IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received May 12, 2020, accepted May 25, 2020, date of publication May 29, 2020, date of current version June 17, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2998463

Process-Quality Evaluation for Wire Bonding With Multiple Gold Wires

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This work was supported in part by the Ministry of Science and Technology, Taiwan, under Grant MOST 108-2622-E-167-008-CC3.

ABSTRACT Taiwan is a world-leader in wafer foundry services and IC packaging and testing. Wire bonding is a crucial process in the overall IC-packaging industry chain. Thus, this paper proposes a process-quality evaluation model for wire bonding with multiple gold wires. We chose process quality indices as a tool of evaluation fully mirroring process yield and quality levels. These indices contain unknown parameters and thus require sample data to estimate. We first derived the uniformly minimum variance unbiased estimator of the indices and calculated the upper confidence limits of the indices based on DeMorgan's theorem and Boole's inequality. The upper confidence limits of the indices were then employed to create a confidence interval-based fuzzy membership function, in order to improve the accuracy of estimation as well as solve the problem of uncertainty of the measured data. Next, we obtained the fuzzy critical value and used index estimates and the fuzzy critical value to establish fuzzy test rules. Next, we marked the fuzzy critical value on the axes of a radar chart, which is a visualization evaluation tool, and connected neighboring critical points to create a critical region in the form of a regular polygon. The observed values of the indices were then marked on the axes to produce a visualized fuzzy radar evaluation chart. This fuzzy radar evaluation chart has a solid foundation in statistical inference, and evaluation rules were established using precise fuzzy test methods. Not only is this fuzzy radar evaluation chart easy to use, but it also reduces the chance of misinterpretations made by sampling errors, so that the accuracy of evaluation can be enhanced.

INDEX TERMS Process quality index, fuzzy critical value, critical region, wire bonding, radar chart.

I. INTRODUCTION

Taiwan's wafer foundry output accounts for approximately 70% of the global market, and its IC packaging output occupies roughly 50% of the global market, marking Taiwan out as a world-leader in the industry. Chen *et al.* [1], Tseng *et al.* [2], and Tunn *et al.* [3] suggested that the cluster effect of Taiwan's electronics industry has resulted in an industrial eco-chain for information-and-communication technology (ICT), which dominates the electronics industry worldwide. In addition, Taiwan has a complete industry chain from IC design, foundry services, and packaging and testing all the way to production and assembly [4]. IC packaging includes wafer dicing, die bonding, wire bonding, encapsulation, packing, singulation and lead forming, marking, and lead finishing, as shown in Figure 1. In this study, we focus on wire bonding, which involves welding gold wires onto the inner leads of chips and lead frames. These wires are vital media for electrical connections and signal transmissions between the internal and external circuits of ICs [5], [6].

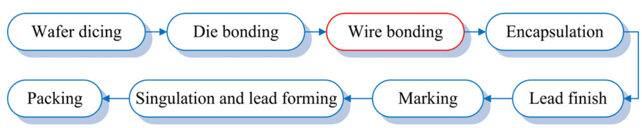

FIGURE 1. IC packaging flow chart.

We therefore propose a fuzzy process-quality evaluation model for wire bonding in IC packaging. Fuzzy evaluation

The associate editor coordinating the review of this manuscript and approving it for publication was F. K. Wang.

models not only assess process quality but also identify which critical-to-quality (CTQ) characteristics require improvement [7]–[10].

Process capability indices (PCIs) are tools for process-quality evaluation commonly applied in the industry. They can help manufacturers evaluate the process quality of their production processes and serve as an effective and convenient means of communication between internal engineers [11]–[13]. Six Sigma, developed in 1986 by Motorola, is widely applied to enhance product quality levels in the manufacturing industry [5], [14]–[16]. Several studies have investigated the correlations between PCIs and Six Sigma quality levels [9], [10], [17]. The wire-bonding process has two significant quality characteristics, both of which are the larger-the-better (LTB) type. Based on the work of Chang *et al.* [18], this study proposes the following process-quality index suitable for assessing LTB quality characteristics. According to Aslam [19], there is a one-to-one mathematical relationship between this index and process yield presenting Six Sigma quality levels.

Obviously, process quality index can completely reflect the process yield and quality level. Therefore, the proposed index is employed to evaluate the quality of the wire-bonding process for IC packaging. This process-quality index involves unidentified parameters, so sample data is needed for estimation [20]–[22]. Many researchers have created fuzzy evaluation models [15]–[23] using the confidence intervals of indices, attempting to improve the accuracy of estimation and overcome uncertainty in the measured data. We first derived the upper confidence limit of the proposed index and employed the method used by Buckley [26] and Chen and Chang [27] to create a confidence-interval-based fuzzy membership function. Then, we obtained the fuzzy critical value and used index estimates and the fuzzy critical value to establish fuzzy test rules. Finally, an easy-to-use visualized radar chart serves as the evaluation interface. The radar chart is a visualization tool widely used in fields such as engineering, management, and education [28]–[31]. Not only is this fuzzy radar evaluation chart easy to use, but it also diminishes the chance of misinterpretation due to sampling errors, thereby increasing the accuracy of evaluation.

The rest of this paper is arranged as follows. Section 2 demonstrates the estimations of process-quality indices and finds a *uniformly minimum variance unbiased estimator* (*UMVUE*) of index *PQIL* and its $100\,(1-\alpha)\,\%$ upper confidence limits. Section 3 applies these upper confidence limits to derive the fuzzy critical value. Next, index estimates and the fuzzy critical value are used to establish fuzzy test rules. Based on these rules, Section 4 constructs an easy-to-use visualized radar evaluation chart to serve as an evaluation interface and an evaluation procedure is established. Section 5 presents our conclusions.

II. ESTIMATIONS OF PROCESS-QUALITY INDICES

As previously mentioned, IC and ceramic packages generally have several gold wires, as shown in Figure 2.

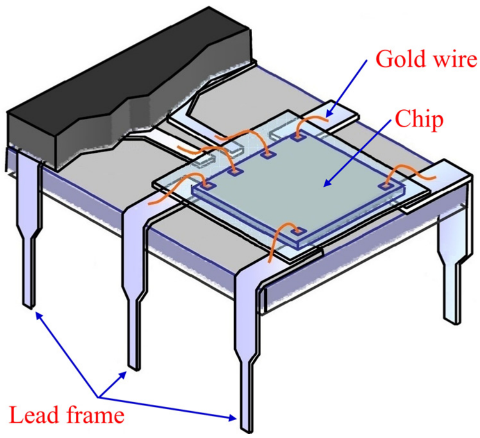

FIGURE 2. Structure of IC package.

In an attempt to prevent loss of generality, we assume that an IC packaging process involves l gold wires, each of which has two LTB quality characteristics: will pull strength ($h = 1$) and ball shear strength ($h = 2$). Let random variable X_{jh} denote the process distributions of quality characteristic h for gold wire j. Then X_{jh} is distributed normally with mean μ_{jh} and standard deviation σ_{jh}, where $h = 1, 2$ and $j = 1, \ldots, l$. The process quality indices can be defined as follows:

$$PQIL_{jh} = \frac{\mu_{jh} - LSL_{jh}}{\sigma_{jh}} \tag{1}$$

where LSL_{jh} stands for the lower specification limit. Based on Chang *et al.* [18], we know that $\mu_{jh} - k\sigma_{jh} = LSL_{jh}$ indicates that the process quality attains the $k - sigma$ level, and therefore,

$$PQIL_{jh} = \frac{\mu_{jh} - LSL_{jh}}{\sigma_{jh}} = \frac{k\sigma_{jh}}{\sigma_{jh}} = k. \tag{2}$$

Clearly, when $PQIL_{jh}$ equals k, the process quality exceeds the $k - sigma$ level. Furthermore, there is a one-to-one mathematical relationship between index $PQIL_{jh}$ and process yield p:

$$\begin{aligned} p_{jh} &= p\left(X_{jh} \geq LSL_{jh}\right) = p\left(Z < \frac{\mu_{jh} - LSL_{jh}}{\sigma_{jh}}\right) \\ &= \Phi\left(PQIL_{jh}\right) \end{aligned} \tag{3}$$

where $Z = \left(X_{jh} - \mu_{jh}\right)/\sigma_{jh}$ follows standard normal distribution and $\Phi(z)$ is the cumulative distribution function for standard normal distribution.

Based on Chang *et al.* [18], we let $\left(X_{jh1}, \cdots, X_{jhi}, \cdots, X_{jhn}\right)$ be a random sample of X_{jh} with sample size n. The estimators of μ_{jh} and σ_{jh} are expressed as follows:

$$\bar{X}_{jh} = \frac{1}{n} \times \sum_{i=1}^{n} X_{jhi} \tag{4}$$

and

$$S_{jh} = \sqrt{\frac{1}{n-1} \times \sum_{i=1}^{n} \left(X_{jhi} - \bar{X}_{jh}\right)^2} \tag{5}$$

Therefore, the estimators of the process-quality indices are as follows:

$$PQIL_{jh}^{*} = b_n \times \left(\frac{\bar{X}_{jh} - LSL_{jh}}{S_{jh}} \right) \tag{6}$$

where b_n is the correction factor expressed as follows:

$$b_n = \frac{\Gamma\left((n-1)/2\right)}{\Gamma\left((n-2)/2\right)} \sqrt{\frac{2}{n}}, \quad n > 2. \tag{7}$$

Obviously, if n approaches infinity, then $b_n = 1$. Under the assumption of normality, $\bar{X}_{jh} - LSL_{jh}$ is distributed as $N\left(\mu_{jh} - LSL_{jh}, \sigma_{jh}^2 / n\right)$ and S_{jh}^{-1} is distributed as $\left(n/\sigma_{jh}\right) K^{-1/2}$, where K is distributed as χ_{n-1}^2. Obviously, the expected value of $\bar{X}_{jh} - LSL_{jh}$ is $\mu_{jh} - LSL_{jh}$ and the expected value of S_{jh}^{-1} can be expressed as follows:

$$\begin{aligned} &E\left[S_{jh}^{-1}\right] \\ &= \left(\sqrt{n}/\sigma_{jh}\right) E\left[K^{-1/2}\right] \\ &= \frac{\sqrt{n}}{\sigma_{jh}} \times \int_0^{\infty} \frac{1}{\Gamma\left((n-1)/2\right) 2^{(n-1)/2}} k^{(n-1)/2-1} exp\left\{\frac{k}{2}\right\} d_k \\ &= \frac{\Gamma\left((n-2)/2\right)}{\Gamma\left((n-1)/2\right)} \sqrt{\frac{n}{2}} \times \sigma_{jh}^{-1} = b_n^{-1} \times \sigma_{jh}^{-1} \end{aligned} \tag{8}$$

Since $\bar{X}_{jh}$ and S_{jh}^2 are mutually independent, the expected value of $PQIL_{jh}^{*}$ can be expressed as follows:

$$\begin{aligned} E\left(PQIL_{jh}^{*}\right) &= b_n \times E\left[\bar{X}_{jh} - LSL_{jh}\right] \times E\left[S_{jh}^{-1}\right] \\ &= \frac{\mu_{jh} - LSL_{jh}}{\sigma_{jh}} = PQIL_{jh}. \end{aligned} \tag{9}$$

Therefore, the unbiased estimator $PQIL_{jh}^{*}$ is only a function of $\left(\bar{X}_{jh}, S_{jh}^2\right)$. Therefore, $PQIL_{jh}^{*}$ is the *UMVUE* of$PQIL_{jh}^{*}$. Obviously, the distribution of $b_n^{-1} \times \sqrt{n} \times PQIL_{jh}^{*}$ is a non-central t-distribution with $n-1$ degrees of freedom and the non-centrality parameter δ is $\sqrt{n} \times PQIL_{jh}^{*}$, denoted as $t'_{n-1}(\delta)$.

As mentioned before, $K = (n-1) S_{jh}^2 / \sigma_{jh}^2$ is distributed as χ_{n-1}^2. Let the random variable Z be as follows:

$$Z = \sqrt{n}\left[PQIL_{jh}^{*} \times b_n^{-1} \times \left(\frac{S_{jh}^{*}}{\sigma_{jh}}\right) - PQIL_{jh}\right]. \tag{10}$$

Then random variable Z follows standard normal distribution. Therefore,

$$\begin{aligned} &1 - \frac{\alpha}{2} \\ &= p\left\{Z \geq -Z_{\alpha/2}\right\} \\ &= p\left\{\sqrt{n}\left[PQIL_{jh}^{*} \times b_n^{-1} \times \left(\frac{S_{jh}^{*}}{\sigma_{jh}}\right) - PQIL_{jh}\right] \geq -Z_{\alpha/2}\right\} \\ &= p\left\{PQIL_{jh} \leq \frac{\bar{X}_{jh} - LSL_{jh}}{\sigma_{jh}} + \frac{Z_{\alpha/2}}{\sqrt{n}}\right\} \end{aligned} \tag{11}$$

and

$$\begin{aligned} 1 - \frac{\alpha}{2} &= p\left\{K \leq \chi_{1-\alpha/2;n-1}^2\right\} \\ &= p\left\{\frac{(n-1) S_{jh}^2}{\sigma_{jh}^2} \leq \chi_{1-\alpha/2;n-1}^2\right\} \\ &= p\left\{\sigma_{jh} \geq \sqrt{\frac{n-1}{\chi_{1-\alpha/2;n-1}^2}} \times S_{jh}\right\} \end{aligned} \tag{12}$$

where $Z_{\alpha/2}$ is the upper $\alpha/2$ quantile of $N(0,1)$ and $\chi_{1-\alpha/2;n-1}^2$ is the lower $1-(\alpha/2)$ quantile of χ_{n-1}^2. Furthermore, let event E_{jh1} and event E_{jh2} be as follows:

$$E_{jh1} = \left\{PQIL_{jh} \leq \frac{\bar{X}_{jh} - LSL_{jh}}{\sigma_{jh}} + \frac{Z_{\alpha/2}}{\sqrt{n}}\right\}$$

and

$$E_{jh2} = \left\{\sigma_{jh} \geq \sqrt{\frac{n-1}{\chi_{1-\alpha/2;n-1}^2}} \times S_{jh}\right\}.$$

Then the complement of event E_{jh1} and event E_{jh2} can be shown as follows:

$$E_{jh1}^{c} = \left\{PQIL_{jh} > \frac{\bar{X}_{jh} - LSL_{jh}}{\sigma_{jh}} - \frac{Z_{\alpha/2}}{\sqrt{n}}\right\}$$

and

$$E_{jh2}^{c} = \left\{\sigma_{jh} < \sqrt{\frac{n-1}{\chi_{1-\alpha/2;n-1}^2}} \times S_{jh}\right\}.$$

Based on DeMorgan's rule and Boole's inequality,

$$p\left(E_{jh1} \cap E_{jh2}\right) \geq 1 - p\left(p\left(E_{jh1}^{c}\right)\right) - p\left(E_{jh2}^{c}\right) = 1 - \alpha. \tag{13}$$

Thus,

$$\begin{aligned} &p\left\{PQIL_{jh} \leq \frac{\bar{X}_{jh} - LSL_{jh}}{\sigma_{jh}} + \frac{Z_{\alpha/2}}{\sqrt{n}}, \sigma_{jh} \geq \sqrt{\frac{n-1}{\chi_{1-\alpha/2;n-1}^2}} \times S_{jh}\right\} \\ &\geq 1 - \alpha. \end{aligned} \tag{14}$$

Equivalently,

$$\begin{aligned} &p\left\{PQIL_{jh} \leq PQIL_{jh}^{*} \times \sqrt{\frac{\chi_{1-\alpha/2;n-1}^2}{n-1}} \times b_n^{-1} + \frac{Z_{\alpha/2}}{\sqrt{n}}\right\} \\ &\geq 1 - \alpha. \end{aligned} \tag{15}$$

Suppose the observed values of $\left(X_{jh1}, \cdots, X_{jhi}, \cdots, X_{jhn}\right)$ are $\left(x_{jh1}, \cdots, x_{jhi}, \cdots, x_{jhn}\right)$. Then, $\bar{x}_{jh}$ and s_{jh} are respectively the observed values of $\bar{x}_{jh}$ and s_{jh} as shown below:

$$\bar{x}_{jh} = \frac{1}{n} \times \sum_{i=1}^{n} x_{jhi} \tag{16}$$

and

$$s_{jh} = \sqrt{\frac{1}{n-1} \times \sum_{i=1}^{n} \left(x_{jhi} - \bar{x}_{jh}\right)^2}. \tag{17}$$

Thus, the observed value of upper confidence limit for index $PQIL_{jh}$ is

$$UPQIL_{jh0} = PQIL_{jh0}^{**} \times \sqrt{\frac{\chi^2_{1-\alpha/2;n-1}}{n-1}} + \frac{Z_{\alpha/2}}{\sqrt{n}} \tag{18}$$

where $PQIL_{jh0}^{**}$ is the observed value of $PQIL_{jh}^{*} \times b_n^{-1}$ as displayed below.

$$PQIL_{jh0}^{**} = \frac{\bar{x}_{jh} - LSL_{jh}}{s_{jh}}. \tag{19}$$

III. FUZZY HYPOTHESIS TESTING

Statistical hypothesis testing is an effective method by which to determine if the process-quality level is legitimate. As stated above, when $PQIL_{jh} = k$, this indicates that the process quality has reached the $k - sigma$ level. Hypothesis testing at the significance level α is expressed as follows:

$$\begin{cases} H_0 : PQIL_{jh} \geq k(meets\ the\ requirement) \\ H_a : PQIL_{jh} < k(does\ not\ meet\ the\ requirement) \end{cases} \tag{20}$$

As mentioned before, the null hypothesis H_0 is $PQIL_{jh} \geq k$ and the alternative hypothesis H_a is $PQIL_{jh} < k$. If $UPQIL_{jh0} \geq k$ then we have $PQIL_{jh0}^{**} \geq k_S$ where

$$k_S = \left(k - \frac{Z_{\alpha/2}}{\sqrt{n}}\right) \Bigg/ \sqrt{\frac{\chi^2_{1-\alpha/2;n-1}}{n-1}}.$$

Therefore, the statistical hypothesis testing rules are as follows:

(1) If $PQIL_{jh0}^{**} \geq k_S$, then do not reject H_0 and conclude that process quality meets requirement.

(2) If $PQIL_{jh0}^{**} < k_S$, then reject H_0 and conclude that process quality does not meet requirement.

Using the abovementioned rules and the methodology proposed by Chen [20], we developed a fuzzy testing method based on the observed value of the upper confidence limit for index $PQIL_{jh}$. As described by Chen [20], the α-cuts of the triangular fuzzy number $\overline{PQIL}_{jh0}^{**}$ are as follows:

$$\overline{PQIL}_{jh0}^{**}[\alpha] = \begin{cases} \left[PQIL_{jh0}^{**}(1), PQIL_{jh0}^{**}(\alpha)\right], \\ \quad \text{for } 0.01 \leq \alpha \leq 1 \\ \left[PQIL_{jh0}^{**}(1), PQIL_{jh0}^{**}(0.01)\right], \\ \quad \text{for } 0 \leq \alpha \leq 0.01 \end{cases} \tag{21}$$

where

$$PQIL_{jh0}^{**}(1) = PQIL_{jh0}^{**} \times \sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}}, \tag{22}$$

$$PQIL_{jh0}^{**}(\alpha) = PQIL_{jh0}^{**} \times \sqrt{\frac{\chi^2_{1-\alpha/2;n-1}}{n-1}} + \frac{Z_{\alpha/2}}{\sqrt{n}}. \tag{23}$$

The half-triangular fuzzy number of $PQIL_{jh}$ is $\overline{PQIL}_{jh0}^{**} = \Delta\left(PQ_{jhM}, PQ_{jhR}\right)$, where

$$PQ_{jhM} = \overline{PQIL}_{jh0}^{**}(1) = PQIL_{jh0}^{**} \times \sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}},$$

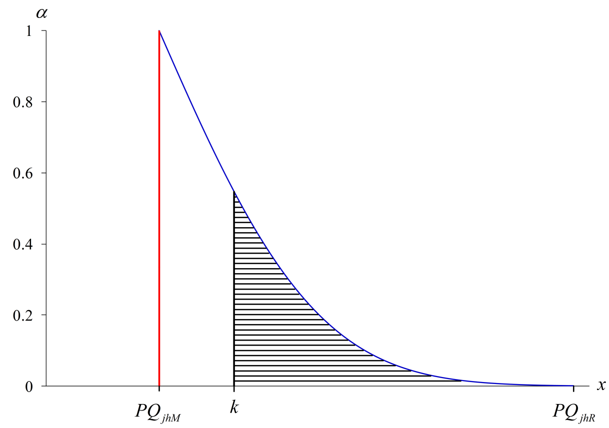

FIGURE 3. Membership function $\eta_{jh}(x)$ with vertical line $x = k$.

$$PQ_{jhR} = \overline{PQIL}_{jh0}^{**}(0.01) \tag{24}$$

$$= PQIL_{jh0}^{**} \times \sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} + \frac{Z_{0.005}}{\sqrt{n}}. \tag{25}$$

Therefore, the membership function of the fuzzy number $\overline{PQIL}_{jh0}^{**}$ is

$$\eta_{jh}(x) = \begin{cases} 0 & if\ x < PQ_{jhM} \\ 1 & if\ x = PQ_{jhM} \\ \alpha & if\ PQ_{jhM} < x < PQ_{jhR} \\ 0 & if\ x \geq PQ_{jhM} \end{cases} \tag{26}$$

where α is determined by $PQIL_{jh0}^{**}(\alpha) = x$. Figure 3 exhibits membership function $\eta_{jh}(x)$ with vertical line $x = k$.

Based on Chen *et al.* [23] and Chen [20], we let set A_{Tjh} be the area of $\eta_{jh}(x)$ and set A_{Rjh} be the area of $\eta_{jh}(x)$ to the right of the vertical line $x = k$. Then,

$$A_{Tjh} = \left\{(x, \alpha) \middle| PQ_{jhM} \leq x \leq PQIL_{jh0}^{**}(\alpha), 0 \leq \alpha \leq 1\right\} \tag{27}$$

and

$$A_{Rjh} = \left\{(x, \alpha) \middle| k \leq x \leq PQIL_{jh0}^{**}(\alpha), 0 \leq \alpha \leq 1\right\}. \tag{28}$$

Thus,

$$d_{Rjh} = PQ_{jhR} - k = PQIL_{jh0}^{**} \times \sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} + \frac{Z_{0.005}}{\sqrt{n}} - k \tag{29}$$

and

$$\begin{aligned} d_{Tjh} &= PQ_{jhR} - PQ_{jhM} \\ &= PQIL_{jh0}^{**} \times \left(\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} - \sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}}\right) + \frac{Z_{0.005}}{\sqrt{n}}. \end{aligned} \tag{30}$$

Based on Chen *et al.* [32],

$$\frac{d_{Rjh}}{2d_{Tjh}} = \frac{PQIL_{jh0}^{**} \times \sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} + \frac{Z_{0.005}}{\sqrt{n}} - k}{2PQIL_{jh0}^{**} \times \left(\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} - \sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}}\right) + \frac{2Z_{0.005}}{\sqrt{n}}}. \tag{31}$$

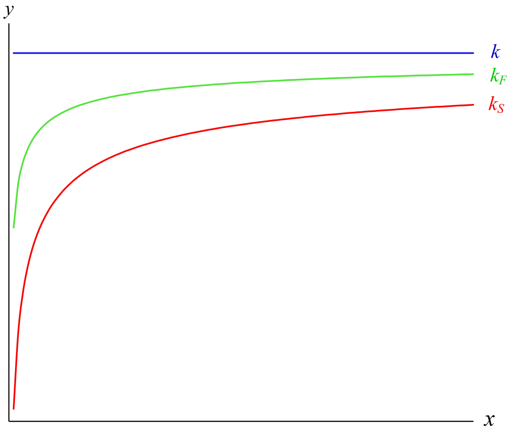

FIGURE 4. Relationship between critical values and sample size.

Obviously, when $PQ_{jhM} = k$, $d_{Rjh}/2d_{Tjh} = 1/2$. On the basis of the approach adopted by Chen *et al.* [23] and Buckley [26], we let $0 < \phi \leq 0.5$ and let the decision value dv_{jh} of quality characteristic h of gold wire j satisfy the following equation:

$$\frac{PQ_{jhR} - dv_{jh}}{2d_{Tjh}} = \phi. \tag{32}$$

Then decision value $dv_{jh} = PQ_{jhR} - 2\phi d_{Tjh}$ can be shown as follows:

$$dv_{jh} = PQIL^{**}_{jh0} \times \left((1-2\phi)\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} + 2\phi\sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}} \right) + (1-2\phi)\frac{Z_{0.005}}{\sqrt{n}}. \tag{33}$$

Based on Chen *et al.* [23], the decision rule for fuzzy testing is

(1) If $k \geq dv_{jh}$ is equivalent to $d_{Rjh}/2d_{Tjh} \leq \phi$, then reject H_0 and conclude that $PQIL_{jh} < k$.
(2) If $k < dv_{jh}$ is equivalent to $d_{Rjh}/2d_{Tjh} > \phi$, then do not reject H_0 and conclude that $PQIL_{jh} \geq k$.

According to Eq. (32), $k \geq dv_{jh}$ is equivalent to $PQIL^{**}_{jh0} \leq k_F$ where

$$k_F = \frac{k - (1-2\phi)\frac{Z_{0.005}}{\sqrt{n}}}{\left((1-2\phi)\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} + 2\phi\sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}} \right)}. \tag{34}$$

We set k_F as a fuzzy critical value. Figure 4 displays the relationship between sample size n and the critical values.

$k_S < k_F < k$ can be distinguished clearly in Figure 4. Obviously, under the situation of the same sample size, k_F will be closer to k value than k_S. Therefore, fuzzy testing is more precise than statistical testing. Besides, whether k_F or k_S is a function of sample size n, and $\lim_{n\to\infty} k_S = \lim_{n\to\infty} k_F = k$. Then, the decision rules for the fuzzy testing are as follows:

(1) If $PQIL^{**}_{jh0} \geq k_F$, then do not reject H_0 and conclude that $PQIL^{**}_{jh0} \geq k$.
(2) If $PQIL^{**}_{jh0} < k_F$, then reject H_0 and conclude that $PQIL^{**}_{jh0} < k$.

IV. EVALUATION PROCESS USING FUZZY RADAR EVALUATION MODEL

As stated above, in order to determine whether the process quality reaches the 5-*sigma* level, hypothesis testing at significance level.005 is as follows:

$$\begin{cases} H_0 : PQIL_{jh} \geq 5(\textit{meets the requirement}) \\ H_a : PQIL_{jh} < 5(\textit{does not meet the requirement}) \end{cases}$$

The fuzzy critical value k_F calculated using Eq. (34) is then marked on the axes and connected to form a polygon radar evaluation chart. Using Eq. (19), $2 \times l$ index values are then derived and marked on the $2 \times l$ axes to form a visualized evaluation chart. The example employed in this study involved an IC package with 6 gold wires ($l = 6$), which means that it has 12 quality characteristics. Table 1 displays the relevant data.

TABLE 1. Quality characteristics for wire bonding with six wires.

Gold wire (l)	Quality characteristic	Specification
l=1	wire pull (h=1)	LSL_{11}= 4g
	ball shear (h=2)	LSL_{12}=30g
l=2	wire pull (h=1)	LSL_{21}= 4g
	ball shear (h=2)	LSL_{22}=30g
l=3	wire pull (h=1)	LSL_{31}= 5g
	ball shear (h=2)	LSL_{32}=40g
l=4	wire pull (h=1)	LSL_{41}= 5g
	ball shear (h=2)	LSL_{42}=40g
l=5	wire pull (h=1)	LSL_{51}= 5g
	ball shear (h=2)	LSL_{52}=40g
l=6	wire pull (h=1)	LSL_{61}= 5g
	ball shear (h=2)	LSL_{62}=40g

Next, we explain this example using numerical methods to show how to apply the theory proposed in this study. Suppose that a company takes 36 random samples for testing. As the IC package has 6 gold wires ($l = 6$), and each gold wire has two quality characteristics ($h = 2$), then there are a total of 12 sets of sample data. To help manufacturers utilize the proposed evaluation model, we established the following evaluation procedure:

Step1: Calculate the mean of samples, the standard deviation of the samples, and index estimates of the 12 samples using Eqs. (16), (17), and (19).

Step2: Based on Eq. (34), with $n = 60$ and $\phi = 0.3$, the fuzzy critical value can be obtained as follows:

$$k_F = \frac{k - (1-2\phi)\frac{Z_{0.005}}{\sqrt{n}}}{\left((1-2\phi)\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} + 2\phi\sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}} \right)} = 4.455.$$

Step3: Draw 12 axes at 30 degrees from each other, mark the fuzzy critical value (k_F) on the axes of the radar chart,

TABLE 2. Sample data of quality characteristics for wire bonding with six wires.

Sample mean ($\bar{x}_{jh}$)	Sample standard deviation(s_{jh})	$PQIL_{jh0}^{**}$
$\bar{x}_{11} = 4.82$	$s_{11} = 0.171$	$PQIL_{110}^{**} = 4.795$
$\bar{x}_{12} = 31.14$	$s_{12} = 0.232$	$PQIL_{120}^{**} = 4.914$
$\bar{x}_{21} = 4.71$	$s_{21} = 0.169$	$PQIL_{210}^{**} = 4.201$
$\bar{x}_{22} = 31.11$	$s_{22} = 0.251$	$PQIL_{220}^{**} = 4.422$
$\bar{x}_{31} = 5.72$	$s_{31} = 0.161$	$PQIL_{310}^{**} = 4.472$
$\bar{x}_{32} = 41.28$	$s_{32} = 0.242$	$PQIL_{320}^{**} = 5.289$
$\bar{x}_{41} = 6.09$	$s_{41} = 0.227$	$PQIL_{410}^{**} = 4.802$
$\bar{x}_{42} = 41.19$	$s_{42} = 0.243$	$PQIL_{420}^{**} = 5.174$
$\bar{x}_{51} = 5.91$	$s_{51} = 0.163$	$PQIL_{510}^{**} = 5.583$
$\bar{x}_{52} = 41.32$	$s_{52} = 0.253$	$PQIL_{520}^{**} = 5.217$
$\bar{x}_{61} = 5.94$	$s_{61} = 0.179$	$PQIL_{610}^{**} = 5.251$
$\bar{x}_{62} = 41.26$	$s_{62} = 0.246$	$PQIL_{620}^{**} = 5.122$

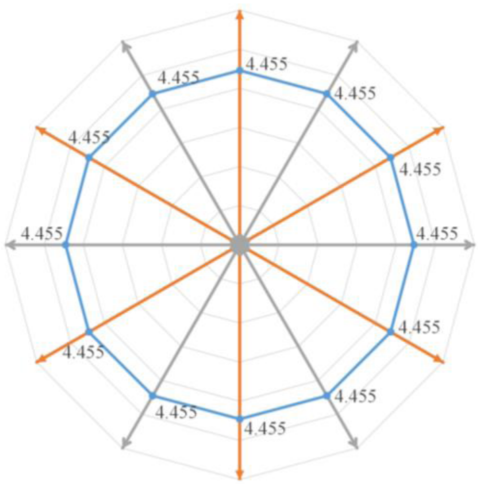

FIGURE 5. Critical region on radar chart.

and connect the neighboring critical points to form a critical region in the form of a regular dodecagon, as shown below:

Step4: Mark the 12 index estimates $PQIL_{jh0}^{**}$ ($j = 1, \ldots, l$ and $h = 1, 2$) in Table 2 on the 12 axes. The two ends of axis j respectively indicate the index estimates $PQIL_{j10}^{**}$ and $PQIL_{j20}^{**}$ of the two quality characteristics of gold wire j, as shown in Figure 6.

Step5: If index estimate $PQIL_{jh0}^{**}$ falls within the twelve-sided critical region and $PQIL_{jh0}^{**} \leq 4.455$, then reject H_0 and conclude that $PQIL_{jh} < 5$. This means that this quality characteristic is in need of improvement. If index estimate $PQIL_{jh0}^{**}$ falls outside of the twelve-sided critical region and $PQIL_{jh0}^{**} > 4.455$, then do not reject H_0 and conclude that $PQIL_{jh} \geq 5$.

Based on the test rules in Step 5, the quality-level index estimates $PQIL_{jh0}^{**}$ for the two quality characteristics of wire 2 both fell within the critical region, which means that they did not meet the five-sigma quality requirement and are in need of improvement. Referring to Eq. (25) and letting $\alpha = 0.05$, the upper confidence limit of the two quality characteristics of gold wire 2 can be calculated as follows:

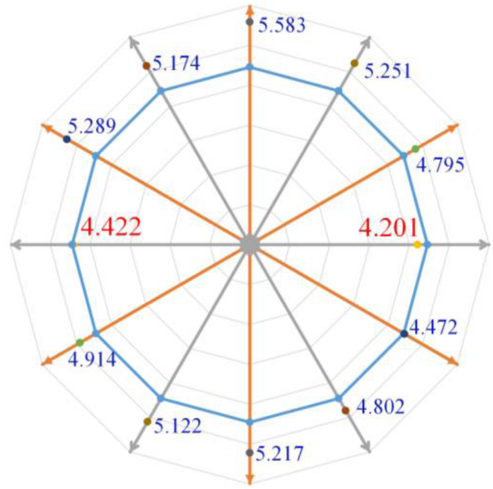

FIGURE 6. Fuzzy radar evaluation chart for gold wires.

$$UPQIL_{21} = 4.201 \times \sqrt{\frac{\chi^2_{0.95;59}}{59}} + \frac{Z_{0.05}}{\sqrt{60}} = 5.040,$$

$$UPQIL_{22} = 4.422 \times \sqrt{\frac{\chi^2_{0.95;59}}{59}} + \frac{Z_{0.05}}{\sqrt{60}} = 5.294.$$

Based on statistical principles, these two upper confidence limits are greater than 5; therefore, we do not reject H_0 and conclude that $PQIL_{jh} \geq 5$. However, $PQIL_{210}^{**} = 4.201$ and $PQIL_{220}^{**} = 4.422$. Both values are obviously smaller than 5. That means the proposed model returns more reasonable results than the previous statistical test results, and is consistent with the discussions made by several studies [20], [24], [33]–[35].

V. CONCLUSIONS

This study proposes a fuzzy quality evaluation model for the wire-bonding process in IC packaging. Both of the quality characteristics of the gold wires are LTB quality characteristics, and for this reason, we propose a process-quality index suitable for LTB quality characteristics based on the concepts presented by Chang *et al.* [18]. This index fully exhibits quality levels and has a one-to-one mathematical relationship with process yield. It is thus a good assessment tool. Since this index involves unknown parameters, sample data are used to find the *UMVUE* of process-quality indices. Based on DeMorgan's rule and Boole's inequality, we developed the $100\,(1-\alpha)\,\%$ upper confidence limits of the indices. Subsequently, we elicited the fuzzy membership function on the basis of these upper confidence limits and obtained the fuzzy critical value to serve as a fuzzy evaluation standard, with which we developed a visualized fuzzy radar evaluation chart. This fuzzy radar evaluation chart has a solid foundation in statistical inference, and evaluation rules were established using precise fuzzy test methods. In addition, this chart constructs a whole picture concerning the important quality characteristics of the product. It facilitates management and aids suppliers in progressing and forming long-term relationships with their partners. Applied in practice, this model will not only benefit industry chains but will also improve the quality of their processes.

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