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l_2 - l_∞ Control for Discrete-Time Descriptor Systems

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ABSTRACT The problem of l_2 - l_∞ controller design for a class of discrete-time descriptor systems is addressed in this paper. In other words, the controller is designed to ensure the admissibility and the prescribed l_2 - l_∞ performance for the descriptor control system. The l_2 - l_∞ performance analysis and the feedback controller design of the descriptor control system are studied by the matrix inequality technique. Novel design conditions of the feedback controller for the descriptor systems are introduced in terms of linear matrix inequality (LMI) representations. An RC circuit is applied to explain the effectiveness of the proposed l_2 - l_∞ control design method.

INDEX TERMS Discrete-time descriptor system, l_2 - l_∞ performance, feedback control, linear matrix inequalities (LMIs).

I. INTRODUCTION

The descriptor system [1]–[3], also known as singular system, has provided a unified and useful modeling method for many real applications, such as electricity, aerospace, and social economy. Therefore, it is clearly observed that the study on descriptor system is active over the recent decades and has made great process [4], [5], in which, as the most important field, the problem of asymptotic stability and feedback stabilization have been attracted more attention from researchers. And some results involving the research on system performance analysis for descriptor system have been reported, for instance, in [6], the problem of a reduced-order $\mathcal{H}_\infty$ controller based on the geometric structures was solved. [7] discussed the problem of the finite-time $\mathcal{H}_\infty$ control for nonlinear descriptor system via state under composed method. In order to handle the effects of unknown inputs, [8] presented a systematical reduced-order observer design method for switched descriptor system. In [9], the design method of the $\mathcal{H}_\infty$ observer for the nonlinear discrete-time descriptor system with time-varying delays and disturbance inputs was investigated. [10] studied the problem of the adaptive $\mathcal{H}_\infty$ sliding mode control for nonlinear system. [11] dealt with the state feedback $\mathcal{H}_\infty$ control problem for discrete-time descriptor system. The output tracking control and filtering problems were investigated for nonlinear descriptor system

in the discrete-time domain in [12]. In [13], the robust $\mathcal{H}_\infty$ controller design scheme for uncertain discrete-time descriptor system was addressed in terms of a set of LMI.

We can find that, among the mentioned references above, the most of the works mainly investigated the $\mathcal{H}_\infty$ performance of the descriptor system. In this work, it takes the advantage to such a degree that obtaining the accurate statistic of external disturbance is not required, which can take any form as long as its energy is bounded. Therefore, it is of significance in theoretical research and practical application [14]–[19]. However, in practice, the other work namely l_2 - l_∞ performance analysis and design for the descriptor system is also of considerable academic value in control theory, its goal is to ensure that the ratio of maximum value of control output (filtering error) to bounded energy of disturbances is required to be less than the given value. That is different from the research on $\mathcal{H}_\infty$ performance [20], so this work becomes very necessary. In the recent years, the study on the l_2 - l_∞ controller (filter) design is reported in much literature. In [21], an output feedback controller design scheme for seismic excitation structures was proposed, which considered the l_2 - l_∞ performance and the saturation of actuator. Based on polyhedral uncertainties in the state-space equations, [22] investigated the l_2 - l_∞ filter design problem for uncertain system in both discrete-time and continuous-time domains. [23] addressed the filter design problem for T-S fuzzy stochastic system, which aimed to find a more convenient filter design condition that ensured the mean square asymptotic stability

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and the l_2 - l_∞ performance. For continuous-time nonlinear uncertain system, [24] discussed the resilient l_2 - l_∞ control problem.

It can be seen that the study on l_2 - l_∞ control (filtering) problem has achieved fruitful results [25]. However, it is worth noting that, to our best knowledge, the l_2 - l_∞ performance has not been developed abundantly in descriptor system. Due to the special structure of the descriptor system, it's difficult to use classical LMI technique to obtain the l_2 - l_∞ controller of descriptor system. Therefore, there are still many problems to be solved in the design of l_2 - l_∞ controller for descriptor system.

Based on the above motivation, the l_2 - l_∞ control problem for the discrete-time descriptor system is dealt with in this paper, and the feasible controller design method is proposed to ensure the resulting descriptor system satisfies the given l_2 - l_∞ performance by using the classical LMI technique. The contributions are described as the following:

- 1) This paper aims at the l_2 - l_∞ performance index for the discrete-time descriptor system, based on this, it proposes the novel performance analysis criteria to ensure the closed-loop descriptor system is admissible and meets l_2 - l_∞ performance;
- 2) For a given closed-loop descriptor system, the proposed l_2 - l_∞ controller design conditions are given under strict LMI presentation.

II. PROBLEM FORMULATION AND PRELIMINARIES

The following discrete-time descriptor system is considered,

$$\begin{aligned} Ex(k+1) &= Ax(k) + Bu(k) + Fw(k) \\ z(k) &= Cx(k) \end{aligned} \quad (1)$$

where $x(k) \in \mathcal{R}^{n_x}$ and $u(k) \in \mathcal{R}^{n_u}$ stand for the state variable and the control input, respectively; $w(k) \in \mathcal{R}^{n_w}$ refers to the noise input which belongs to $l_2[0, \infty)$; $z(k) \in \mathcal{R}^{n_z}$ stands for the controlled output; A , B , F , and C are given system matrices with appropriate dimensions given by $f_\star$ which will be introduced in the following of this section. In particular, E is singular but nonzero and satisfies $\text{rank}(E) = r < n$.

Assumption 1: In this paper, the practical discrete-time system described by the descriptor system model (1) is assumed as:

$$\begin{aligned} x_1(k+1) &= f_1[x_1(k), x_2(k), \dots, x_n(k), u(k), w(k)] \\ x_2(k+1) &= f_2[x_1(k), x_2(k), \dots, x_n(k), u(k), w(k)] \\ &\vdots \\ x_r(k+1) &= f_r[x_1(k), x_2(k), \dots, x_n(k), u(k), w(k)] \\ 0 &= f_{r+1}[x_1(k), x_2(k), \dots, x_n(k), u(k), w(k)] \\ &\vdots \\ 0 &= f_n[x_1(k), x_2(k), \dots, x_n(k), u(k), w(k)] \\ z(k) &= f_z[x_1(k), x_2(k), \dots, x_r(k), w(k)] \end{aligned}$$

where $f_\star$ represents a linear relationship of $x(k)$ and $w(k)$. The form means that a property on the system parameter matrices

is $E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ and $C = [C_r \quad 0]$ in the descriptor system (1). In fact, such a form has a wide range of universality for the descriptor system (1) such as the RC circuit system [26] and the RLC circuit systems [27], the Leontief systems [28], the inverted pendulum balancing systems [29], the open flow canal systems [30], etc.

Definition 1 ([1]): Given a scalar $s > 0$, the descriptor system (1) is admissible if $\det(sE - A)$ is not identically zero, $\deg(\det(sE - A)) = \text{rank}(E)$ and all the roots of $\det(sE - A) = 0$ lie in the interior of unit disk (i.e., regular, casual, and stable).

Further, in order to achieve the control tasks for the descriptor system (1), the state feedback controller can be considered as

$$u(k) = Kx(k) \quad (2)$$

where K is controller gain matrix to be determined.

The main objective of this work is to construct controller (2) to ensure the following requirements are guaranteed:

- 1) When $w(k) = 0$, the descriptor system (1) with the controller (2) is admissible;
- 2) Under the zero initial condition, the descriptor system (1) with the controller (2) satisfies the prescribed l_2 - l_∞ performance γ , i.e., for any $w(k) \in l_2[0, \infty)$, the output $z(k)$ guarantees the condition $\|z(k)\|_\infty < \gamma \|w(k)\|_2$ where $\|z(k)\|_\infty = \sup_k |z(k)|$ and $\|w(k)\|_2^2 = \sum_{k=0}^\infty w^T(k)w(k)$.

Due to the rank deficiency of matrix E , the l_2 - l_∞ control problem is difficult to solve. In fact, via using results given in the literature [21]–[24], a nonlinear matrix inequality can be obtained for the controller design directly. In order to describe this problem, we introduce the following system:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Fw(k) \\ z(k) &= Cx(k) \end{aligned} \quad (3)$$

Then, by importing the feedback controller described in (2), the closed-loop system is obtained as

$$\begin{aligned} x(k+1) &= (A + BK)x(k) + Fw(k) \\ z(k) &= Cx(k) \end{aligned} \quad (4)$$

Furthermore, considering the Lyapunov function $V(x(k)) = x^T(k)P^{-1}x(k)$ with $P > 0$, the system (4) satisfies the l_2 - l_∞ performance if the following inequality conditions hold:

$$\begin{bmatrix} -P^{-1} & * \\ 0 & -I \end{bmatrix} + [A + BK \quad F]^T P^{-1} [A + BK \quad F] < 0 \quad (5)$$

$$\frac{1}{\gamma^2} [C \quad 0]^T [C \quad 0] < \begin{bmatrix} P^{-1} & * \\ 0 & I \end{bmatrix}. \quad (6)$$

Hence, by applying the matrix inequality congruence property and Schur complement lemma to the matrix inequalities

(5) and (6), the corresponding LMI-based l_2 - l_∞ controller design conditions for system (4) can be summarized as:

$$\begin{bmatrix} -P & * & * \\ 0 & -I & * \\ AP + BN & F & -P \end{bmatrix} < 0 \quad (7)$$

$$\begin{bmatrix} -P & * & * \\ 0 & -I & * \\ CP & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (8)$$

where $N = KP$. If there exist some variables P and N which satisfy the above inequalities (7) and (8), we will get the l_2 - l_∞ controller gain matrix as $K = NP^{-1}$ for system (4).

We should note that above approach of analysing the l_2 - l_∞ performance and designing controller for system (4) is inapplicable to the descriptor system (1) with the feedback controller (2). As a matter of fact, the Lyapunov function is often selected by $V(x(k)) = x^T(k)E^T P E x(k)$ with $E^T P E \geq 0$ for the descriptor system (1) with the feedback controller (2), then the l_2 - l_∞ performance analysis criteria corresponding to one in (5) and (6) can be described as

$$\begin{bmatrix} -E^T P E & * \\ 0 & -I \end{bmatrix} + [A + BK \ F]^T P [A + BK \ F] < 0 \quad (9)$$

$$\frac{1}{\gamma^2} [C \ 0]^T [C \ 0] < \begin{bmatrix} E^T P E & * \\ 0 & I \end{bmatrix}. \quad (10)$$

Based on the l_2 - l_∞ performance analysis criteria in (9) and (10), it is difficult to acquire design conditions for the descriptor system (1) with the controller in (2). The key to this problem is that

- 1) Since $E^T P E \geq 0$, it leads to that the Schur complement lemma cannot be applied on (9) and (10) as (7) and (8), respectively;
- 2) If the matrix inequality decoupling property [31] is adopted to (9) and separate the matrix P for the term $[A + BK \ F]^T P [A + BK \ F]$, it is difficult to get a linear coupling between decision variables because E is singular.

We can clearly observe if these presented results on l_2 - l_∞ performance research in literature [21]–[24] are directly applied to the design of the controller for descriptor system, corresponding design conditions are difficult to be obtained via strict LMI framework. Based on this, the feedback control problem for the descriptor system is first addressed, and then, the design conditions for the feedback controller are deduced to satisfy the prescribed l_2 - l_∞ performance by the solvability of a set of LMIs.

For the sake of convenience, the following lemma is presented, which is important to the results of this paper.

Lemma 1 ([31]): The two next problems are equivalent.

- i) Find $P = P^T$ such that

$$T + A^T P A < 0. \quad (11)$$

- ii) Find $P = P^T$, L , and G such that

$$\begin{bmatrix} T + A^T L^T + LA & * \\ -L^T + GA & -G - G^T + P \end{bmatrix} < 0. \quad (12)$$

III. MAIN RESULTS

In the following, we introduce the augmented descriptor representation approach to represent the descriptor system (1) with the controller (2). First of all, based on the form of the feedback controller presented in (2), one can effortlessly give

$$0 = Kx(k) - u(k). \quad (13)$$

Furthermore, combining the descriptor system (1) with (13), one leads to

$$\begin{aligned} Ex(k+1) &= Ax(k) + Bu(k) + Fw(k) \\ 0 \cdot u(k+1) &= Kx(k) - u(k) \\ z(k) &= Cx(k) \end{aligned} \quad (14)$$

Then, denoting an augmented vector as $\eta(k) = [x^T(k) \ u^T(k)]^T$, (14) can be represented by the following augmented descriptor system:

$$\begin{aligned} \mathcal{E}\eta(k+1) &= \mathcal{A}\eta(k) + \mathcal{F}w(k) \\ z(k) &= \mathcal{C}\eta(k) \end{aligned} \quad (15)$$

where

$$\begin{aligned} \mathcal{E} &= \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} A & B \\ K & -I \end{bmatrix} \\ \mathcal{F} &= \begin{bmatrix} F \\ 0 \end{bmatrix}, \quad \mathcal{C} = [C \ 0]. \end{aligned}$$

Then, based on such an augmented system, the final goal of this paper can be determined to find the controller gain matrix K introduced in (2) to ensure that the augmented descriptor system (15) is admissible with prescribed l_2 - l_∞ performance.

Without loss of generality, the corresponding performance analysis criteria are given before the controller design conditions are presented.

Theorem 1: Consider the descriptor system (15), for a known scalar $\gamma > 0$, if there exist matrices $\mathcal{P}$, $\mathcal{L}$, $\mathcal{G}$, and K , such that

$$\begin{aligned} \mathcal{E}^T \mathcal{P} \mathcal{E} &\geq 0 \\ \begin{bmatrix} \Omega & * \\ -[\mathcal{L}^T \ 0] + \mathcal{G}[\mathcal{A} \ \mathcal{F}] & -\mathcal{G} - \mathcal{G}^T + \mathcal{P} + \mathcal{C}^T \mathcal{C} \end{bmatrix} &< 0 \end{aligned} \quad (16)$$

where $\Omega = \text{diag}\{-\mathcal{E}^T \mathcal{P} \mathcal{E} - \mathcal{C}^T \mathcal{C}, -\gamma^2 I\} + [\mathcal{L}^T \ 0]^T [\mathcal{A} \ \mathcal{F}] + [\mathcal{A} \ \mathcal{F}]^T [\mathcal{L}^T \ 0]$, the descriptor system (15) is admissible with the prescribed l_2 - l_∞ performance γ .

Proof: Firstly, selecting a special Lyapunov function as

$$V(\eta(k)) = \eta^T(k) \mathcal{E}^T (\mathcal{P} + \mathcal{C}^T \mathcal{C}) \mathcal{E} \eta(k), \quad \mathcal{E}^T \mathcal{P} \mathcal{E} \geq 0 \quad (18)$$

we have

$$\begin{aligned} V(\eta(k+1)) - V(\eta(k)) &= \eta^T(k+1) \mathcal{E}^T (\mathcal{P} + \mathcal{C}^T \mathcal{C}) \mathcal{E} \eta(k+1) \\ &\quad - \eta^T(k) \mathcal{E}^T (\mathcal{P} + \mathcal{C}^T \mathcal{C}) \mathcal{E} \eta(k). \end{aligned} \quad (19)$$

Let's focus on the following inequality condition:

$$\eta^T(k+1)\mathcal{E}^T(\mathcal{P} + \mathcal{C}^T\mathcal{C})\mathcal{E}\eta(k+1) - \eta^T(k)\mathcal{E}^T(\mathcal{P} + \mathcal{C}^T\mathcal{C})\mathcal{E}\eta(k) < \gamma^2 w^T(k)w(k). \quad (20)$$

And we can easily find from (20), when the external disturbance $w(k) = 0$, one follows

$$V(\eta(k+1)) - V(\eta(k)) < 0 \quad (21)$$

which means that the descriptor system (15) is admissible.

Therefore, from the above, we know that under the condition of $w(k) = 0$, if (20) holds, the system is admissible. Next, in order to verify the l_2 - l_∞ performance with $w(k) \neq 0$ from (20), considering the special system structure, we rewrite (20) as

$$\begin{aligned} & \eta^T(k+1)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(k+1) - \eta^T(k)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(k) \\ & + \eta^T(k+1)\mathcal{E}^T\mathcal{C}^T\mathcal{C}\mathcal{E}\eta(k+1) - \eta^T(k)\mathcal{E}^T\mathcal{C}^T\mathcal{C}\mathcal{E}\eta(k) \\ & = \eta^T(k+1)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(k+1) - \eta^T(k)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(k) \\ & + \eta^T(k+1)\mathcal{C}^T\mathcal{C}\eta(k+1) - \eta^T(k)\mathcal{C}^T\mathcal{C}\eta(k) \\ & < \gamma^2 w^T(k)w(k). \end{aligned} \quad (22)$$

Then, consider the structure of system (15), one can be obtained that

$$\eta^T(k+1)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(k+1) - \eta^T(k)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(k) + z^T(k+1)z(k+1) - z^T(k)z(k) < \gamma^2 w^T(k)w(k). \quad (23)$$

Further, taking sum on both sides of (23) from 0 to $k-1$ yields

$$\begin{aligned} & \sum_{j=0}^{k-1} \left(\eta^T(j+1)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(j+1) - \eta^T(j)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(j) \right) \\ & + \sum_{j=0}^{k-1} \left(z^T(j+1)z(j+1) - z^T(j)z(j) \right) \\ & < \gamma^2 \sum_{j=0}^{k-1} w^T(j)w(j) \end{aligned} \quad (24)$$

which is

$$\eta^T(k)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(k) - \eta^T(0)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(0) + z^T(k)z(k) - z^T(0)z(0) < \gamma^2 \sum_{j=0}^{k-1} w^T(j)w(j). \quad (25)$$

In fact, under the zero initial condition, (25) means

$$\eta^T(k)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(k) + z^T(k)z(k) < \gamma^2 \sum_{j=0}^{k-1} w^T(j)w(j). \quad (26)$$

Because of the fact $\sum_{j=0}^{k-1} w^T(j)w(j) \leq \sum_{k=0}^{\infty} w^T(k)w(k)$, (26) gives

$$\eta^T(k)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(k) + z^T(k)z(k) < \gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k). \quad (27)$$

From the condition that $\mathcal{E}^T\mathcal{P}\mathcal{E} \geq 0$ in (18), it leads to $\eta^T(k)\mathcal{E}^T\mathcal{P}\mathcal{E}\eta(k) \geq 0$. Based on this, consider the inequality (27), the following inequality condition will be easily established:

$$z^T(k)z(k) < \gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k) \quad (28)$$

which means $\|z(k)\|_\infty < \gamma \|w(k)\|_2$ for any external disturbance $w(k) \in l_2[0, \infty)$ when $\eta(0) = 0$.

The above discussion shows that the inequality condition (20) can be viewed as an analysis condition to detect whether or not the considered feedback descriptor system with known parameters is admissible and meets required l_2 - l_∞ performance, which can be organized into

$$\begin{aligned} & \mu^T(k) \begin{bmatrix} \mathcal{A} & \mathcal{F} \end{bmatrix}^T (\mathcal{P} + \mathcal{C}^T\mathcal{C}) \begin{bmatrix} \mathcal{A} & \mathcal{F} \end{bmatrix} \mu(k) \\ & + \mu^T(k) \begin{bmatrix} -\mathcal{E}^T(\mathcal{P} + \mathcal{C}^T\mathcal{C})\mathcal{E} & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \mu(k) < 0 \end{aligned} \quad (29)$$

$$\text{where } \mu(k) = \begin{bmatrix} \eta(k) \\ w(k) \end{bmatrix}.$$

Then, a sufficient condition that ensures the above inequality holds is described as follows,

$$\begin{aligned} & \begin{bmatrix} -\mathcal{E}^T(\mathcal{P} + \mathcal{C}^T\mathcal{C})\mathcal{E} & * \\ 0 & -\gamma^2 I \end{bmatrix} \\ & + \begin{bmatrix} \mathcal{A} & \mathcal{F} \end{bmatrix}^T (\mathcal{P} + \mathcal{C}^T\mathcal{C}) \begin{bmatrix} \mathcal{A} & \mathcal{F} \end{bmatrix} < 0. \end{aligned} \quad (30)$$

So far, we have that (30) can ensure that the preparatory condition (20) holds, and thus guarantee the l_2 - l_∞ performance γ for the descriptor system (15). Then, by applying the Lemma 1 to (30), we have the strict LMI condition (17). The proof is completed. $\square$

Remark 1: In the l_2 - l_∞ performance analysis for the descriptor system (15), a useful Lyapunov function is constructed in (18). In contrast to traditional function $\mathcal{E}^T\mathcal{P}\mathcal{E}$, the function (18) has changed the Lyapunov matrix $\mathcal{P}$ for the establishment of the l_2 - l_∞ performance, under which the novel performance analysis criteria of the l_2 - l_∞ performance for the descriptor system (15) are presented by two matrix inequalities (16) and (17). It should be noted that a constraint $\mathcal{E}^T\mathcal{P}\mathcal{E} \geq 0$ is necessary for deducing the l_2 - l_∞ performance analysis condition (see (28)). In fact, since $\mathcal{C}^T\mathcal{C} \geq 0$, which with $\mathcal{E}^T\mathcal{P}\mathcal{E} \geq 0$ can ensure $\mathcal{E}^T(\mathcal{P} + \mathcal{C}^T\mathcal{C})\mathcal{E} \geq 0$ for the Lyapunov function (18).

Such a construction form of the augmented system (15) with feedback controller (2) is helpful to the analysis of l_2 - l_∞ performance for the feedback system. If we adopt the construction method similar to system (4), it will bring certain difficulties to the subsequent controller design. Our ultimate goal is to find the optimal l_2 - l_∞ controller based on the solvability of the strict LMIs, so the construction method of this augmented system is necessary. Moreover, due to the particularity of equation (13), the structure of system (15) retains the characteristics of descriptor system, which is the extension of system (1), so that the analysis of the system (1) with the controller (2) is not affected.

Next, since we already know that the criterion for the l_2 - l_∞ performance analysis for the descriptor system (15) is introduced in Theorem 1, so we will propose the corresponding design condition for the controller given in (2), i.e., determine the controller K , Lyapunov matrix $\mathcal{P}$, and relevant matrix variables $\mathcal{L}$ and $\mathcal{G}$ in (16) and (17) to ensure the descriptor system (15) guarantees the given l_2 - l_∞ performance γ .

Theorem 2: Consider the descriptor system (15), for a known scalar $\gamma > 0$, if there exist matrices $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, N, L, L_1, L_2, G_1$, and G_2 satisfying the following two matrix inequalities:

$$\begin{aligned} E^T \mathcal{P}_1 E &\geq 0 \\ \begin{bmatrix} \Psi_1 & * & * & * & * \\ \Psi_2 & He\{L_2 B - L\} & * & * & * \\ (L_1 F)^T & (L_2 F)^T & -\gamma^2 I & * & * \\ \Psi_3 & \Psi_4 & G_1 F & \Psi_5 & * \\ \Psi_6 & \Psi_7 & G_2 F & \Psi_8 & \Psi_9 \end{bmatrix} &< 0 \end{aligned} \quad (31) \quad (32)$$

where

$$\begin{aligned} \Psi_1 &= -E^T \mathcal{P}_1 E - C^T C + He\{L_1 A + BN\} \\ \Psi_2 &= L_2 A + N + (L_1 B - BL)^T \\ \Psi_3 &= G_1 A + BN - L_1^T \\ \Psi_4 &= G_1 B - BL - L_2^T \\ \Psi_5 &= -G_1 - G_1^T + \mathcal{P}_1 + C^T C \\ \Psi_6 &= G_2 A + N - (BL)^T \\ \Psi_7 &= G_2 B - L - L^T \\ \Psi_8 &= -G_2 - (BL)^T + \mathcal{P}_2 \\ \Psi_9 &= -L - L^T + \mathcal{P}_3 \end{aligned}$$

the descriptor system (15) is admissible with the prescribed l_2 - l_∞ performance γ . Further, the l_2 - l_∞ controller gain matrix is constructed as

$$K = L^{-1}N. \quad (33)$$

Proof: According to the inequality conditions (16) and (17) in Theorem 1, we can detect whether the given l_2 - l_∞ performance level γ for the descriptor system (15) is guaranteed through the solvability of the matrix inequalities (16) and (17).

Now, let's consider these matrix variables $\mathcal{P}$, $\mathcal{L}$, and $\mathcal{G}$ by the following form:

$$\mathcal{P} = \begin{bmatrix} \mathcal{P}_1 & * \\ \mathcal{P}_2 & \mathcal{P}_3 \end{bmatrix}, \quad \mathcal{L} = \begin{bmatrix} L_1 & BL \\ L_2 & L \end{bmatrix}, \quad \mathcal{G} = \begin{bmatrix} G_1 & BL \\ G_2 & L \end{bmatrix}. \quad (34)$$

Substitute (34) into (16) and (17) and defining $N = LK$, the LMI-based l_2 - l_∞ controller design conditions (31) and (32) can be acquired directly. $\square$

Remark 2: From the inequality (17), we can see that the system matrix $\mathcal{A}$ is multiplied by $\mathcal{L}$ and $\mathcal{G}$, respectively. By taking into account the matrix form of $\mathcal{A} = \begin{bmatrix} A & B \\ K & -I \end{bmatrix}$ in (15), the structure of the matrix variables $\mathcal{L}$ and $\mathcal{G}$ is considered in (34) where B in $\mathcal{L}$ and $\mathcal{G}$ is used to

adjust the matrix dimension. Indeed, the structure in $\mathcal{L}$ and $\mathcal{G}$ inevitably leads to a certain degree of design conservatism due to the repetition of the matrix variable L , however, the matrix variables L_1, L_2, G_1 , and G_2 are free that can also produce some relaxation for the controller design. The structure of the matrix variables $\mathcal{L}$ and $\mathcal{G}$ has been widely used in the design of the descriptor system [32], [33].

Remark 3: One should point out that if the constraint condition on the Lyapunov matrix (31) is not counted, the l_2 - l_∞ controller design condition in Theorem 2 residues (32). Relative to the conventional design results (7) and (8), the proposed l_2 - l_∞ controller design condition has been summarized as a single matrix inequality, it can bring convenience to the design.

Remark 4: Theorem 2 provides useful design methods for l_2 - l_∞ control problem of the descriptor system (15), especially, the l_2 - l_∞ controller design conditions (31) and (32) are strict LMIs (the semi-definite matrix inequality (31) can be organized into a strict LMI through constructing the matrix $\mathcal{P}_1$ with a reasonable structure, see [34] for the details). Relative to the conventional design results (7) and (8), the proposed l_2 - l_∞ controller design conditions are more effective for the descriptor system, it can bring convenience to the design.

Remark 5: In Theorem 2, the LMI-based design schemes are introduced to guarantee l_2 - l_∞ performance for the descriptor system (15). From the proof of Theorem 2, one can know that the LMI-based design schemes are obtained with the help of Lemma 1 because $-\mathcal{E}^T \mathcal{P} \mathcal{E} - \mathcal{C}^T \mathcal{C} \leq 0$. In the study on the problem of controller design for descriptor system via LMI technique, Lemma 1 has been effectively applied [32], [34]. The focus of this paper is to give LMI representation of the design conditions of the l_2 - l_∞ controller for discrete-time descriptor system. In fact, the extensive research has been devoted to extended LMI characterizations of Lemma 1 for the problem of stability and system performance in [35], which will provide further analysis and design for discrete-time descriptor system with the l_2 - l_∞ performance.

Remark 6: It should be noted that matrix inequality conditions in Theorem 2 can be solved in polynomial time with complexity proportional to $\mathbb{C} = \mathbb{D}^3 \mathbb{L}$ where $\mathbb{D}$ and $\mathbb{L}$ means the number of undetermined variables and lines of the matrix inequalities to be solved. Therefore, we can analyse the numerical complexity by calculating $\mathbb{D}$ and $\mathbb{L}$. The numbers of variables and the numbers of lines in Theorem 2 are $\mathbb{D}_{Th2} = \frac{5}{2}n_x n_x + \frac{3}{2}n_u n_u + \frac{1}{2}n_x + \frac{1}{2}n_u$, $\mathbb{L}_{Th2} = 2n_x + 2n_u + n_w + r$.

IV. EXAMPLE

In this section, we aim to validate the feasibility and the effectiveness of the proposed methods by introducing the RC circuit shown in Fig. 1. From the Kirchhoff's laws, one can obtain $C\dot{v} = -Gv + i_R$ and $0 = -v - m(i_R)$ where $\underline{m} \leq m(i_R)/i_R \leq \bar{m}$ with $\underline{m} \geq 0$. That has appeared in [26], [36], in which $m(i_R)$ is only sector constrained. In summary,

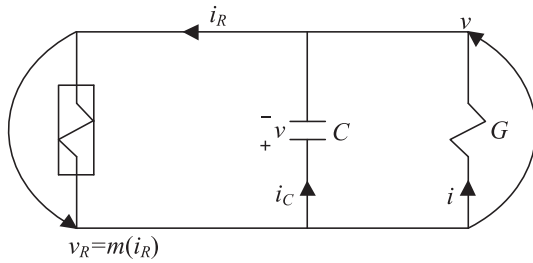


FIGURE 1. The RC circuit.

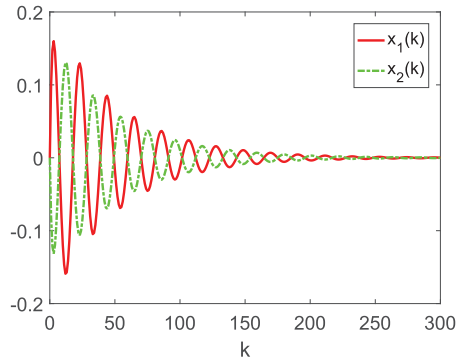


FIGURE 2. Responses of system states.

an augmented equation can be described as

$$\begin{aligned} \dot{v} &= -\frac{G}{C}v + \frac{1}{C}i_R \\ 0 &= -v - \Phi(i_R) - \underline{m}i_R \end{aligned} \quad (35)$$

where $\Phi(i_R) = m(i_R) - \underline{m}i_R$. Denote $x(t) = [x_1(t) \ x_2(t)]^T = [v(t) \ i_R(t)]^T$, $z(t) = x_1(t) = v(t)$, $u(t) = -\Phi(i_R)$, and consider the disturbance, (54) may be expressed as

$$\begin{aligned} E_c \dot{x}(t) &= A_c x(t) + B_c u(t) + F_c w(t) \\ z(t) &= C_c x(t) \end{aligned} \quad (36)$$

In (55), considering $\underline{m} = 1$ and choose these parameters by $C = 0.2\text{F}$ and $G = 1\text{S}$ respectively, we have

$$\begin{aligned} E_c &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_c = \begin{bmatrix} -1/0.2 & 1/0.2 \\ -1 & -1 \end{bmatrix} \\ B_c &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad F_c = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad C_c = [5 \ 0] \end{aligned} \quad (37)$$

and for the convenience of demonstrating the different algorithms.

For the above continuous-time system, by using the discretization process with $h = 0.05\text{s}$, a discrete-time descriptor system with the following parameters can be obtained

$$\begin{aligned} E &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.75 & 0.25 \\ -0.05 & -0.05 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.05 \end{bmatrix} \\ F &= \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad C = [5 \ 0]. \end{aligned} \quad (38)$$

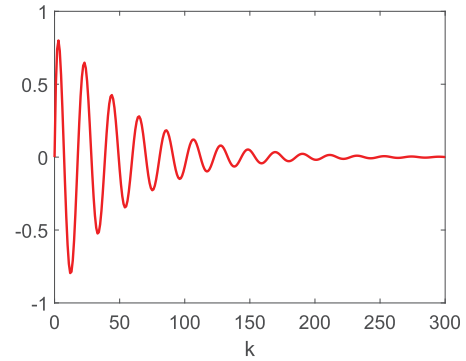


FIGURE 3. Response of $z(k)$.

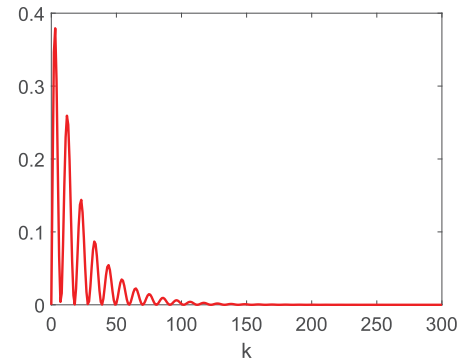


FIGURE 4. Ratio of $\sqrt{z^T(k)z(k)/\sum_{k=0}^{\infty} w^T(k)w(k)}$.

This detailed discretization process can be seen in Appendix at end of this paper.

In the next, we will design the controller defined in (2) for system (38) by using the design schemes provided in Theorem 2. It ensures the system (38) with the determined controller meets the given l_2 - l_∞ index γ .

By choosing $\gamma = 2.0$ and applying the MATLAB LMI Control Toolbox to solve (31) and (32) for the considered system (38), the related matrix variables can be obtained by $L = -3.9640$, $N = [-10.9829 \ -12.5189]$. Moreover, by (33), one gives the l_2 - l_∞ controller gain as

$$K = [2.7706 \ 3.1581]. \quad (39)$$

By considering $x(0) = [0 \ 0]^T$ and $w(k) = \cos(0.3k)e^{-0.02k}$, we will respectively simulate the descriptor system (38) with the controller.

Based on the controller gain (39) for the descriptor system (38): The simulation results of $x_1(k)$ and $x_2(k)$ are illustrated in Fig. 2. Fig. 3 shows the simulation result of $z(k)$. Fig. 4 shows the trend of ratio of $\sqrt{z^T(k)z(k)/\sum_{k=0}^{\infty} w^T(k)w(k)}$, which clearly appears to be decreasing at a macro level. And we can see the maximum value 0.3793 is less than the given performance index 2.0.

The results indicate the designed l_2 - l_∞ controllers achieve the prescribed control tasks for the descriptor system (38). In addition, it should be noted that when Lemma 1 is applied in Theorem 1, a structure of the auxiliary matrix variable

$[\mathcal{L}^T \ 0]^T$ is selected. Of course, the auxiliary matrix variable with the structure $[\mathcal{L}^T \ \mathcal{M}^T]^T$ can also be considered in the controller design. One notes that the matrix variable $\mathcal{M}$ also is coupled with the system matrix $\mathcal{A}$, then which can certainly be constructed as $\mathcal{M} = [M \ L]$. However, if we use the structure of the matrix variable to be $[M \ L]$ in Theorem 2 and design the l_2 - l_∞ controller for the above system (38), the corresponding condition is feasible only for larger γ , compared with the γ in our current Theorem 2. That means the structure of $[\mathcal{L}^T \ 0]^T$ used in the presented Theorem 2 is better for design conservatism.

A. COMPARATIVE EXPLANATIONS

This paper gives useful l_2 - l_∞ controller design schemes for discrete-time descriptor system. Compared with the existing design methods for l_2 - l_∞ performance [21]–[24], the main advantages of the proposed results in this paper can be summarized as the next: 1) The conditions presented in Theorem 2 are effective and feasible for the l_2 - l_∞ controller design of descriptor system (38); 2) The conditions (31) and (32) are strict LMIs, which can easily solved by using the MATLAB LMI Control Toolbox.

V. CONCLUSION

The l_2 - l_∞ control problem for discrete-time descriptor system has been investigated in this work. The resultant control system is formulated via augmented descriptor representation approach. And in this paper, for the known controller gain matrices, we have given the analysis for the l_2 - l_∞ performance for the descriptor control system to detect whether or not the known controllers can ensure the feedback descriptor system guarantees the admissibility and meets the required l_2 - l_∞ performance index γ . Then, based on the analysis criteria, the corresponding controller design scheme is proposed according to the solution of the strict LMIs, which can ensure the discrete-time descriptor system is admissible with the prescribed l_2 - l_∞ performance. The RC circuit has been presented to explain the effectiveness of the proposed method. So in the future work, we will extend the proposed results to the l_2 - l_∞ feedback control for the nonlinear descriptor systems [37]–[42].

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