, leading to ineffective data transmission, thus affecting the regular work of the system. However, most of the studies of DoS attacks are focused on researching DoS attacks signal detection [6]–[8] and system security control [9]–[11]. The network attacks could be detected effectively by the attacks detection algorithm in [7], but missed signal detection is inevitable for the change of network attacks. Therefore, it is crucial to design the proper robust control strategy to preserve the stability of networked control systems with network attacks. In fact, the assumption that the energy of DoS attacks is limited is significant for researching the security of networked control systems. For one thing, in [12], based on the asymptotic stability of the system, it is proved that DoS attacks tolerated for the system are limited in energy and duration. For another, it is pointed out that DoS attacks are limited in power from the perspective of attackers in [13].

Actually, the researches of networked control systems are about network security and how to improve the utilization of network bandwidth. Some event-triggered strategies have been intensively worked on [14]–[17] to solve limited network system resources. Only when a specific event occurs can the control rate be updated, or the transmitted signal

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keeps the latest value. Compared with the traditional time-triggered strategy, the event-triggered strategy saves the bandwidth resource of the networked control systems in [15]. Research into event triggering should pay attention to the fact that the minimum interval time between events must be strictly greater than zero to avoid Zeno behavior. One way to overcome this problem is to employ an event-triggered strategy based on sampled data [16]. To further save bandwidth resources, [18] proposes a dynamic event-triggered mechanism. By introducing an additional variable, the event-triggered number is reduced while holding the stability of the system. A new dynamic event-triggered mechanism is designed in [19], and by setting the threshold value of the dynamic parameter, the parameter neither decays prematurely nor increases excessively, which generates fewer transmissions. The analysis and synthesis of networked control systems based on dynamic event triggering have also been discussed in [20]–[23]. Under the same control performance as the static counterpart, the dynamic event-triggered strategies generate fewer events. Besides, [19] researching dynamic event-triggered mechanism based on sampling instant can also avoid the Zeno behavior.

There are growing focus on event triggering and security control of networked control systems. In [24], using the active period and dormant period of DoS attacks, the resilient event-triggered strategy based on output is designed, and the strategy and DoS attacks are integrated into a unified framework by switching system theory. In [25], considering the practical engineering applications, and the state of the system is hard to get all, the networked control systems with periodic DoS attacks based on state observer are studied for resilient event-triggered control, and the observer and controller of the system are designed collaboratively. Although some static event-triggered methods have been developed with the DoS attacks, only a few results considered dynamic event-triggered strategy into account. In [26], resilient dynamic event-triggered strategy is designed to study the control problem of power systems under DoS attacks and transmission delay. Based on [25], to reduce the control update frequency, the parameters of dynamic event triggering, observer, and controller are designed collaboratively in [27]. Considering that the information transmission between the sensor and the controller may be subject to asynchronous DoS attacks, [28] proposes a resilient control method for dynamic event triggering, and the designed dynamic event-triggered strategy generates fewer events than static event-triggered strategy.

To sum up, this paper mainly assumes that DoS attacks energy are limited. Compared with [25], this paper discusses more common attacks with the unknown period, while the above resilient control strategy mainly adopts the zero-input method to compensate for the impact of DoS attacks on the system in [25]–[28]. Considering that the performance of the hold-input strategy may be better for the compensation of the unstable system, this paper takes the hold-input method to design the controller and studies the secure dynamic event-triggered control of the networked control systems with uncertainty and external disturbance. The novelty of this paper lies in two aspects:

1) Different from [25], [28], [29], considering a more complex networked control systems with DoS attacks, external disturbance, and uncertainty, sufficient conditions are given for robust asymptotic stability of the system without disturbance, and with disturbance, the sufficient condition for the closed-loop system with H_∞ performance index is deduced.
2) Comparing with results in [28], by introducing an integral inequality to deal with the integrals with derivatives in Lyapunov-Krasovskii functional, a low conservatism condition is obtained with an additional parameter, using the hold-input strategy to realize faster and more smoothly the stability of the system.

Notation: $\mathbf{R}^n$ denotes the n-dimensional Euclidean space and $\mathbf{R}^{n\times m}$ is the set of $n \times m$ real matrices; the superscript T stands for matrix transposition; the symbol $*$ represents each of the symmetric blocks; $\mathcal{L}_2[0,\infty)$ denotes the space of square-integrable vector functions defined on $[0,\infty)$; I and 0 represent identify matrix and zero matrix with proper dimensions, respectively.

II. PROBLEM FORMULATION AND MODELING

Consider a class of networked control systems with uncertainty and disturbance:

$$\begin{cases}\dot{x}(t) = [A + \Delta A]x(t) + [B_1 + \Delta B]\,u(t) + Bw(t), \\ y(t) = Cx(t),\end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the plant state, $y(t) \in \mathbf{R}^q$ is the plant output, $u(t) \in \mathbf{R}^m$ is the control input, $w(t) \in \mathbf{R}^p$ denotes the disturbance input belonging to $\mathcal{L}_2[0,\infty)$, A, B_1, B, C are the known parameter matrices with suitable dimensions, and $\Delta A, \Delta B$ are the unknown time-variant matrices representing norm-bounded parameter uncertainties, which are assumed to be of the form $\begin{bmatrix}\Delta A & \Delta B\end{bmatrix} = HF(t)\begin{bmatrix}E_1 & E_2\end{bmatrix}$, where H, E_1 and E_2 are known constant matrices with suitable dimensions, and $F(t)$ is an unknown time-varying matrix function, which satisfies the inequality condition $F^T(t)F(t) \leq I$ for all t.

In Fig.1, the unknown periodic DoS attacks can be modeled as the following switching signals

$$D(t) = \begin{cases}1, & t \in [g_n, g_n + h_n)\,, \\ 0, & t \in [g_n + h_n, g_{n+1})\,,\end{cases} \tag{2}$$

where $[g_n, g_n + h_n)$ denotes the sleep interval of the *nth* DoS attacks with normal transmission, and $[g_n + h_n, g_{n+1})$ denotes the active interval of the *nth* DoS attacks with interrupted transmission. $\{h_n\}$ represents the time instants when DoS attacks are sleeping, and $\{g_n + h_n\}$ represents the active start time of the *nth* DoS attacks which end at g_{n+1}.

In order to simplify the derivation, let $\mathrm{H}_{1,n} = [g_n, g_n + h_n)$, $\mathrm{H}_{2,n} = [g_n + h_n, g_{n+1})$, $t_{1,n} = g_n$, $t_{2,n} = g_n + h_n$.

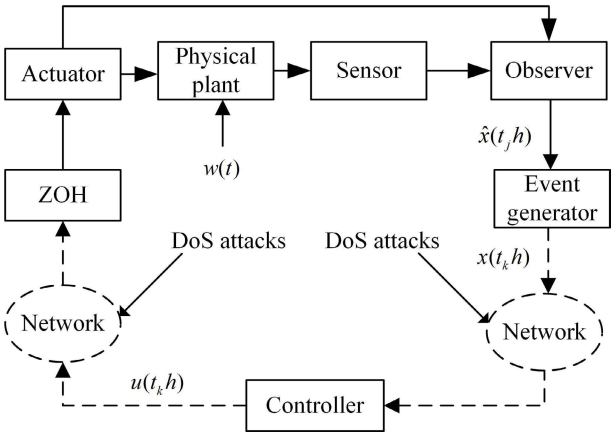

FIGURE 1. Observer-based dynamic event-triggered control for the network system with unknown periodic DoS attacks.

Assumption 1: Same as [29], $n(t)$ is the number of the positive edge triggering of DoS attacks in the current period $[t, \infty)$. There are two real scalars $\nu \geq 0$ and $\Gamma > 0$ for all $t \geq 0$ that satisfy

$$n(t) \leq \nu + \frac{t}{\Gamma}. \tag{3}$$

Assumption 2: Same as [29], $\Xi(t)$ is the total duration of DoS attacks within $n(t)$ of the current positive edge triggering. There are two real scalars $\omega \geq 0$ and $\Gamma_D > 0$ for all $t \geq 0$ that satisfy

$$|\Xi(t)| \leq \omega + \frac{t}{\Gamma_D}. \tag{4}$$

Considering the following full-dimensional state observer for system (1)

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(t)[\hat{y}(t) - y(t)], \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{5}$$

where $\hat{x} \in \mathbf{R}^n$ is the observer state, $\hat{y} \in \mathbf{R}^q$ is the observer output, and $L(t) \in \mathbf{R}^{n\times q}$ is the observer gain to be designed as

$$L(t) = \begin{cases} L_1, & t \in \mathrm{H}_{1,n}, \\ L_2, & t \in \mathrm{H}_{2,n}. \end{cases}$$

From Fig.1, $h > 0$ is the sampling period, $t_j h$ is the sampling time, and $t_k h$ is the latest trigger time, $t_j, t_k \in \mathbf{N}$. During the nth dormant DoS attacks, the jth sampling is defined as t_j^n, $n, t_j^n, j \in \mathbf{N}$, and $t_j^n h$ is the current sampling time. If $t_j^n h$ satisfies (6) as follows

$$\begin{aligned} &\theta\eta\left(t_k h\right) + \sigma\hat{x}^T\left(t_k h\right)\Omega\hat{x}\left(t_k h\right) \\ &\quad - \left[\hat{x}\left(t_j^n h\right) - \hat{x}\left(t_k h\right)\right]^T \Omega \left[\hat{x}\left(t_j^n h\right) - \hat{x}\left(t_k h\right)\right] < 0, \end{aligned} \tag{6}$$

the data will be transmitted and the trigger time will be undated.

To compensate for the impact of the system from the active DoS attacks, we designed the data transmitted at g_{n+1} which represents the end of the nth DoS attacks. Therefore, the transmission instant of observer state $\hat{x}$ is determined by dynamic event generator which is as follows

$$t_{k,n}h \in \left\{t_j^n h \ \ satisfying\ (6) \mid t_j^n h \in \mathrm{H}_{1,n}\right\} \bigcup \{g_{n+1}\}. \tag{7}$$

In (6), $\theta > 0$, $\sigma \in (0, 1)$, Ω is a positive definite symmetric matrix to be designed, and the dynamic parameter $\eta(t)$ satisfies

$$\dot{\eta}(t) = \begin{cases} -\beta(\eta(t)) - \theta\eta\left(t_k h\right) \\ \quad +\hat{x}^T\left(t_k h\right)\Lambda\hat{x}\left(t_k h\right), \eta(t) < \tilde{\eta}, \\ \beta(\eta(t)) - \theta\eta\left(t_k h\right), \eta(t) \geq \tilde{\eta}, \end{cases} \tag{8}$$

where $t \in [t_{k,n}h, t_{k+1,n}h)$, Λ is a positive definite weighted matrix to be designed, β is a function of class K_∞ and satisfies Lipschitz continuity, and $\tilde{\eta}$ is an adjustable parameter according to the actual condition.

Remark 1: In order to avoid Zeno behavior, the designed trigger condition is based on the sampling time of the system, so the minimum event-triggered interval of the system is an integer multiple of the sampling period.

Lemma 1 [19]: Suppose β is a K_∞ class function and satisfies Lipschitz continuity, let $\sigma \in (0, 1)$, $\eta_0 > 0$, $\theta > 0$. If h, θ satisfy $h \leq -\frac{1}{\lambda}\ln\frac{\theta}{\lambda+\theta}$, $\eta > 0$ for any $t \in [0, \infty)$.

Considering the switching signals of DoS attacks, the controller of the system (1) is designed as follows

$$u(t) = \begin{cases} K_1\hat{x}\left(t_{k,n}h\right), & t \in \left[t_{k,n}h, t_{k+1,n}h\right) \cap \mathrm{H}_{1,n}, \\ K_2\hat{x}\left(t_{k(n),n}h\right), & t \in \left[t_{k(n),n}h, t_{k(n)+1,n}h\right) \cap \mathrm{H}_{2,n}, \end{cases} \tag{9}$$

where K_1, K_2 are the controller gain matrices, the sequence $t_{k,n}h$ is determined by iteration of the trigger condition (7), $t_{0,n} = g_n$, $h > 0$, $k \in \{0, 1, \cdots, k(n)\} = K(n)$, $k(n) = \sup\left\{k \in N \mid t_{k,n} \leq g_n + h_n\right\}$, $t_{k(n)+1,n}h > g_n + h_n$. From the above definition, it can be known that $t_{k(n),n}h$ is the $k(n)th$ normal triggering within the time of the nth dormant DoS attacks.

In order to simplify the derivation, similar to [25], let $R_{k,n} = \left[t_{k,n}h, t_{k+1,n}h\right)$. Substitute (9) into (1), we can obtain

$$\begin{cases} \dot{x}(t) = [A + \Delta A]x(t) + [B_1 + \Delta B]K_1\hat{x}\left(t_{k,n}h\right) \\ \qquad +B_2 w(t), \quad t \in R_{k,n} \cap \mathrm{H}_{1,n}, \\ \dot{x}(t) = [A + \Delta A]x(t) + [B_1 + \Delta B]K_2\hat{x}\left(t_{k(n),n}h\right) \\ \qquad +B_2 w(t), \quad t \in R_{k(n),n} \cap \mathrm{H}_{2,n}. \end{cases} \tag{10}$$

$R_{k,n}$ is divided into inter cell according to sampling interval as follows

$$R_{k,n} = \bigcup_{m=1}^{\rho_{k,n}} \left[t_{k,n}h + (m-1)h, t_{k,n}h + mh\right), \tag{11}$$

where $k \in K(n)$, $n \in N$, $\rho_{k,n} = \inf\{m \in N \mid t_{k,n}h + mh \geq t_{k+1,n}h\}$.

For $k \in K(n)$, $n \in N$, same as [29], two piecewise functions are defined,

$$\tau_{k,n}(t) = \begin{cases} t - t_{k,n}h, t \in \left[t_{k,n}h, t_{k,n}h + h\right), \\ t - t_{k,n}h - h, \\ \quad t \in \left[t_{k,n}h + h, t_{k,n}h + 2h\right), \\ \vdots \\ t - t_{k,n}h - \left(\rho_{k,n-1}\right)h, \\ \quad t \in \left[t_{k,n}h + \left(\rho_{k,n-1}\right)h, t_{k+1,n}h\right), \end{cases} \tag{12}$$

$$e_{k,n}(t) = \begin{cases} 0, t \in \left[t_{k,n}h, t_{k,n}h + h\right), \\ \hat{x}\left(t_{k,n}h\right) - \hat{x}\left(t_{k,n}h + h\right), \\ \quad t \in \left[t_{k,n}h + h, t_{k,n}h + 2h\right), \\ \vdots \\ \hat{x}\left(t_{k,n}h\right) - \hat{x}\left(t_{k,n}h + \left(\rho_{k,n-1}\right), h\right), \\ \quad t \in \left[t_{k,n}h + \left(\rho_{k,n-1}\right)h, t_{k+1,n}h\right), \end{cases} \tag{13}$$

it can be concluded that $\tau_{k,n}(t) \in [0, h), t \in R_{k,n}$, and $\hat{x}\left(t_{k,n}h\right) = \hat{x}\left(t - \tau_{k,n}(t)\right) + e_{k,n}(t), t \in R_{k,n}$.

The error state is defined as $e(t) = \hat{x}(t) - x(t)$, and the augmented state is $\xi^T(t) = \left[x^T(t)\ \ e^T(t)\right]$, then the augmented system is derived as follows:

$$\begin{cases} \dot{\xi}(t) = A_1\xi(t) + \bar{B}_1\xi\left(t - \tau_{k,n}(t)\right) + \tilde{B}_1 e_{k,n}(t) \\ \qquad + \bar{B}w(t), \quad t \in R_{k,n} \cap \mathrm{H}_{1,n}, \\ \dot{\xi}(t) = A_2\xi(t) + \bar{B}_2\xi\left(t - \tau_{k(n),n}(t)\right) + \tilde{B}_2 e_{k(n),n}(t) \\ \qquad + \bar{B}w(t), t \in R_{k(n),n} \cap \mathrm{H}_{2,n}, \\ \xi(t) = \phi(t), \quad t \in [-h, 0), \end{cases} \tag{14}$$

where

$$A_1 = \begin{bmatrix} A + \Delta A & 0 \\ \Delta A & A - L_1C \end{bmatrix},$$
$$A_2 = \begin{bmatrix} A + \Delta A & 0 \\ \Delta A & A - L_2C \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ -B \end{bmatrix},$$
$$\tilde{B}_1 = \begin{bmatrix} BK_1 + \Delta BK_1 \\ \Delta BK_1 \end{bmatrix}, \quad \tilde{B}_2 = \begin{bmatrix} BK_2 + \Delta BK_2 \\ \Delta BK_2 \end{bmatrix},$$
$$\bar{B}_1 = \begin{bmatrix} BK_1 + \Delta BK_1 & BK_1 + \Delta BK_1 \\ \Delta BK_1 & \Delta BK_1 \end{bmatrix},$$
$$\bar{B}_2 = \begin{bmatrix} BK_2 + \Delta BK_2 & BK_2 + \Delta BK_2 \\ \Delta BK_2 & \Delta BK_2 \end{bmatrix}.$$

In this paper, we design a robust dynamic event-triggered controller (9) based on the observer (5) for the networked control systems (1) with unknown periodic DoS attacks (2) so that the closed-loop switched system (14) satisfies the following two conditions:

1) When the external disturbance $w(t) = 0$, the closed-loop switched system (14) is robust asymptotically stable.
2) When the external disturbance $w(t) \neq 0$, the system in its initial state $x(t) = 0$ satisfies the following H_∞ performance

$$\|y(t)\|_2 \le \gamma \|w(t)\|_2, \quad \forall w(t) \in L_2[0, \infty). \tag{15}$$

In order to facilitate, the lemmas are given for the treatment of uncertainty and integral item in Lyapunov functional.

Lemma 2 [30]: If there are two known matrices D and E of proper dimensions and $F(t)$ satisfies $F^T(t)F(t) \le I$, there is a scalar $\lambda > 0$ such that the following inequality holds

$$DF(t)E + E^T F^T(t) D^T \le \lambda DD^T + \lambda^{-1}E^T E.$$

Lemma 3 [31]: For constant matrices $T, R = R^T > 0$, scalars $d_1 \le d(t) \le d_2$, a vector function $\dot{x} : [-d_2, -d_1] \to \mathbb{R}^n$ such that the integration in the following inequality is well defined, then it holds that

$$(d_1 - d_2)\int_{t-d_2}^{t-d_1} \dot{x}^T(\alpha)R\dot{x}(\alpha)d\alpha \le \vartheta^T(t)\begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix}^T W \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix}\vartheta(t), \tag{16}$$

where $\vartheta^T(t) = \left[x^T(t-d_1)\ \ x^T(t-d(t))\ \ x^T(t-d_2)\right]$, $W = \begin{bmatrix} -R & T \\ * & -R \end{bmatrix} \le 0$.

III. MAIN RESULTS

Lemma 4: Given fixed DoS attacks parameters $\nu \ge 0, \Gamma \ge 0$, $\omega \ge 0$, $\Gamma_D \ge 0$, the feedback control gain matrices K_1, K_2, the observer gain matrices L_1, L_2, and the parameters $\alpha_1 > 0$, $\alpha_2 > 0$, $\sigma > 0$, $h > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, if there exist positive definite symmetric matrices $P_1 > 0$, $Q_1 > 0$, $R_1 > 0$, $P_2 > 0$, $Q_2 > 0$, $R_2 > 0$, $\Omega_1 > 0$, $\Lambda > 0$, $\Omega_2 > 0$ and matrices T_1, T_2 of appropriate dimensions, such that the following matrix inequalities

$$\hat{\Phi}_1 = \begin{bmatrix} \hat{\Phi}_{11}^1 & * & * & * \\ hR_1\hat{F}_1 & -R_1 & * & * \\ \hat{E}_1 & 0 & -\varepsilon_1 I & * \\ 0 & 0 & \hat{H}^T & -\varepsilon_1^{-1}I \end{bmatrix} < 0, \tag{17}$$

$$\hat{\Phi}_2 = \begin{bmatrix} \hat{\Phi}_{11}^2 & h\hat{F}_2^T R_2 & \hat{E}_2^T & 0 \\ hR_2\hat{F}_2 & -R_2 & 0 & 0 \\ \hat{E}_2 & 0 & -\varepsilon_2 I & \hat{H} \\ 0 & 0 & \hat{H}^T & -\varepsilon_2^{-1}I \end{bmatrix} < 0, \tag{18}$$

where

$$\hat{F}_1 = \begin{bmatrix} \hat{F}_{11} & \hat{F}_{12} & B_1K_1 & B_1K_1 & 0 & 0 & B_1K_1 \\ \hat{F}_{13} & \hat{F}_{14} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\hat{F}_{11} = A + \varepsilon_1 HH^T P_{11}, \quad \hat{F}_{12} = \varepsilon_1 HH^T P_{12},$$
$$\hat{F}_{13} = \varepsilon_1 HH^T P_{11}, \quad \hat{F}_{14} = A - L_1C + \varepsilon_1 HH^T P_{12},$$
$$\hat{E}_1 = \left[E_1\ \ 0\ \ E_2K_1\ \ E_2K_1\ \ 0\ \ 0\ \ E_2K_1\right],$$
$$\hat{F}_2 = \begin{bmatrix} \hat{F}_{21} & \hat{F}_{22} & B_2K_2 & B_2K_2 & 0 & 0 & B_2K_2 \\ \hat{F}_{23} & \hat{F}_{24} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\hat{F}_{21} = A + \varepsilon_2 HH^T P_{21}, \quad \hat{F}_{22} = \varepsilon_2 HH^T P_{22},$$
$$\hat{F}_{23} = \varepsilon_2 HH^T P_{21}, \quad \hat{F}_{24} = A - L_2C + \varepsilon_2 HH^T P_{22},$$
$$\hat{H} = \left[H^T\ \ H^T\right]^T,$$

$$\hat{\Phi}^1_{11}=\begin{bmatrix}\hat{\Gamma}^1_{11} & * & * & * & * & * & * \\ \hat{\Gamma}^1_{21} & \hat{\Gamma}^1_{22} & * & * & * & * & * \\ \hat{\Gamma}^1_{31} & -T^T_{13} & \hat{\Gamma}^1_{33} & * & * & * & * \\ \hat{\Gamma}^1_{41} & \hat{\Gamma}^1_{42} & \hat{\Gamma}^1_{43} & \hat{\Gamma}^1_{44} & * & * & * \\ T^T_{11} & T^T_{13} & \hat{\Gamma}^1_{53} & -T^T_{13} & \hat{\Gamma}^1_{55} & * & * \\ T^T_{12} & T^T_{14} & -T^T_{12} & \hat{\Gamma}^1_{64} & 0 & \hat{\Gamma}^1_{66} & * \\ \hat{\Gamma}^1_{71} & 0 & \sigma\Omega_1 & \sigma\Omega_1 & 0 & 0 & \hat{\Gamma}^1_{77}\end{bmatrix},$$

where

$$\begin{aligned}\hat{\Gamma}^1_{11} &= 2\alpha_1 P_{11} + He\{P_{11}A\} + Q_{11} - e^{-2\alpha_1 h}R_{11} \\ &\quad + \varepsilon_1 P_{11}HH^T P_{11}, \\ \hat{\Gamma}^1_{21} &= \varepsilon_1 P_{12}HH^T P_{11}, \\ \hat{\Gamma}^1_{31} &= K_1^T B^T P_{11} - T^T_{11} + e^{-2\alpha_1 h}R_{11}, \\ \hat{\Gamma}^1_{41} &= K_1^T B^T P_{11} - T^T_{12}, \quad \hat{\Gamma}^1_{71} = K_1^T B_1^T P_{11}, \\ \hat{\Gamma}^1_{22} &= 2\alpha_1 P_{12} + He\{P_{12}A - P_{12}L_1C\} + Q_{12} \\ &\quad - e^{-2\alpha_1 h}R_{12} + \varepsilon_1 P_{12}HH^T P_{12}, \\ \hat{\Gamma}^1_{33} &= \sigma\Omega_1 + \Lambda + He\left\{T_{11} - e^{-2\alpha_1 h}R_{11}\right\}, \\ \hat{\Gamma}^1_{42} &= \hat{\Gamma}^1_{64} = e^{-2\alpha_1 h}R_{12} - T^T_{14}, \\ \hat{\Gamma}^1_{43} &= \sigma\Omega_1 + \Lambda + T^T_{12} + T_{13}, \\ \hat{\Gamma}^1_{44} &= \sigma\Omega_1 + \Lambda + He\left\{T_{14} - e^{-2\alpha_1 h}R_{12}\right\}, \\ \hat{\Gamma}^1_{53} &= e^{-2\alpha_1 h}R_{11} - T^T_{11}, \\ \hat{\Gamma}^1_{55} &= -e^{-2\alpha_1 h}Q_{11} - e^{-2\alpha_1 h}R_{11}, \\ \hat{\Gamma}^1_{66} &= -e^{-2\alpha_1 h}Q_{12} - e^{-2\alpha_1 h}R_{12}, \quad \hat{\Gamma}^1_{77} = (\sigma-1)\Omega_1,\end{aligned}$$

and

$$\hat{\Phi}^2_{11}=\begin{bmatrix}\hat{\Gamma}^2_{11} & * & * & * & * & * & * \\ 0 & \hat{\Gamma}^2_{22} & * & * & * & * & * \\ \hat{\Gamma}^2_{31} & -T^T_{23} & \hat{\Gamma}^2_{33} & * & * & * & * \\ \hat{\Gamma}^2_{41} & \hat{\Gamma}^2_{42} & \hat{\Gamma}^2_{43} & \hat{\Gamma}^2_{44} & * & * & * \\ T^T_{21} & T^T_{23} & \hat{\Gamma}^2_{53} & -T^T_{23} & \hat{\Gamma}^2_{55} & * & * \\ T^T_{22} & T^T_{24} & -T^T_{22} & \hat{\Gamma}^2_{64} & 0 & \hat{\Gamma}^2_{66} & * \\ \hat{\Gamma}^2_{71} & 0 & \sigma\Omega_2 & \sigma\Omega_2 & 0 & 0 & \hat{\Gamma}^2_{77}\end{bmatrix},$$

where

$$\begin{aligned}\hat{\Gamma}^2_{11} &= -2\alpha_2 P_{11} + He\{P_{21}A\} + Q_{21} - e^{2\alpha_2 h}R_{21} \\ &\quad + \varepsilon_2 P_{21}HH^T P_{21}, \\ \hat{\Gamma}^2_{31} &= K_2^T B^T P_{21} - T^T_{21} + e^{2\alpha_2 h}R_{21}, \\ \hat{\Gamma}^2_{41} &= K_2^T B^T P_{21} - T^T_{22}, \quad \hat{\Gamma}^2_{71} = K_2^T B_1^T P_{21}, \\ \hat{\Gamma}^2_{22} &= -2\alpha_2 P_{22} + He\{P_{22}A - P_{22}L_2C\} \\ &\quad + Q_{22} - e^{2\alpha_2 h}R_{22} + \varepsilon_2 P_{22}HH^T P_{22}, \\ \hat{\Gamma}^2_{33} &= \sigma\Omega_2 + He\left\{T_{21} - e^{-2\alpha_2 h}R_{21}\right\}, \\ \hat{\Gamma}^2_{42} &= \hat{\Gamma}^2_{64} = e^{2\alpha_2 h}R_{22} - T^T_{24}, \quad \hat{\Gamma}^2_{43} = \sigma\Omega_2 + T^T_{22} + T_{23}, \\ \hat{\Gamma}^2_{44} &= \sigma\Omega_2 + He\left\{T_{24} - e^{2\alpha_2 h}R_{22}\right\}, \\ \hat{\Gamma}^2_{53} &= e^{2\alpha_2 h}R_{21} - T^T_{21}, \quad \hat{\Gamma}^2_{55} = -e^{2\alpha_2 h}Q_{21} - e^{2\alpha_2 h}R_{21}, \\ \hat{\Gamma}^2_{66} &= -e^{2\alpha_2 h}Q_{22} - e^{2\alpha_2 h}R_{22}, \quad \hat{\Gamma}^2_{77} = (\sigma-1)\Omega_2,\end{aligned}$$

it can be obtained

$$W(t) \le \begin{cases} e^{-2\alpha_1(t-t_{1,n})}W_1\left(t_{1,n}\right), & t \in R_{k,n} \cap \mathrm{H}_{1,n}, \\ e^{2\alpha_2(t-t_{2,n})}W_2\left(t_{2,n}\right), & t \in R_{k(n),n} \cap \mathrm{H}_{2,n}. \end{cases} \tag{19}$$

Proof: An candidate Lyapunov-Krasovskii functional for the closed-loop switched system (14) is constructed as follows

$$W_i(t) = \begin{cases} V_i(t) + \eta(t), & t \in R_{k,n} \cap \mathrm{H}_{1,n}, i = 1, \\ V_i(t), & t \in R_{k(n),n} \cap \mathrm{H}_{2,n}, i = 2. \end{cases} \tag{20}$$

$\eta(t)$ is defined by (8), $\beta(\eta(t)) = -2\alpha_1\eta(t)$, and $V_i(t)$ is constructed as

$$\begin{aligned}V_i &= \xi^T(t)P_i\xi(t) \\ &\quad + \int_{t-h}^{t} \xi^T(s)e^{2(-1)^i\alpha_i(t-s)}Q_i\xi(s)ds \\ &\quad + h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{\xi}^T(s)e^{2(-1)^i\alpha_i(t-s)}R_i\dot{\xi}(s)dsd\theta,\end{aligned} \tag{21}$$

where $P_i > 0$, $Q_i > 0$, $R_i > 0$, $i \in \{1, 2\}$.

When $i = 1$, $w(t) = 0$,

$$W(t) = W_1(t), \tag{22}$$

by taking the derivative of $V_1(t)$ along the trajectory of the system (14) at $t \in R_{k,n} \cap \mathrm{H}_{1,n}$,

$$\begin{aligned}\dot{V}_1(t) &\le \dot{\xi}^T(t)P_1\xi(t) + \xi^T(t)P_1\dot{\xi}(t) + \xi^T(t)Q_1\xi(t) \\ &\quad - \xi^T(t-h)e^{-2\alpha_1 h}Q_1\xi(t-h) + h^2\dot{\xi}^T(t)R_1\dot{\xi}(t) \\ &\quad - h\int_{t-h}^{t}\dot{\xi}^T(s)e^{-2\alpha_1 h}R_1\dot{\xi}(s)ds,\end{aligned} \tag{23}$$

applying Lemma 3 to the integral term

$$-h\int_{t-h}^{t}\dot{\xi}^T(s)e^{-2\alpha_1 h}R_1\dot{\xi}(s)ds,$$

we can get

$$-h\int_{t-h}^{t}\dot{\xi}^T(s)e^{-2\alpha_1 h}R_1\dot{\xi}(s)ds \le \vartheta^T(t)Z_1\vartheta(t), \tag{24}$$

where

$$Z_1 = \begin{bmatrix} -e^{-2\alpha_1 h}R_1 & * & * \\ Z_{11} & Z_{12} & * \\ T_1^T & e^{-2\alpha_1 h}R - T_1^T & -e^{-2\alpha_1 h}R_1 \end{bmatrix},$$

$$Z_{11} = e^{-2\alpha_1 h}R_1 - T_1^T, \quad Z_{12} = He\left\{T_1 - e^{-2\alpha_1 h}R_1\right\},$$

$$\vartheta^T(t) = \begin{bmatrix}\xi^T(t) & \xi^T\left(t-\tau_{k,n}(t)\right) & \xi^T(t-h)\end{bmatrix}.$$

Substituting (24) for (23), we can get

$$\begin{aligned}\dot{V}_1(t) &\le -2\alpha_1 V_1(t) + 2\alpha_1\xi^T(t)P_1\xi(t) + \dot{\xi}^T(t)P_1\xi(t) \\ &\quad + \xi^T(t)P_1\dot{\xi}(t) + \xi^T(t)Q_1\xi(t) + h^2\dot{\xi}^T(t)R_1\dot{\xi}(t) \\ &\quad - \xi^T(t-h)e^{-2\alpha_1 h}Q_1\xi(t-h) + \vartheta^T(t)Z_1\vartheta(t).\end{aligned} \tag{25}$$

Define

$$\zeta^T(t) = \begin{bmatrix}\xi^T(t) & \xi^T\left(t-\tau_{k,n}(t)\right) & \xi^T(t-h) & e^T_{k,n}(t)\end{bmatrix}.$$

According to (8) and (25) we can obtain

$$\dot{W}_1(t) \le -2\alpha_1 V_1(t) - 2\alpha_1\eta(t) + \zeta^T(t)[\Phi_{11}^1 + h^2 F_1^T R_1 F_1]\zeta(t),$$

let $\Phi_1 = \Phi_{11}^1 + h^2 F_1^T R_1 F_1$, and $\Phi_1 < 0$. To apply Schur's Complement Lemma [30], we can obtain

$$\Phi_1 = \begin{bmatrix} \Phi_{11}^1 & * \\ hR_1F_1 & -R_1 \end{bmatrix} < 0, \tag{26}$$

where $F_1 = \begin{bmatrix} A_1 & \bar{B}_1 & 0 & \tilde{B}_1 \end{bmatrix}$ and

$$\Phi_{11}^1 = \begin{bmatrix} \Gamma_{11}^1 & * & * & * \\ \Gamma_{12}^1 & \Gamma_{13}^1 & * & * \\ T_1^T & \Gamma_{14}^1 & \Gamma_{15}^1 & * \\ \tilde{B}_1^T P_1 & \sigma\Omega_1 G & 0 & (\sigma-1)\Omega_1 \end{bmatrix} < 0,$$

where

$$\begin{aligned}
\Gamma_{11}^1 &= 2\alpha_1 P_1 + A_1^T P_1 + P_1 A_1 + Q_1 - e^{-2\alpha_1 h} R_1, \\
\Gamma_{12}^1 &= \bar{B}_1^T P_1 - T_1^T + e^{-2\alpha_1 h} R_1, \\
\Gamma_{13}^1 &= \sigma G^T \Omega_1 G + \Lambda + He\left\{T_1 - e^{-2\alpha_1 h} R_1\right\}, \\
\Gamma_{14}^1 &= e^{-2\alpha_1 h} R_1 - T_1^T, \quad \Gamma_{15}^1 = -e^{-2\alpha_1 h} Q_1 - e^{-2\alpha_1 h} R_1, \\
G &= \begin{bmatrix} I & I \end{bmatrix}.
\end{aligned}$$

To eliminate the uncertainty in Φ_1, we obtain $\Phi_1 \le \hat{\Phi}_1$, and the form of $\hat{\Phi}_1$ is shown in (17) of Lemma 4.

Obviously, if $\hat{\Phi}_1 < 0$ then $\Phi_1 < 0$, $\dot{W}_1(t) < -2\alpha_1 W_1(t)$, when $t \in R_{k,n} \cap \mathrm{H}_{1,n}$, we can obtain

$$W(t) \le e^{-2\alpha_1(t-t_{1,n})} W_1\left(t_{1,n}\right).$$

When $i = 2$, $w(t) = 0$,

$$W(t) = W_2(t), \tag{27}$$

in the same way by taking the derivative of $V_2(t)$ along the trajectory of the system (14) at $t \in R_{k(n),n} \cap \mathrm{H}_{2,n}$, we can obtain

$$\dot{W}_2(t) = \dot{V}_2(t) \le 2\alpha_2 V_2(t) + \zeta^T(t)\left[\Phi_{11}^2 + h^2 F_2^T R_2 F_2\right]\zeta(t),$$

let $\Phi_2 = \Phi_{11}^2 + h^2 F_2^T R_2 F_2$, taking Schur complement of $\Phi_2 < 0$, and it can be obtained to

$$\Phi_2 = \begin{bmatrix} \Phi_{11}^2 & * \\ hR_2F_2 & -R_2 \end{bmatrix} < 0, \tag{28}$$

where $F_2 = \begin{bmatrix} A_2 & \bar{B}_2 & 0 & \tilde{B}_2 \end{bmatrix}$, and

$$\Phi_{11}^2 = \begin{bmatrix} \Gamma_{11}^2 & * & * & * \\ \Gamma_{21}^2 & \Gamma_{22}^2 & * & * \\ T_2^T & \Gamma_{32}^2 & \Gamma_{33}^2 & * \\ \tilde{B}_2^T P_2 & \sigma\Omega_2 G & 0 & (\sigma-1)\Omega_2 \end{bmatrix} < 0,$$

where

$$\begin{aligned}
\Gamma_{11}^2 &= -2\alpha_2 P_2 + A_2^T P_2 + P_2 A_2 + Q_2 - e^{2\alpha_2 h} R_2, \\
\Gamma_{21}^2 &= \bar{B}_2^T P_2 - T_2^T + e^{2\alpha_2 h} R_2, \\
\Gamma_{22}^2 &= \sigma G^T \Omega_2 G + He\left\{T_2 - e^{2\alpha_2 h} R_2\right\}, \\
\Gamma_{32}^2 &= e^{2\alpha_2 h} R_2 - T_2^T, \quad \Gamma_{33}^2 = -e^{2\alpha_2 h} Q_2 - e^{2\alpha_2 h} R_2.
\end{aligned}$$

Similarly, the uncertainty of (28) is eliminated to obtain $\Phi_2 \le \hat{\Phi}_2$, and the form of $\hat{\Phi}_2$ is shown in (18) of Lemma 4.

Obviously, if $\hat{\Phi}_2 < 0$ then $\Phi_2 < 0$, $\dot{W}_2(t) < 2\alpha_2 W_2(t)$, when $t \in R_{k(n),n} \cap \mathrm{H}_{2,n}$, we can obtain

$$W(t) \le e^{2\alpha_2(t-t_{2,n})} W_2\left(t_{2,n}\right).$$

The proof is now completed.

Remark 2: Since the closed-loop switched system has time delay, Lyapunov-Krasovskii functional with the integral term is designed to deal with the delay effect of the system. In [25], the integral term is processed by the free weight matrix, and an additional constraint is introduced. In this paper, the integral term is processed by Lemma 3 in [31], and no redundant constraint is introduced, thus providing a low conservative condition.

(19) is a sufficient condition that will be used to prove the asymptotic stability of (14) in Theorem 1.

Theorem 1: Given fixed DoS attacks parameters $\nu \ge 0$, $\Gamma \ge 0$, $\omega \ge 0$, $\Gamma_D \ge 0$, feedback controller gain matrices K_1, K_2, observer gain matrices L_1, L_2, and parameters $\alpha_1 > 0$, $\alpha_2 > 0$, $\sigma > 0$, $h > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\mu_1 > 0$, $\mu_2 > 0$, if there exist positive definite symmetric matrices $P_1 > 0$, $Q_1 > 0$, $R_1 > 0$, $P_2 > 0$, $Q_2 > 0$, $R_2 > 0$, $\Omega_1 > 0$, $\Lambda > 0$, $\Omega_2 > 0$ and matrices T_1, T_2 of appropriate dimensions satisfying the inequalities (17) and (18), and satisfying the following conditions

$$\begin{cases} P_1 - \mu_2 P_2 \le 0, \\ Q_1 - \mu_2 Q_2 \le 0, \\ R_1 - \mu_2 R_2 \le 0, \end{cases} \tag{29}$$

$$\begin{cases} P_2 - \mu_1 e^{2(\alpha_1+\alpha_2)h} P_1 \le 0, \\ Q_2 - \mu_1 Q_1 \le 0, \\ R_2 - \mu_1 R_1 \le 0, \end{cases} \tag{30}$$

$$\frac{\ln(\mu_1\mu_2)/2(\alpha_1+\alpha_2) + h}{\Gamma} + \frac{1}{\Gamma_D} < \frac{\alpha_1}{\alpha_1+\alpha_2}, \tag{31}$$

then the closed-loop switched system (14) is robust asymptotically stable under unknown periodic DoS attacks (2).

Proof: From (29) and (30) we can get

$$\begin{cases} \mu_2 W_2\left(t_{1,n}^-\right) + \eta\left(t_{1,n}\right) \ge W_1\left(t_{1,n}\right), & t \in \left[t_{1,n}, t_{2,n}\right), \\ e^{2(\alpha_1+\alpha_2)h}\mu_1 W_1\left(t_{2,n}^-\right) \ge W_2\left(t_{2,n}\right), & t \in \left[t_{2,n}, t_{1,n+1}\right). \end{cases} \tag{32}$$

When $t \in [t_{1,n}, t_{2,n}]$, from the DoS attacks assumptions (3) and (4) we can obtained

$$\begin{aligned}
&W(t) \\
&\le e^{-2\alpha_1(t-t_{1,n})} W_1\left(t_{1,n}\right) \\
&\le \ldots \le e^{n(t)[2(\alpha_1+\alpha_2)h + \ln(\mu_1\mu_2) + (2\alpha_1+2\alpha_2)\Xi(t) - 2\alpha_1]t} W_1(0)
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{p=0}^{n(t)} e^{n(t-pT)[2(\alpha_1+\alpha_2)h+\ln(\mu_1\mu_2)+(2\alpha_1+2\alpha_2)\Xi(t-pT)]}\\
&\times e^{-2\alpha_1(t-pT)}\eta(pT).
\end{aligned}\tag{33}
$$

In the same way, when $t \in [t_{2,n}, t_{1,n+1}]$, we can get

$$
\begin{aligned}
&W(t)\\
&\leq e^{2\alpha_2(t-t_{2,n})} W_2\left(t_{2,n}\right) \leq \ldots\\
&\leq \frac{1}{\mu_2} e^{n(t)[2(\alpha_1+\alpha_2)h+\ln(\mu_1\mu_2)+(2\alpha_1+2\alpha_2)\Xi(t)-2\alpha_1]t} W_1(0)\\
&\quad+\frac{1}{\mu_2}\sum_{p=0}^{n(t)} e^{n(t-pT)[2(\alpha_1+\alpha_2)h+\ln(\mu_1\mu_2)+(2\alpha_1+2\alpha_2)\Xi(t-pT)]}\\
&\quad\times e^{-2\alpha_1(t-pT)}\eta(pT).
\end{aligned}\tag{34}
$$

In order to simplify, let $\mu = \begin{cases} 1, & t \in H_{1,n} \\ 1/\mu_2, & t \in H_{2,n} \end{cases}$, we can get

$$
W(t) \leq \mu\left[W_1(0)e^{\beta_1} + \sum_{p=0}^{n(t)} \eta(pT)e^{\bar{\beta}_1(pT)}\right] e^{\beta_2 t}, \tag{35}
$$

where

$$
\begin{aligned}
\beta_1 &= v(\ln(\mu_1\mu_2) + 2(\alpha_1+\alpha_2)h) + (2\alpha_1+2\alpha_2)\omega,\\
\bar{\beta}_1(pT) &= (v - \frac{pT}{\Gamma})(\ln(\mu_1\mu_2) + 2(\alpha_1+\alpha_2)h)\\
&\quad +(2\alpha_1+2\alpha_2)(\omega - \frac{pT}{\Gamma_D}),\\
\beta_2 &= t/\Gamma((\ln\mu_1\mu_2) + 2\left(\alpha_1+\alpha_2\right)h)\\
&\quad +t/\Gamma_D\left(2\alpha_1+2\alpha_2\right) - 2\alpha_1 t.
\end{aligned}
$$

From (31) and (35), when $t \to \infty$ we can get $\beta_2 \to 0$, and from (8) we can get $\eta(pT) < \infty$. To sum up, when $t \to \infty$, we can get $W(t) \to 0$. It can be known from Lyapunov functional theory that closed-loop switched system (14) is robust asymptotically stable.

The proof is now completed.

Theorem 2: Given fixed DoS attacks parameters $v \geq 0$, $\Gamma \geq 0$, $\omega \geq 0$, $\Gamma_D \geq 0$, if for some parameters $\alpha_1 > 0$, $\alpha_2 > 0$, $\sigma > 0$, $h > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\mu_1 > 0$, $\mu_2 > 0$, there exist positive definite symmetric matrices $X_1 \in \mathbf{R}^{2n\times 2n} > 0$, $\bar{Q}_1 \in \mathbf{R}^{2n\times 2n} > 0$, $\bar{R}_1 \in \mathbf{R}^{2n\times 2n} > 0$, $X_2 \in \mathbf{R}^{2n\times 2n} > 0$, $\bar{Q}_2 \in \mathbf{R}^{2n\times 2n} > 0$, $\bar{R}_2 \in \mathbf{R}^{2n\times 2n} > 0$, $\bar{\Omega}_1 \in \mathbf{R}^{n\times n} > 0$, $\bar{\Lambda} \in \mathbf{R}^{n\times n} > 0$, $\bar{\Omega}_2 \in \mathbf{R}^{n\times n} > 0$ and matrices $\bar{T}_1, \bar{T}_2$ of appropriate dimensions satisfying the inequality (31), and satisfying the following LMIs

$$
\bar{\Phi}_1 = \begin{bmatrix}
\bar{\Phi}_{11}^1 & * & * & * & * \\
h\bar{F}_1 & \lambda_1^2\bar{R}_1 - 2\lambda_1 X_1 & * & * & * \\
\bar{E}_1 & 0 & -\varepsilon_1 I & * & * \\
0 & \bar{H}^T & 0 & -\varepsilon_1^{-1} I & * \\
\bar{C} & 0 & 0 & 0 & -I
\end{bmatrix} < 0, \tag{36}
$$

$$
\bar{\Phi}_2 = \begin{bmatrix}
\bar{\Phi}_{11}^2 & * & * & * & * \\
h\bar{F}_2 & \lambda_2^2\bar{R}_2 - 2\lambda_2 X_2 & * & * & * \\
\bar{E}_2 & 0 & -\varepsilon_2 I & * & * \\
0 & \bar{H}^T & 0 & -\varepsilon_2^{-1} I & * \\
\bar{C} & 0 & 0 & 0 & -I
\end{bmatrix} < 0, \tag{37}
$$

$$
\begin{bmatrix} -\mu_2 X_2 & * \\ X_2 & -X_1 \end{bmatrix} \leq 0, \tag{38}
$$

$$
\begin{bmatrix} -\mu_1 e^{2(\alpha_1+\alpha_2)h} X_1 & * \\ X_1 & -X_2 \end{bmatrix} \leq 0, \tag{39}
$$

$$
\begin{bmatrix} -\mu_2 \bar{Q}_2 & * \\ X_2 & \lambda_3^2\bar{Q}_1 - 2\lambda_3 X_1 \end{bmatrix} \leq 0, \tag{40}
$$

$$
\begin{bmatrix} -\mu_2 \bar{R}_2 & * \\ X_2 & \lambda_1^2\bar{R}_1 - 2\lambda_1 X_1 \end{bmatrix} \leq 0, \tag{41}
$$

$$
\begin{bmatrix} -\mu_1 \bar{Q}_1 & * \\ X_1 & \lambda_4^2\bar{Q}_2 - 2\lambda_4 X_2 \end{bmatrix} \leq 0, \tag{42}
$$

$$
\begin{bmatrix} -\mu_1 \bar{R}_1 & * \\ X_1 & \lambda_2^2\bar{R}_2 - 2\lambda_2 X_2 \end{bmatrix} \leq 0, \tag{43}
$$

where

$$
\bar{\Phi}_{11}^1 = \begin{bmatrix}
\bar{\Gamma}_{11}^1 & * & * & * & * & * & * & * \\
\bar{\Gamma}_{21}^1 & \bar{\Gamma}_{22}^1 & * & * & * & * & * & * \\
\bar{\Gamma}_{31}^1 & -\bar{T}_{13}^T & \bar{\Gamma}_{33}^1 & * & * & * & * & * \\
\bar{\Gamma}_{41}^1 & \bar{\Gamma}_{42}^1 & \bar{\Gamma}_{43}^1 & \bar{\Gamma}_{44}^1 & * & * & * & * \\
\bar{T}_{11}^T & \bar{T}_{13}^T & \bar{\Gamma}_{53}^1 & -\bar{T}_{13}^T & \bar{\Gamma}_{55}^1 & * & * & * \\
\bar{T}_{12}^T & \bar{T}_{14}^T & -\bar{T}_{12}^T & \bar{\Gamma}_{64}^1 & 0 & \bar{\Gamma}_{66}^1 & * & * \\
\bar{\Gamma}_{71}^1 & 0 & \sigma\bar{\Omega}_1 & \sigma\bar{\Omega}_1 & 0 & 0 & \bar{\Gamma}_{77}^1 & * \\
B^T & -B^T & 0 & 0 & 0 & 0 & 0 & \bar{\Gamma}_{88}^1
\end{bmatrix},
$$

$$
\begin{aligned}
\bar{F}_1 &= \begin{bmatrix} \bar{F}_{11} & \bar{F}_{12} & B_1Y_1 & B_1Y_1 & 0 & 0 & B_1Y_1 & B \\ \bar{F}_{13} & \bar{F}_{14} & 0 & 0 & 0 & 0 & 0 & -B \end{bmatrix},\\
\bar{E}_1 &= \begin{bmatrix} E_1X_{11} & 0 & E_2Y_1 & E_2Y_1 & 0 & 0 & E_2Y_1 & 0 \end{bmatrix},\\
\bar{H} &= \begin{bmatrix} H^T & H^T \end{bmatrix}^T,
\end{aligned}
$$

$$
\bar{\Phi}_{11}^2 = \begin{bmatrix}
\bar{\Gamma}_{11}^2 & * & * & * & * & * & * & * \\
\bar{\Gamma}_{21}^2 & \bar{\Gamma}_{22}^2 & * & * & * & * & * & * \\
\bar{\Gamma}_{31}^2 & -\bar{T}_{23}^T & \bar{\Gamma}_{33}^2 & * & * & * & * & * \\
\bar{\Gamma}_{41}^2 & \bar{\Gamma}_{42}^2 & \bar{\Gamma}_{43}^2 & \bar{\Gamma}_{44}^2 & * & * & * & * \\
\bar{T}_{21}^T & \bar{T}_{23}^T & \bar{\Gamma}_{53}^2 & -\bar{T}_{23}^T & \bar{\Gamma}_{55}^2 & * & * & * \\
\bar{T}_{22}^T & \bar{T}_{24}^T & -\bar{T}_{22}^T & \bar{\Gamma}_{64}^2 & 0 & \bar{\Gamma}_{66}^2 & * & * \\
\bar{\Gamma}_{71}^2 & 0 & \sigma\bar{\Omega}_2 & \sigma\bar{\Omega}_2 & 0 & 0 & \bar{\Gamma}_{77}^2 & * \\
B^T & -B^T & 0 & 0 & 0 & 0 & 0 & \bar{\Gamma}_{88}^2
\end{bmatrix},
$$

$$
\begin{aligned}
\bar{F}_2 &= \begin{bmatrix} \bar{F}_{21} & \bar{F}_{22} & B_1Y_2 & B_1Y_2 & 0 & 0 & B_1Y_2 & B \\ \bar{F}_{23} & \bar{F}_{24} & 0 & 0 & 0 & 0 & 0 & -B \end{bmatrix},\\
\bar{E}_2 &= \begin{bmatrix} E_1X_{21} & 0 & E_2Y_2 & E_2Y_2 & 0 & 0 & E_2Y_2 & 0 \end{bmatrix},
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{\Gamma}_{11}^1 &= 2\alpha_1 X_{11} + He\{AX_{11}\} + \bar{Q}_{11}\\
&\qquad -e^{-2\alpha_1 h}\bar{R}_{11} + \varepsilon_1 HH^T,\\
\bar{\Gamma}_{21}^1 &= \varepsilon_1 HH^T, \quad \bar{\Gamma}_{31}^1 = Y_1^T B^T - \bar{T}_{11}^T + e^{-2\alpha_1 h}\bar{R}_{11},\\
\bar{\Gamma}_{41}^1 &= Y_1^T B^T - \bar{T}_{12}^T, \quad \bar{\Gamma}_{71}^1 = Y_1^T B_1^T,\\
\bar{\Gamma}_{22}^1 &= 2\alpha_1 X_{12} + He\{AX_{12} - S_1 C\} + \bar{Q}_{12}\\
&\qquad -e^{-2\alpha_1 h}\bar{R}_{12} + \varepsilon_1 HH^T,\\
\bar{\Gamma}_{42}^1 &= e^{-2\alpha_1 h}\bar{R}_{12} - \bar{T}_{14}^T,\\
\bar{\Gamma}_{33}^1 &= \sigma\bar{\Omega}_1 + \bar{\Lambda} + He\left\{\bar{T}_{11} - e^{-2\alpha_1 h}\bar{R}_{11}\right\},\\
\bar{\Gamma}_{43}^1 &= \sigma\bar{\Omega}_1 + \bar{\Lambda} + \bar{T}_{12}^T + \bar{T}_{13}, \quad \bar{\Gamma}_{53}^1 = e^{-2\alpha_1 h}\bar{R}_{11} - \bar{T}_{11}^T,\\
\bar{\Gamma}_{44}^1 &= \sigma\bar{\Omega}_1 + \bar{\Lambda} + He\left\{\bar{T}_{14} - e^{-2\alpha_1 h}\bar{R}_{12}\right\},
\end{aligned}
$$

$$
\begin{aligned}
\bar{\Gamma}^1_{64} &= e^{-2\alpha_1 h}\bar{R}_{12} - \bar{T}^T_{14},\\
\bar{\Gamma}^1_{55} &= -e^{-2\alpha_1 h}\bar{Q}_{11} - e^{-2\alpha_1 h}\bar{R}_{11},\\
\bar{\Gamma}^1_{66} &= -e^{-2\alpha_1 h}\bar{Q}_{12} - e^{-2\alpha_1 h}\bar{R}_{12},\\
\bar{\Gamma}^1_{77} &= (\sigma - 1)\bar{\Omega}_1, \quad \bar{\Gamma}^1_{88} = -\gamma^2 I,\\
\bar{F}_{11} &= AX_{11} + \varepsilon_1 HH^T, \quad \bar{F}_{12} = \varepsilon_1 HH^T, \ \bar{F}_{13} = \varepsilon_1 HH^T,\\
\bar{F}_{14} &= AX_{12} - S_1 C + \varepsilon_1 HH^T,\\
\bar{\Gamma}^2_{11} &= 2\alpha_2 X_{21} + He\{AX_{21}\} + \bar{Q}_{21}\\
&\qquad - e^{2\alpha_2 h}\bar{R}_{21} + \varepsilon_2 HH^T,\\
\bar{\Gamma}^2_{21} &= \varepsilon_2 HH^T, \quad \bar{\Gamma}^2_{31} = Y_2^T B^T - \bar{T}^T_{21} + e^{2\alpha_2 h}\bar{R}_{21},\\
\bar{\Gamma}^2_{41} &= Y_2^T B^T - \bar{T}^T_{22}, \ \bar{\Gamma}^2_{71} = Y_2^T B_1^T,\\
\bar{\Gamma}^2_{22} &= 2\alpha_2 X_{22} + He\{AX_{22} - S_2 C\} + \bar{Q}_{22}\\
&\qquad - e^{2\alpha_2 h}\bar{R}_{22} + \varepsilon_2 HH^T,\\
\bar{\Gamma}^2_{42} &= e^{2\alpha_2 h}\bar{R}_{22} - \bar{T}^T_{24},\\
\bar{\Gamma}^2_{33} &= \sigma\bar{\Omega}_2 + He\left\{\bar{T}_{21} - e^{2\alpha_2 h}\bar{R}_{21}\right\},\\
\bar{\Gamma}^2_{43} &= \sigma\bar{\Omega}_2 + \bar{T}^T_{22} + \bar{T}_{23}, \ \bar{\Gamma}^2_{53} = e^{2\alpha_2 h}\bar{R}_{21} - \bar{T}^T_{21},\\
\bar{\Gamma}^2_{44} &= \sigma\bar{\Omega}_2 + He\left\{\bar{T}_{24} - e^{2\alpha_2 h}\bar{R}_{22}\right\},\\
\bar{\Gamma}^2_{64} &= e^{2\alpha_2 h}\bar{R}_{22} - \bar{T}^T_{24},\\
\bar{\Gamma}^2_{55} &= -e^{2\alpha_2 h}\bar{Q}_{21} - e^{2\alpha_2 h}\bar{R}_{21},\\
\bar{\Gamma}^2_{66} &= -e^{2\alpha_2 h}\bar{Q}_{22} - e^{2\alpha_2 h}\bar{R}_{22},\\
\bar{\Gamma}^2_{77} &= (\sigma - 1)\bar{\Omega}_2, \quad \bar{\Gamma}^2_{88} = -\gamma^2 I,\\
\bar{F}_{21} &= AX_{21} + \varepsilon_2 HH^T, \quad \bar{F}_{22} = \varepsilon_2 HH^T, \ \bar{F}_{23} = \varepsilon_2 HH^T,\\
\bar{F}_{24} &= AX_{22} - S_2 C + \varepsilon_2 HH^T.
\end{aligned}
$$

Then, the closed-loop switched system (14) satisfies H_∞ performance. The gain matrices of controller are $K_1 = Y_1 X_{11}^{-1}$ and $K_2 = Y_2 X_{21}^{-1}$, and the gain matrices of observer are $L_1 = S_1 \bar{X}_{11}^{-1}$ and $L_2 = S_2 \bar{X}_{21}^{-1}$.

Proof: When $w(t) \neq 0$, the H_∞ performance index is constructed as

$$
J_{yw} = \int_0^\infty \left[y^T(t)y(t) - \gamma^2 w^T(t)w(t)\right] dt, \tag{44}
$$

when initial state $x(t) = 0$, the Lyapunov-Krasovskii functional satisfies $W(0) = 0$, $W(\infty) = 0$, then

$$
\begin{aligned}
J_{yw} &= \int_0^\infty [y^T(t)y(t) - \gamma^2 w^T(t)w(t) + \dot{W}(\zeta(t))]dt\\
&\qquad - W(\infty)\\
&= \int_0^\infty [x^T(t)C^T Cx(t) - \gamma^2 w^T(t)w(t) + \dot{W}(\zeta(t))]dt
\end{aligned}
$$

where $\dot{W}(\zeta(t))$ is $\dot{W}(t)$ in Lemma 4, if

$$
x^T(t)C^T Cx(t) - \gamma^2 w^T(t)w(t) + \dot{W}(\zeta(t)) < 0,
$$

we can obtain $y^T(t)y(t) \leq \gamma^2 w^T(t)w(t)$.

When $i = 1$, the alternative Lyapunov-Krasovskii functional form same as Lemma 4 is selected. Define the augmented state $\tilde{\zeta}^T(t) = \begin{bmatrix}\zeta^T(t) & w^T(t)\end{bmatrix}$, the same proof process as Lemma 4 can be obtained

$$
x^T(t)C^T Cx(t) - \gamma^2 w^T(t)w(t) + \dot{W}(\zeta(t)) < \tilde{\zeta}^T(t)\breve{\Phi}_1\tilde{\zeta}(t),
$$

and

$$
\breve{\Phi}_1 = \begin{bmatrix} \breve{\Phi}^1_{11} & * & * & * \\ hR_1\breve{F}_1 & -R_1 & * & * \\ \breve{E}_1 & 0 & -\varepsilon_1 I & * \\ 0 & \breve{H}^T & 0 & -\varepsilon_1^{-1} I \end{bmatrix}, \tag{45}
$$

where

$$
\breve{\Phi}^1_{11} = \begin{bmatrix}
\breve{\Gamma}^1_{11} & * & * & * & * & * & * & * \\
\breve{\Gamma}^1_{21} & \breve{\Gamma}^1_{22} & * & * & * & * & * & * \\
\breve{\Gamma}^1_{31} & -T^T_{13} & \breve{\Gamma}^1_{33} & * & * & * & * & * \\
\breve{\Gamma}^1_{41} & \breve{\Gamma}^1_{42} & \breve{\Gamma}^1_{43} & \breve{\Gamma}^1_{44} & * & * & * & * \\
T^T_{11} & T^T_{13} & \breve{\Gamma}^1_{53} & -T^T_{13} & \breve{\Gamma}^1_{55} & * & * & * \\
T^T_{12} & T^T_{13} & -T^T_{12} & \breve{\Gamma}^1_{64} & 0 & \breve{\Gamma}^1_{66} & * & * \\
\breve{\Gamma}^1_{71} & T^T_{14} & \sigma\Omega_1 & \sigma\Omega_1 & 0 & 0 & \breve{\Gamma}^1_{77} & * \\
\breve{\Gamma}^1_{81} & \breve{\Gamma}^1_{82} & 0 & 0 & 0 & 0 & 0 & \breve{\Gamma}^1_{88}
\end{bmatrix},
$$

$$
\breve{F}_1 = \begin{bmatrix} \breve{F}_{11} & \breve{F}_{13} & \breve{F}_{15} & \breve{F}_{15} & 0 & 0 & \breve{F}_{15} & B \\ \breve{F}_{12} & \breve{F}_{14} & 0 & 0 & 0 & 0 & 0 & -B \end{bmatrix},
$$

$$
\breve{E}_1 = \begin{bmatrix} E_1 & 0 & E_2K_1 & E_2K_1 & 0 & 0 & E_2K_1 & 0 \end{bmatrix},
$$

where

$$
\begin{aligned}
\breve{\Gamma}^1_{11} &= 2\alpha_1 P_{11} + He\{P_{11}A\} + Q_{11} - e^{-2\alpha_1 h}R_{11}\\
&\qquad + \varepsilon_1 P_{11}HH^T P_{11} + C^T C,\\
\breve{\Gamma}^1_{21} &= \varepsilon_1 P_{12}HH^T P_{11},\\
\breve{\Gamma}^1_{31} &= K_1^T B^T P_{11} - T^T_{11} + e^{-2\alpha_1 h}R_{11},\\
\breve{\Gamma}^1_{41} &= K_1^T B^T P_{11} - T^T_{12}, \quad \breve{\Gamma}^1_{71} = K_1^T B_1^T P_{11},\\
\breve{\Gamma}^1_{81} &= B^T P_{11},\\
\breve{\Gamma}^1_{22} &= 2\alpha_1 P_{12} + \mathrm{He}\{P_{12}A - P_{12}L_1C\} + Q_{12}\\
&\qquad - e^{-2\alpha_1 h}R_{12} + \varepsilon_1 P_{12}HH^T P_{12},\\
\breve{\Gamma}^1_{42} &= e^{-2\alpha_1 h}R_{12} - T^T_{14}, \quad \breve{\Gamma}^1_{82} = -B^T P_{12},\\
\breve{\Gamma}^1_{33} &= \sigma\Omega_1 + \Lambda + He\left\{T_{11} - e^{-2\alpha_1 h}R_{11}\right\},\\
\breve{\Gamma}^1_{43} &= \sigma\Omega_1 + \Lambda + T^T_{12} + T_{13}, \quad \breve{\Gamma}^1_{53} = e^{-2\alpha_1 h}R_{11} - T^T_{11},\\
\breve{\Gamma}^1_{44} &= \sigma\Omega_1 + \Lambda + He\left\{T_{14} - e^{-2\alpha_1 h}R_{12}\right\},\\
\breve{\Gamma}^1_{64} &= e^{-2\alpha_1 h}R_{12} - T^T_{14},\\
\breve{\Gamma}^1_{55} &= -e^{-2\alpha_1 h}Q_{11} - e^{-2\alpha_1 h}R_{11},\\
\breve{\Gamma}^1_{66} &= -e^{-2\alpha_1 h}Q_{12} - e^{-2\alpha_1 h}R_{12},\\
\breve{\Gamma}^1_{77} &= (\sigma - 1)\Omega_1, \quad \breve{\Gamma}^1_{88} = -\gamma^2 I,\\
\breve{F}_{11} &= A + \varepsilon_1 HH^T P_{11}, \quad \breve{F}_{12} = \varepsilon_1 HH^T P_{11},\\
\breve{F}_{13} &= \varepsilon_1 HH^T P_{12}, \quad \breve{F}_{14} = A - L_1C + \varepsilon_1 HH^T P_{12},\\
\breve{F}_{15} &= B_1 K_1, \quad \breve{H} = \begin{bmatrix} H^T & H^T \end{bmatrix}^T.
\end{aligned}
$$

Similarly, when $i = 2$, we can obtain

$$
x^T(t)C^T Cx(t) - \gamma^2 w^T(t)w(t) + \dot{W}(\zeta(t)) < \tilde{\zeta}^T(t)\breve{\Phi}_2\tilde{\xi}(t),
$$

and

$$
\breve{\Phi}_2 = \begin{bmatrix} \breve{\Phi}^2_{11} & * & * & * \\ hR_2\breve{F}_2 & -R_2 & * & * \\ \breve{E}_2 & 0 & -\varepsilon_2 I & * \\ 0 & \breve{H}^T & 0 & -\varepsilon_2^{-1} I \end{bmatrix}, \tag{46}
$$

where

$$\breve{\Phi}_{11}^2 = \begin{bmatrix} \breve{\Gamma}_{11}^2 & * & * & * & * & * & * & * \\ \breve{\Gamma}_{21}^2 & \breve{\Gamma}_{22}^2 & * & * & * & * & * & * \\ \breve{\Gamma}_{31}^2 & -T_{23}^T & \breve{\Gamma}_{33}^2 & * & * & * & * & * \\ \breve{\Gamma}_{41}^2 & \breve{\Gamma}_{42}^2 & \breve{\Gamma}_{43}^2 & \breve{\Gamma}_{44}^2 & * & * & * & * \\ T_{21}^T & T_{23}^T & \breve{\Gamma}_{53}^2 & -T_{23}^T & \breve{\Gamma}_{55}^2 & * & * & * \\ T_{22}^T & T_{24}^T & -T_{22}^T & \breve{\Gamma}_{64}^2 & 0 & \breve{\Gamma}_{66}^2 & * & * \\ \breve{\Gamma}_{71}^2 & 0 & \sigma\Omega_2 & \sigma\Omega_2 & 0 & 0 & \breve{\Gamma}_{77}^2 & * \\ \breve{\Gamma}_{81}^2 & \breve{\Gamma}_{81}^2, & 0 & 0 & 0 & 0 & 0 & \breve{\Gamma}_{88}^2 \end{bmatrix},$$

$$\breve{F}_2 = \begin{bmatrix} \breve{F}_{21} & \breve{F}_{23} & \breve{F}_{25} & \breve{F}_{25} & 0 & 0 & \breve{F}_{25} & B \\ \breve{F}_{22} & \breve{F}_{24} & 0 & 0 & 0 & 0 & 0 & -B \end{bmatrix},$$

$$\breve{E}_2 = \begin{bmatrix} E_1 & 0 & E_2K_2 & E_2K_2 & 0 & 0 & E_2K_2 & 0 \end{bmatrix},$$

where

$$\begin{aligned}
\breve{\Gamma}_{11}^2 &= 2\alpha_2 P_{21} + He\{P_{21}A\} + Q_{21} - e^{2\alpha_2 h}R_{21} \\
&\quad + \varepsilon_2 P_{21}HH^T P_{21} + C^T C, \\
\breve{\Gamma}_{21}^2 &= \varepsilon_2 P_{22}HH^T P_{21}, \\
\breve{\Gamma}_{31}^2 &= K_2^T B^T P_{21} - T_{21}^T + e^{2\alpha_2 h}R_{21}, \\
\breve{\Gamma}_{41}^2 &= K_2^T B^T P_{21} - T_{22}^T, \quad \breve{\Gamma}_{71}^2 = K_2^T B_1^T P_{21}, \\
\breve{\Gamma}_{81}^2 &= B^T P_{21}, \\
\breve{\Gamma}_{22}^2 &= -2\alpha_2 P_{22} + \mathrm{He}\{P_{22}A - P_{22}L_2C\} + Q_{22} \\
&\quad - e^{2\alpha_2 h}R_{22} + \varepsilon_2 P_{22}HH^T P_{22}, \\
\breve{\Gamma}_{42}^2 &= e^{2\alpha_2 h}R_{22} - T_{24}^T, \quad \breve{\Gamma}_{82}^2 = -B^T P_{21}, \\
\breve{\Gamma}_{33}^2 &= \sigma\Omega_2 + He\{T_{21} - e^{2\alpha_2 h}R_{21}\}, \\
\breve{\Gamma}_{43}^2 &= \sigma\Omega_2 + T_{22}^T + T_{23}, \quad \breve{\Gamma}_{53}^2 = e^{2\alpha_2 h}R_{21} - T_{21}^T, \\
\breve{\Gamma}_{44}^2 &= \sigma\Omega_2 + He\{T_{24} - e^{2\alpha_2 h}R_{22}\}, \\
\breve{\Gamma}_{64}^2 &= e^{2\alpha_2 h}R_{22} - T_{24}^T, \quad \breve{\Gamma}_{55}^2 = -e^{2\alpha_2 h}Q_{21} - e^{2\alpha_2 h}R_{21}, \\
\breve{\Gamma}_{66}^2 &= -e^{2\alpha_2 h}Q_{22} - e^{2\alpha_2 h}R_{22}, \quad \breve{\Gamma}_{77}^2 = (\sigma - 1)\Omega_2, \\
\breve{\Gamma}_{88}^2 &= -\gamma^2 I, \quad \breve{F}_{21} = A + \varepsilon_2 HH^T P_{21}, \\
\breve{F}_{22} &= \varepsilon_2 HH^T P_{21}, \quad \breve{F}_{23} = \varepsilon_2 HH^T P_{22}, \\
\breve{F}_{24} &= A - L_2C + \varepsilon_2 HH^T P_{22}, \quad \breve{F}_{25} = B_1K_2.
\end{aligned}$$

By repeatedly deriving the stability theory of the augmented system in Theorem 1, the conditions (29) - (31) are obtained. Multiply J_1 by the left and right sides of (45), and $J_1 = \mathrm{diag}\{X_1, J_3, J_3, X_{11}, X_1, I, I, I, I\}$, where $J_3 = \mathrm{diag}\{X_{11}, X_{11}\}$, multiply X_1 by the left and right sides of (29), let $X_1 = P_1^{-1}$, $\bar{R}_1 = X_1R_1X_1$, $\bar{Q}_1 = X_1Q_1X_1$, $\bar{T}_1 = J_3T_1X_1$, $\bar{\Omega}_1 = X_{11}\Omega_1X_{11}$, $\bar{\Lambda} = X_{11}\Lambda X_{11}$, $S_1 = L_1\bar{X}_{12}$, $Y_1 = K_1X_{11}$, through the inequality $R_1^{-1} = X_1\bar{R}_1^{-1}X_1 \geq 2\lambda_1X_1 - \lambda_1^2\bar{R}_1$, $\lambda_1 \in R_+$, finally we can obtain (36), (39), (42) and (43). Similarly, we can obtain (37), (38), (40) and (41).

IV. ILLUSTRATIVE EXAMPLES AND SIMULATION

Consider an unstable batch reactor system [32], whose system parameters are as follows

$$A = \begin{bmatrix} 1.38 & -0.2007 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.1 \\ 0.5 \\ 1 \\ 0.7 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.1 & -0.2 & 0.2 & -0.1 \\ -0.2 & -4.29 & 0 & 0.675 \\ 0.067 & 0.273 & -0.254 & 0.093 \\ 0.048 & 0.273 & 0.343 & -0.04 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0.1 & 0.179 & 0.136 & 0.136 \\ 0.1 & 0.1 & -0.146 & 0.2 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.1 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.1 & 0.175 \\ 0.2 & 0.273 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.343 & 0.04 \end{bmatrix}.$$

Chose $\nu = 15$, $\Gamma = 5$, $\omega = 20$, $\Gamma_D = 5$ satisfying the assumptions of DoS attacks, take $h = 0.001$, $\lambda = 0.01$, $\theta = 1$, $\sigma = 0.03$, $\tilde{\eta} = 1.5$, take $\mu_1 = \mu_2 = 0.1$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, let $\lambda_1 = \lambda_2 = 0.1$, $\lambda_3 = \lambda_4 = 0.2$, $\varepsilon_1 = \varepsilon_2 = 30$, $F(t) = \sin(t)$, $w(t) = 0.1\sin(2t)$, $\gamma = 1.5$. The following can be obtained by solving LMIs in Theorem 2

$$K_1 = \begin{bmatrix} -0.2692 & -0.6510 & -0.3791 & -0.3883 \\ 2.0151 & 0.1300 & 1.4625 & -1.1605 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.2599 & -0.5850 & -0.3691 & -0.3351 \\ 1.8299 & 0.1628 & 1.3461 & -1.0652 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 62.3060 & -11.4265 \\ 18.6272 & 3.7095 \\ -7.9987 & -0.0178 \\ -11.1734 & 17.0843 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 48.7177 & -5.9805 \\ 12.2175 & 3.0929 \\ -5.9061 & 0.6505 \\ -8.1897 & 14.8573 \end{bmatrix},$$

$$\bar{\Lambda} = 10^3 \times \begin{bmatrix} 2.3553 & -0.3120 & -1.8956 & 1.3475 \\ -0.3120 & 1.9100 & 0.5961 & 0.05736 \\ -1.8956 & 0.5961 & 1.7927 & -0.8754 \\ 1.3475 & 0.05736 & -0.8754 & 1.1097 \end{bmatrix},$$

$$\bar{\Omega}_1 = 10^4 \times \begin{bmatrix} 3.1319 & 0.4510 & -1.1348 & 0.4174 \\ 0.4510 & 4.3719 & 1.4267 & 1.0779 \\ -1.1348 & 1.4267 & 2.2304 & 7.3887 \\ 0.4174 & 1.0779 & 7.3887 & 2.1503 \end{bmatrix}.$$

In order to verify the effectiveness of Theorem 2, the closed-loop switched system (14) is simulated as shown in Fig. 2 - Fig. 5. As shown in Fig. 2 and Fig. 3, when there is no disturbance, the state of the closed-loop batch reactor system with unknown periodic DoS attacks converges to zero in finite time. When there is a disturbance, the states of the closed-loop system converge to zero in finite time as well as the system still can effectively restrain disturbance with H_∞

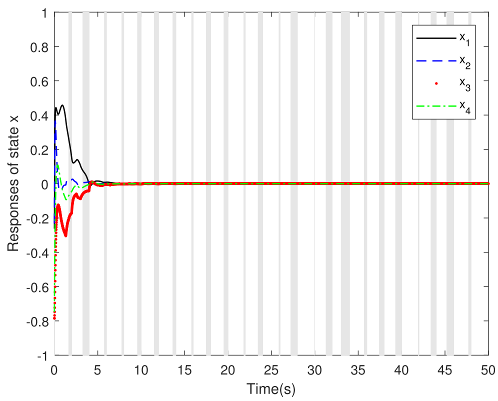

FIGURE 2. Response of closed-loop system without disturbance.

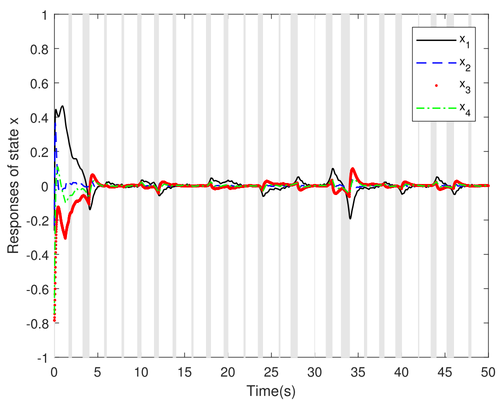

FIGURE 3. Response of closed-loop system with disturbance.

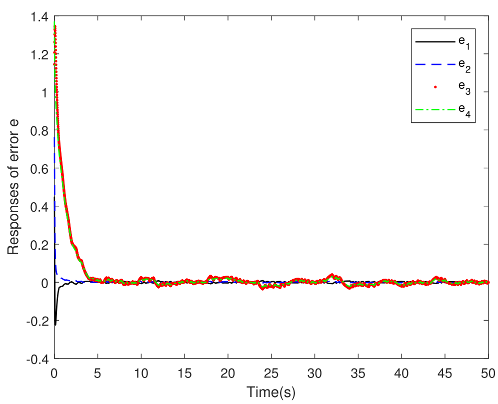

FIGURE 4. Response of closed-loop system observation error.

performance. According to Fig. 4, the observation errors converge to zero before the state of the closed-loop system is stable, which verifies the effectiveness of the full-dimensional state observer designed in this paper.

In addition, instants and intervals of dynamic event triggering are shown in Fig. 5, and we can derive that the average triggering interval is 0.14s, which is much longer than the sampling period 0.001s. Thus, system resources such as networked bandwidth can be saved. The minimum triggering

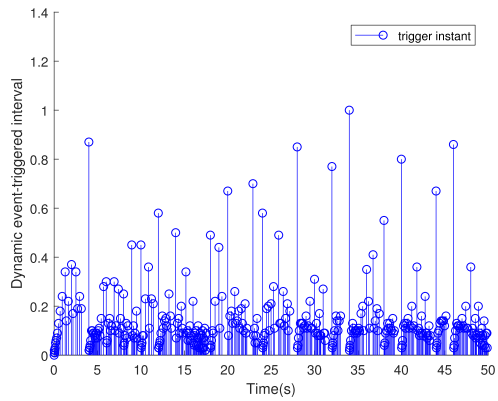

FIGURE 5. Dynamic event-triggered sequence.

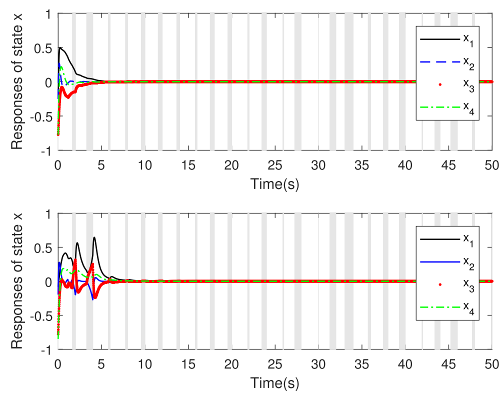

FIGURE 6. Responses of state x with the method in Theorem 1 (upper) and the method in [28](lower).

interval equals to the sampling period 0.001s, which excludes Zeno behavior. These observations above confirm Remark 1. Significantly, by comparing Fig. 2 and Fig. 3, the system can suppress the disturbance with short-term DoS attacks, such as the 5s to 15s. However, the disturbance suppression ability of the system is reduced with long-term DoS attacks, such as the 30s to 40s. It can be known that high energy DoS attacks will worsen the adverse effects of disturbance on the system and interfere with the normal operation of the system.

Here we compare the results of the proposed method with [26]. The responses of state show in Fig.6 employ the two different control strategies without disturbance and uncertainty. Apparently, Theorem 1 can achieve satisfactory control performance faster and more smoothly. This result is mainly because the hold-input strategy adopted in this paper compensates for the impact of DoS attacks on the system. In contrast, the zero-input strategy is adopted in [28] to compensate for the impact of DoS attacks on the system.

To further optimize the H_∞ performance of the system, we use MATLAB to solve the following optimization problem

$$\begin{cases} \min \gamma \\ \text{s.t. } (31), (36)-(43) \end{cases} \tag{47}$$

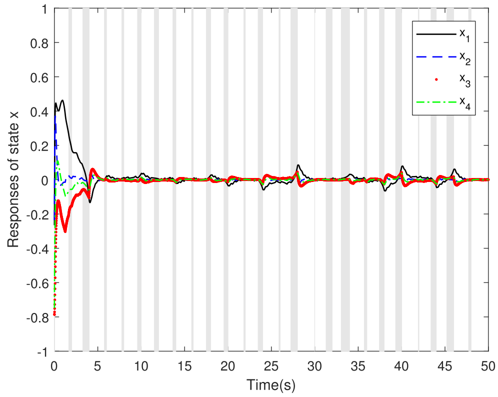

FIGURE 7. Response of closed-loop with optimized γ.

The optimal value $\gamma = 0.36$ of H_∞ performance parameter is obtained through *mincx* solver of the LMI toolbox, and the simulation result is shown in Fig. 7. By comparing Fig. 3 and Fig. 7, it can be seen that the disturbance suppression degree of the closed-loop system increases after the H_∞ performance parameter γ is optimized. According to (15), the H_∞ performance parameter γ determines the degree of disturbance suppression, and the smaller γ is, the greater the degree of disturbance suppression. However, if the given γ is smaller, the LMIs in Theorem 2 will be more challenging to solve. In this article, the control parameters of the closed-loop system and minimum γ can be effectively found through solving optimization problem (47).

V. CONCLUSION

This paper studied an observer-based dynamic event-triggered control for networked control systems with unknown periodic DoS attacks, external disturbance, and uncertainty. Firstly, we have analyzed the case without the disturbance and obtained sufficient conditions for the robust asymptotic stability of the closed-loop switched system by using Lyapunov functional theory, improved Jensen's inequality, Schur's Complement Lemma, etc. Secondly, we have considered the case with the disturbance. By introducing the H_∞ performance index, we obtained the sufficient conditions that the closed-loop switched system has the H_∞ performance with the disturbance. Finally, an example of batch reactor has been used to verify the correctness of the theory studied in this paper, and H_∞ performance parameter optimization has been realized to increase the disturbance suppression level of the system.

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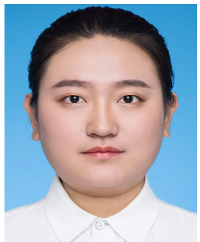

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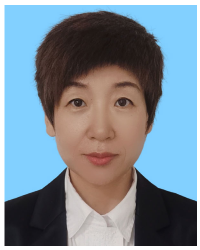

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