

A New Method for Harmonic Impedance Estimation

Fang-Wei Xu, Lu Wang, Jin-Shuai Zhao

College of Electrical Engineering and Information
Technology,
Sichuan University
Chengdu, China

Ying Chen, Lin Xu, Can Hu

State Grid Sichuan Electric Power Research Institute
Chengdu, China

Abstract—Determining the harmonic responsibilities of utility and customer side at a PCC has been a great concern in harmonic research, and utility harmonic impedance is a key factor for assessing harmonic emission levels. When background harmonic is unstable, traditional methods have a large error. A new method is proposed in this paper to assess harmonic impedance based on minimum energy fluctuation. The method can calculate harmonic impedance by constructing an object function which minimizes the background harmonic energy fluctuation, without assuming background harmonic is stable. Simulation analysis indicates that the proposed method is more reliable to calculate harmonic impedance, which can reduce the error caused by the fluctuation of background harmonic.

Index Terms—harmonic impedance; harmonic emission level; background harmonics; minimum of energy fluctuation; power quality

I. INTRODUCTION

With a large number of nonlinear loads integrated into the electric network and nonlinear components in the power system itself, harmonic distortion has become one of the great concerns for power quality. Since harmonics at the point of common coupling (PCC) are produced by both utility side and customer side, it is important to evaluate the harmonic emission level on both sides reasonably [1].

Calculating the utility harmonic impedance is a key step for determining harmonic responsibilities. The traditional methods for this can be divided into two classes at present. One is called invasive method and the other non-invasive method. The former is based on injecting harmonic currents into PCC or switching some branches at PCC to measure harmonic impedance. It may cause harmful influence on power system though the results are accurate. The latter is widely used for its safety and easy-implementation.

The typical non-invasive method mainly includes liner regression method [2], fluctuation method [3], etc. Linear regression method makes regression analysis of data of voltage and current at the PCC to calculate harmonic impedance. Fluctuation method is based on symbolic characteristics of ration of harmonic voltage and current fluctuation, it works only if the background harmonic is stable. When background harmonics fluctuate a lot, the above

methods can't meet the accuracy requirement in engineering. Reference [6] proposed independent random vector method. According to the independent random vector covariance is zero, the influence of background harmonic fluctuation can be weakened. However, when background harmonics have large fluctuation, the calculation results still have large errors. Reference [7] proposed a method based on fast independent component analysis, which can effectively suppress the background harmonic interference, but this method can produce analytical error in the process of whitening the data.

Although the above methods can reduce the influence of background harmonic fluctuation on the calculation result to some extent, the error still cannot be ignored when background harmonics fluctuate greatly. This paper proposes a new method to estimate harmonic impedance based on minimizing the background harmonic energy fluctuation without assuming background harmonic is stable. The simulation analysis indicates that this method can evaluate harmonic emission level accurately when the fluctuation of background harmonic is large.

II. THE THEORY OF HARMONIC EMISSION LEVEL EVALUATION

The evaluation index of harmonic emission level for two sides can be considered from harmonic voltage and current. Since harmonic current cannot be compared under different voltage level, it is more reasonable to determine harmonic responsibilities by comparing the harmonic voltage at the PCC. According to the standard IEC61000-3-6, the harmonic voltage emission level is defined by the vector difference between the harmonic voltage before and after the customer has been connected to the network. The equivalent circuit is shown in figure 1.

In Fig.1, $\dot{I}_u$ and Z_u are the utility-side equivalent harmonic current and impedance; $\dot{I}_c$ and Z_c are the customer-side equivalent harmonic current and impedance. The harmonic voltage at the PCC before and after the customer installation connecting to the network can be written respectively as

$$\dot{V}_{PCC-pre} = \dot{I}_s Z_s \quad (1)$$

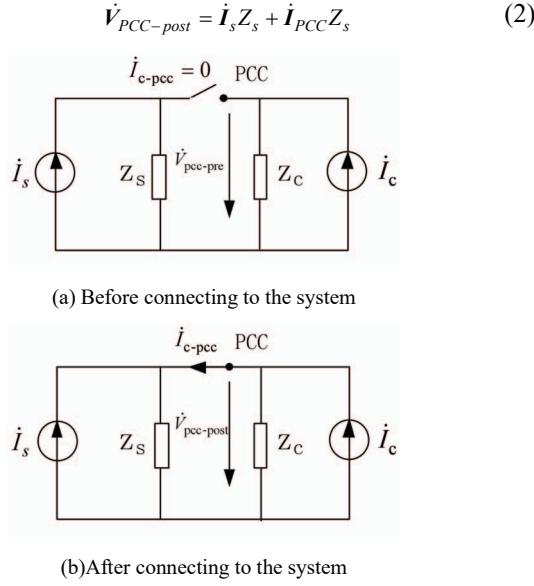


Figure 1. Before and after the customer connecting to the system

where $\dot{V}_{PCC-pre}$ and $\dot{V}_{PCC-post}$ are harmonic voltage of the PCC before and after loads accessing to the system respectively.

We can write

$$\dot{V}_c = \dot{I}_{PCC} Z_s \quad (3)$$

where $\dot{V}_c$ is the customer-side harmonic emission level at the PCC, and $\dot{V}_s$ is the utility-side harmonic emission level at the PCC.

Since $\dot{I}_{pcc}$ can be measured directly at the PCC, the key problem of assessing the customer-side harmonic voltage emission level is how to calculate the utility-side harmonic impedance. The equivalent harmonic impedance of utility side is related to the operation of system and network topology structure, Z_s can be regarded as a constant if the system operation mode doesn't change during a period of time.

III. HARMONIC IMPEDANCE CALCULATION

A. Fundamental principles

As in Fig.1(b),

$$\dot{V}_{PCC}(n) = Z_s [\dot{I}_{PCC}(n) + \dot{I}_s(n)] \quad (1)$$

In the process of sampling, the harmonic voltage, current and the mean of the utility side harmonic current are respectively $\dot{V}_{PCC,m}$, $\dot{I}_{PCC,m}$ and $\dot{I}_{s,m}$.

$$\begin{cases} \dot{V}_{PCC,m} = \frac{1}{N} \sum_{n=1}^N \dot{V}_{PCC}(n) \\ \dot{I}_{PCC,m} = \frac{1}{N} \sum_{n=1}^N \dot{I}_{PCC}(n) \\ \dot{I}_{s,m} = \frac{1}{N} \sum_{n=1}^N \dot{I}_s(n) \end{cases} \quad (2)$$

where N is the total number of sampling points.

One can obtain

$$\dot{V}_{PCC,m} = Z_s (\dot{I}_{PCC,m} + \dot{I}_{s,m}) \quad (3)$$

In this paper, the fluctuation of voltage is defined as the difference between the voltage at each sampling and the mean of all the samples. So is the fluctuation of current. Then

$$\begin{cases} \Delta \dot{V}_{PCC}(n) = \dot{V}_{PCC}(n) - \dot{V}_{PCC,m} \\ \Delta \dot{I}_{PCC}(n) = \dot{I}_{PCC}(n) - \dot{I}_{PCC,m} \\ \Delta \dot{I}_s(n) = \dot{I}_s(n) - \dot{I}_{s,m} \end{cases} \quad (4)$$

Then

$$\Delta \dot{V}_{PCC}(n) = Z_s [\Delta \dot{I}_{PCC}(n) + \Delta \dot{I}_s(n)] \quad (5)$$

After transformation:

$$Z_s = \frac{\Delta \dot{V}_{PCC}(n)}{\Delta \dot{I}_{PCC}(n) + \Delta \dot{I}_s(n)} \quad (6)$$

As in (16), since $\Delta \dot{V}_{PCC}(n)$ and $\Delta \dot{I}_{PCC}(n)$ are known, we should only calculate $\Delta \dot{I}_s(n)$. Since $\Delta \dot{I}_s(n)$ is the background harmonic fluctuation of the utility side, the smaller the corresponding fluctuation energy is, the more accurate Z_s is obtained.

From (5), one can obtain

$$\Delta \dot{V}_{PCC}(n+1) = Z_s [\Delta \dot{I}_{PCC}(n+1) + \Delta \dot{I}_s(n+1)] \quad (7)$$

Multiply $\Delta \dot{V}_{PCC}(n+1)$ on both sides of (5), (5) can be developed as follows:

$$\begin{aligned} \Delta \dot{V}_{PCC}(n+1) \cdot \Delta \dot{V}_{PCC}(n) \\ = Z_s [\Delta \dot{V}_{PCC}(n+1) \cdot \Delta \dot{I}_{PCC}(n) + \Delta \dot{V}_{PCC}(n+1) \cdot \Delta \dot{I}_s(n)] \end{aligned} \quad (8)$$

Multiply $\Delta \dot{V}_{PCC}(n)$ on both sides of (7), (7) can be written as

$$\begin{aligned} \Delta \dot{V}_{PCC}(n) \cdot \Delta \dot{V}_{PCC}(n+1) \\ = Z_s [\Delta \dot{V}_{PCC}(n) \cdot \Delta \dot{I}_{PCC}(n+1) + \Delta \dot{V}_{PCC}(n) \cdot \Delta \dot{I}_s(n+1)] \end{aligned} \quad (9)$$

Subtracting (9) from (8), one can obtain

$$\begin{aligned} 0 = Z_s [\Delta \dot{V}_{PCC}(n+1) \cdot \Delta \dot{I}_{PCC}(n) - \Delta \dot{V}_{PCC}(n) \cdot \Delta \dot{I}_{PCC}(n+1) \\ + \Delta \dot{V}_{PCC}(n+1) \cdot \Delta \dot{I}_s(n) - \Delta \dot{V}_{PCC}(n) \cdot \Delta \dot{I}_s(n+1)] \end{aligned} \quad (10)$$

Since $Z_s \neq 0$, construct a variable:

$$\begin{aligned}\dot{\mathbf{b}}(n) &= \Delta \dot{\mathbf{V}}_{PCC}(n+1) \cdot \Delta \dot{\mathbf{I}}_{PCC}(n) - \Delta \dot{\mathbf{V}}_{PCC}(n) \cdot \Delta \dot{\mathbf{I}}_{PCC}(n+1) \\ &= -\Delta \dot{\mathbf{V}}_{PCC}(n+1) \cdot \Delta \dot{\mathbf{I}}_s(n) + \Delta \dot{\mathbf{V}}_{PCC}(n) \cdot \Delta \dot{\mathbf{I}}_s(n+1)\end{aligned}\quad (11)$$

So

$$\dot{\mathbf{b}} = \begin{pmatrix} \dot{\mathbf{b}}(1) \\ \dot{\mathbf{b}}(2) \\ \vdots \\ \dot{\mathbf{b}}(N) \end{pmatrix} = \begin{pmatrix} -\Delta \dot{\mathbf{V}}_{PCC}(2) & \Delta \dot{\mathbf{V}}_{PCC}(1) & 0 & \cdots & \cdots & 0 \\ 0 & -\Delta \dot{\mathbf{V}}_{PCC}(3) & \Delta \dot{\mathbf{V}}_{PCC}(2) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & -\Delta \dot{\mathbf{V}}_{PCC}(N) & \Delta \dot{\mathbf{V}}_{PCC}(N-1) & 0 \\ 0 & \cdots & \cdots & 0 & -\Delta \dot{\mathbf{V}}_{PCC}(N+1) & \Delta \dot{\mathbf{V}}_{PCC}(N) \end{pmatrix} \begin{pmatrix} \Delta \dot{\mathbf{I}}_s(1) \\ \Delta \dot{\mathbf{I}}_s(2) \\ \vdots \\ \Delta \dot{\mathbf{I}}_s(N+1) \end{pmatrix} = \mathbf{A} \times \Delta \dot{\mathbf{I}}_s \quad (12)$$

$\dot{\mathbf{b}}$ and matrix $\mathbf{A}$ can be calculated by the voltage and current measured by the PCC, and $\Delta \dot{\mathbf{I}}_s$ is unknown. Since the numbers of equations and unknowns are N and $N+1$, there are multiple solutions to the equation. In these solutions, we can find the solution that minimizes the fluctuation of the background harmonic energy, which is the optimal solution for equation (12). For this purpose, set the target function as

$$\begin{aligned}\min J &= \sum_{n=1}^N \left(\Delta \dot{\mathbf{I}}_s(n) \right)^2 \\ &+ \dot{\lambda}^H (\mathbf{A} \Delta \dot{\mathbf{I}}_s - \dot{\mathbf{b}}) + (\mathbf{A} \Delta \dot{\mathbf{I}}_s - \dot{\mathbf{b}})^H \dot{\lambda}\end{aligned}\quad (13)$$

where $\dot{\lambda}$ is the N dimensional column vector, the first part represents the energy fluctuation of background harmonic. Thus, the key problem is to solve equation (12) for target equation (13). And then we can solve for Z_s . When $\Delta \dot{\mathbf{I}}_s$ meets the minimum of J , the utility side is the most stable, and Z_s is also the most accurate.

B. Calculation procedure

In this paper, the proposed method constructs a target function to calculate harmonic impedance of utility side by solving the minimum value of background harmonic fluctuation energy. It can effectively weaken the influence of background harmonic fluctuation. The steps of this method are as follows:

- (1) Collect the harmonic voltage and current data at the PCC and calculate the fluctuation of voltage and current at each sampling time.
- (2) Calculate $\dot{\mathbf{b}}$ and matrix $\mathbf{A}$ from (11) and (12).
- (3) Calculate $\Delta \dot{\mathbf{I}}$ when the background harmonic energy fluctuation gets to minimum.
- (4) Take $\Delta \dot{\mathbf{I}}$ into (6), Z_s can be obtained.

IV. SIMULATION ANALYSIS

A. Simulation parameter setting

In order to verify the validity of the proposed method, this paper makes a comparison between the proposed method and the independent random vector method and the binary linear regression method.

The simulation data is set as follows:

- 1) Harmonic current source: the amplitude of $\dot{I}_c$ is 100 A; the amplitude of $\dot{I}_s$ is 0.1-0.3 times that of $\dot{I}_c$. The phase angle of $\dot{I}_c$ is 0° , the phase angle of $\dot{I}_s$ is 60° . Add random disturbance of $\pm 20\%$ to the amplitude of $\dot{I}_c$, add random disturbance of $\pm 5\%$ to the amplitude of $\dot{I}_s$. Add 10% sine wave to the amplitude and phase angle of $\dot{I}_s$.
- 2) Harmonic impedance: Z_s is $(15 + j20) \Omega$. Z_c is $(80 + j160) \Omega$, and add random disturbance of $\pm 5\%$ to the angle and the amplitude of impedance respectively.

B. Simulation Result analysis

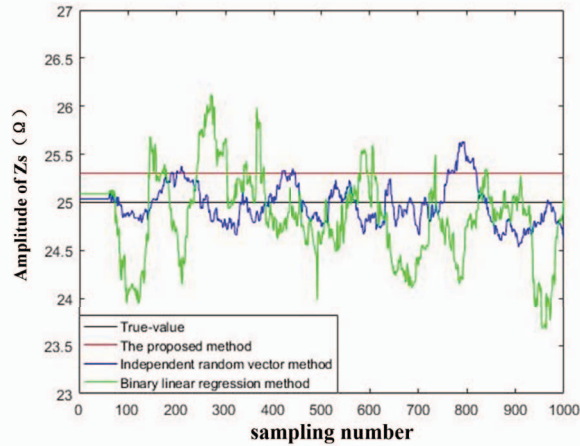
Use the three methods to obtain Z_s . The calculation results are shown in Table 1, Table 2 and Figure 2.

TABLE I. CALCULATION RESULTS AND ERROR ANALYSIS OF AMPLITUDE BY THREE METHODS

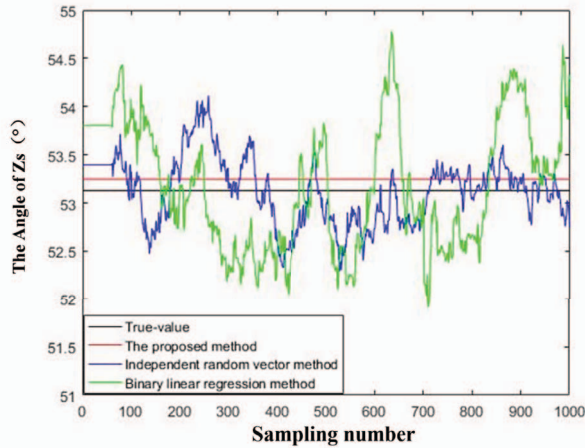
Methods	The proposed method		Independent random vector method		Binary linear regression method	
	Results (Ω)	Error (%)	Results (Ω)	Error (%)	Results (Ω)	Error (%)
k=0.1	25.33	1.33	27.81	11.36	30.24	20.95
k=0.2	25.92	3.61	28.70	14.92	42.66	70.68
k=0.3	27.32	9.41	31.83	27.28	56.51	126.19

TABLE II. CALCULATION RESULTS AND ERROR ANALYSIS OF ANGLE BY THREE METHODS

Methods	The proposed method		Independent random vector method		Binary linear regression method	
	Results (Ω)	Error (%)	Results (Ω)	Error (%)	Results (Ω)	Error (%)
k=0.1	52.96	-0.32	49.41	-6.95	39.40	-25.81
k=0.2	52.83	-0.57	48.58	-8.59	14.50	-72.70
k=0.3	52.95	-0.31	44.83	-15.64	-6.19	-111.67



(a) Amplitude of impedance



(b) The Angle of the impedance

Figure 2. Simulation results of three methods when $k=0.1$

According to Table 1, the calculation error of this method is lower than the other two, no matter the amplitude of impedance or the angle. According to Fig.2, the result calculated by this method is closer to the true value and more stable. The calculation of independent random vector method and the binary regression method has large fluctuation, especially the latter one. It indicates that this method can obtain higher calculation accuracy. It can suppress the influence of background harmonic fluctuation on utility side. Thus the results can be obtained more accurately and stably.

V. CONCLUSION

When background harmonic fluctuation is large, traditional methods cannot calculate the harmonic impedance accurately. In this paper, a novel method of harmonic impedance estimation is proposed which is based on minimum energy fluctuation of background harmonics. The conclusions are as follows:

(1) The method can calculate harmonic impedance by constructing an object function which minimizes the background harmonic energy fluctuation, without assuming background harmonic is stable. Therefore, it is still valid when the background harmonic fluctuation is large.

(2) Compared with the independent random vector method and the binary linear regression method by simulation, the results show that the harmonic impedance error is smaller and the calculation result is more stable in this paper. The harmonic impedance can be calculated accurately in the background harmonic fluctuation.

REFERENCES

- [1] T Pfajfar, B Blazic, I Papic, "Harmonic Contributions Evaluation with the Harmonic Current Vector Method," *IEEE Trans. Power Delivery*, vol.23, pp. 425-433, Jan. 2008.
- [2] Xu W, Liu Y. "A method for determining customer and utility harmonic contributions at the point of common coupling," *IEEE Trans. Power Delivery*, vol.15, pp. 804-811, Apr. 2000.
- [3] Hui J, Freitas W, Vieira J C.M., "Utility Harmonic Impedance Measurement Based on Data Selection," *IEEE Trans. Power Delivery*, vol.27, pp. 2193-2202, Oct. 2012.
- [4] Xu Y, Huang S, Liu Y. "Partial Least-Squares Regression Based Harmonic Emission Level Assessing at the Point of Common Coupling," in *Power Con 2006. International Conference on Power System Technology*. pp. 1-5.
- [5] Yang H, Pirotte P, Robert A, "Harmonic emission levels of industrial loads statistical assessment," in *1996. International Council on Large Electric systems*. pp 36-306.
- [6] Hui J, Yang H, Lin S. "Assessing Utility Harmonic Impedance Based on the Covariance Characteristic of Random Vectors," *IEEE Trans. Power Delivery*, vol.25, pp.1778-1786, July.2010.
- [7] Zhao X, Yang H. "A New Method to Calculate the Utility Harmonic Impedance Based on FastICA," *IEEE Trans. Power Delivery*, vol.31, pp. 381-388, Feb. 2016.