Testing and Validation of FTR Results in NZEM

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***Abstract*— This paper presents an auction and pricing model for both obligation and option Financial Transmission Rights (FTRs). A number of cases are established and run using our model and the FTR auction results are compared with those using a new FTR software from a reputed vendor. Two methods of FTR clearing price calculation are proposed for each studied case. Though some of the results are known and in use by a few system operators, the proposed model and case studies demonstrate clearly the FTR clearing mechanism and price formation, using varieties of cases. The case studies presented in this paper are very helpful for testing and pre-auditing a new FTR program from a vendor and for tutorial purpose.**

***Index Terms*—Electricity Markets, Financial Transmission Rights, FTR awards, Locational Marginal Price, LMP, obligation FTRs, Option FTRs, Shift Factors, System Operator, Transmission Congestion.**

I. INTRODUCTION

THE New Zealand Electricity Authority established the Financial Transmission Rights (FTR) Market under part 13 and 14 of the Electricity Industry Participation code [2, 7]. The discussion in this paper is based on New Zealand Electricity Market (NZEM). In NZEM the LMP has been in place since 1996 [2, 10]. The FTR market started in June 2013 with two FTR points at Otahuhu and Benmore. In November 2014, the FTR market expanded to include new FTR points at Haywards, Invercargill and Islington [2]. The FTRs are managed by a FTR manager. We as system operator required to test the FTR software that we bought from a reputed vendor, before it is operational. This paper describes the experience we learned while testing the FTR program.

New Zealand (NZ) FTR participants submit buy and sell bids into forward auction markets for a pre-defined start and an end date [2]. FTRs are base load instruments spanning over a calendar month. Each bid specifies a buy or sell bid, obligation or option bid (detailed below), maximum quantity in MW, price in $/MWh, point of injection (source) and point of withdrawal (sink). Participants may submit multiple price/quantity pairs for each path. The obligation FTR holder is entitled to receive (LMP at sink – LMP at source) x FTR MW held x number of hours in the period covered by the FTR contract. The option FTR holder is entitled to receive MAX (0, LMP at sink – LMP at source) x FTR MW held x number of hours in the period covered by the FTR contract.

Hogan [1] was the first to introduce both obligation and option FTRs and the revenue adequacy. Auction methodology of both obligation and option FTRs for PJM market was presented in [3, 4]. Contingent transmission rights were discussed in [5]. Joint energy and transmission rights auction was presented in [6]. Grant Read presented experience with FTRs and related concepts in Australia and New Zealand in [7]. In this paper, we derive and discuss the FTR auction in terms of cleared FTR, and the clearing price of FTRs. This paper discusses a number of FTR cases to test and pre-audit a new FTR software. Two methods of FTR clearing price calculation are presented.

A. *New Zealand Context*

This paper is structured as follows. Section 2 presents an auction and pricing optimization model for both obligation and option FTRs. We implement it in the GAMS optimisation system [11], set up a test system and run different FTR scenarios. In Section 3 a large number of cases are analysed. We benchmark the results in order to test and pre-audit the new FTR program. Results for 10 cases, in terms of FTR awards and prices, are summarised in Section 4. Two methods of price determination, for each case, are presented. The cases include a few standard cases to check whether the basic results from our model are matched with standard results. The results for the remaining cases are then compared with the results derived from a FTR program from a reputed FTR provider. Next follows acknowledgements, disclaimer, references, and appendix.

II. MATHEMATICAL MODEL

A. *Shift Factor*

For a lossless network with a given injection (reference) node and a withdrawal node, the matrix of shift factors is fixed. The

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studied network data and shift factor matrix are given in the Appendix.

B. *Mathematical Model for FTR Auction*

A 3 bus triangular network, Fig. 1, with 3 transmission lines is used for illustrating the auction model. Each transmission line has two directional links (LKAB, LKBA, LKBC, LKCB, LKAC, and LKCA). In this paper we assume the lossless case and balanced FTRs – i.e. injection and offtake quantities are equal for all FTRs. Since FTRs are auctioned in advance (up to 24 months ahead in the NZEM), the exact state of the transmission grid is not known with certainty at the time of the auction. This uncertainty is handled by effectively releasing FTRs progressively over multiple auctions by progressively increasing the auction grid capacity as the state of the transmission grid becomes better known closer and closer to real time.

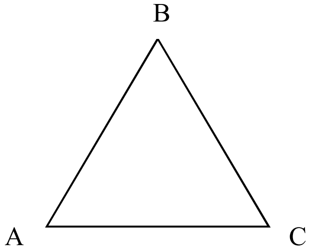

Figure 1. A 3-bus network

1) *Nomenclature*

FO_i	i^{th} Obligation FTR bid cleared. It is a decision variable, MW.
FO_i^{max}	i^{th} Obligation FTR maximum bid limit.
FP_i	i^{th} Option FTR bid cleared. It is a decision variable, MW.
FP_i^{max}	i^{th} Option FTR maximum bid limit.
PO_i	i^{th} Obligation FTR bid price, \$/MW.
PPi	i^{th} option FTR bid price, \$/MW.
SO, SP	Shift factor matrices (Appendix)
$LKFO_k$	k^{th} Link flow due to obligation FTRs.
$LKFP_k$	k^{th} Link flow due to option FTRs
$FLOW_k^{max}$	Maximum Flow limit of k^{th} link
Greek letters	These are the dual variables or shadow price of the constraints.

2) *Mathematical Model - Primal Problem*

a) *Objective Function*

The objective is to maximize the auction revenue (benefit) with subject to FTR limits and flow capacity limits

$$\text{Max. } Z = \sum_i FO_i * PO_i + \sum_i FP_i * PP_i \qquad (1)$$

Subject to following constraints,

- i^{th} FTR bid limit constraints

$$FO_i \le FO_i^{\max} \qquad (2) \qquad \lambda_{oi}^{+}$$

$$-FO_i \le 0. \qquad (3) \qquad \lambda_{oi}^{-}$$

$$FP_i \le FP_i^{\max} \qquad (4) \qquad \lambda_{pi}^{+}$$

$$-FP_i \le 0 \qquad (5) \qquad \lambda_{pi}^{-}$$

- Flow limit constraint in link k

$$\sum_i (SO_{i,k} * FO_i) + \sum_i (SP_{i,k} * FP_i) \le FLOW_k^{\max} \qquad (6) \qquad \tau_k$$

The model consists of constraints (1) to (6). All dual variables associated with ≤ inequality constraints (associated with maximization problem) are positive.

Equation 1 is the objective function. Constraints (2) to (5) are the upper and lower bid limits of the obligation and option FTRs. Left hand Side of Constraint (6) is the total link flow in the links (k) due to both cleared obligation and option FTRs (i). This constraint sets flow limits in link k, $(\forall k)$. Notice that $\boldsymbol{\tau}_k$ captures the combined effect of link flows due to worst option FTRs as well as obligation FTRs, when link limit constraint is binding.

- *Flows in the kth Link due to FTRs*

$$LKFO_k = \sum_i SO_{i,k} * FO_i \qquad (7)$$

$$LKFP_k = \sum_i SP_{i,k} * FP_i \qquad (8)$$

$$\text{Total } FLOW_k = LKFO_k + LKFP_k \qquad (9)$$

At the solution, link flows for all the combinations of option FTRs and the flow due to obligation FTRs satisfy the flow constraint thus preserving simultaneous feasibility.

Notice that we have indexed both obligation and option FTRs using a single index (i) in the model but it could have been different.

3) *FTR Prices*

A Lagrangian function (L) is formed and differentiating L with respect to FO_i, i^{th} obligation/option FTR Price becomes,

$$PO_i - (\lambda_{o,i}^{+} - \lambda_{o,i}^{-}) = \sum_k (SO_{i,k} . \tau_k) \qquad (11)$$

$$PP_i - (\lambda_{p,i}^{+} - \lambda_{p,i}^{-}) = \sum_k (SP_{i,k} . \tau_k) \qquad (12)$$

4) *Determination of FTR clearing price*

a) Method 1

We can see that the i^{th} FTR clearing price can be obtained by:
FTRs which have not been cleared at all.
FTR clearing price = FTR offer price –FTR reduced cost
FTRS fully cleared.
FTR clearing price = Bid price - bid limit constraint shadow price
FTRS partly cleared: FTR clearing price = Bid price

(1) Reduced Cost

The reduced cost value is only non-zero when the optimal value of a variable is zero and appears when the FTR is not cleared. If the optimal value of a variable is positive (not zero), then the reduced cost or variable shadow price is always zero [9]. This value is directly available from the variable-solution report.

b) *Method 2*

The price of i^{th} FTR can be expressed in terms of sum of elements of SO (i, k) or SP (i, k) times the dual variable of the binding link flow limit constraints, τ_k.

III. FTR CASES AND RESULTS

A. FTR Cases

The table 1 depicts the 10 cases selected for testing the stated model.

TABLE I
DESCRIPTION OF FTR CASES

Cases	Case Description
1	Two obligation bids and two option bids are on different paths; obAB, obCA, opAB and opAC
2	Two obligation bids, one with a negative bid price; obAC, obCA.
3	Two obligation bids, one in the forward and the other in the reverse direction; obAB and obBA
4	One obligation and one option FTR on different paths; obAB and opAC.
5	The clearing price of Option FTR can never be less than that of obligation FTR for the same path.
6	Two obligation bids and one option bids; obAB, opAB, opAC.
7	Under-subscribed obligation bids in two directions; obAB and obAC
8	Two option bids with same bid price (tied bids), in the same direction; opAB, opAB1.
9	Two obligation bids with same price (tied bids), in the same direction; obAB, obAB1.
10	Two obligation bids on different paths, one bid is very low.

Each case describes different scenario involving obligation and option bids and direction of the power flows.

1) *Input data*

a) Maximum limits of different FTRs in MW (case 1)

Table 2 shows the maximum FTR bid limits and buying bids, used for case 1.

TABLE II.
FTR BID LIMITS AND PRICES

FTRs	Max bid limit, MW	Buying bids, $/MW	FTRs	Max bid limit, MW	Buying bids, $/MW
obAB	100	10	opBA	0	8
obCA	140	1	opAC	1000	7
opAB	1000	12	opCA	0	20

FTRs are modelled as injection at the source point and load at the sink point; for example, FTR AB 100 MW indicates that an FTR from A to B, injected 100 MW at A and withdrawal of 100 MW at B. Given an injection of 1 MW at A, into a system and withdrawal of 1 MW at B, for a balanced obligation type FTR, obAB from A to B, perfectly hedges the marginal cost of congestion associated with this transaction.

b) Maximum limits of different links, in MW

The table-3 shows the maximum capacity limit in MW, of different links.

TABLE III
LINK CAPACITY MW

Link	Rating, MW	Link	Rating, MW
LKAB	100	LKCB	146
LKBA	100	LKAC	114
LKBC	146	LKCA	114

c) Exercising combinations for option FTRs

An option FTR will only contribute an implied flow to the transmission grid if it is exercised, i.e. has a positive payout – i.e. LMP at sink > LMP at source. But it is not known at the time of the auction which options will be exercised. One way to guarantee feasibility would be to consider all possible combinations of exercising of option FTRs. For example,two option FTRs can be exercised in 4 different ways as shown in table 4.

TABLE IV
EXERCISING COMBINATIONS

n1	opAB	opBA
1	0	0
2	0	1
3	1	0
4	1	1

1-exercising, 0- not exercising

Fortunately, for each link in the transmission grid one can determine the worst case option exercise pattern – i.e. the set of option FTRs that, if exercised, would maximize the implied flow on the link. This occurs when only option FTRs with positive shift factors are exercised. Hence replacing the negative entries in the option FTR shift factor matrix with zeros, means that the flow limit for each link is automatically tested against its worst case option exercise pattern [6]. This is the method we have used to preserve feasibility.

2) Interpretation of results

Before presenting the results of different cases, we interpret the results using Case1.

Case 1. Two obligation and two option bids are on different paths; obAB, obCA, opAB and opAC.

In this case two obligation bids on different paths, obAB is not cleared, and obCA is fully cleared and binds at the upper bid limit. Option bids are marginal on both paths. The links LKAB and LKAC are binding with shadow prices of $15.334, and $ 2.8338. The objective cost is $ 2976.423. The results are shown in table 5.

The 1st column of the table shows the products (FTRs) the participant bids. 2nd and 3rd columns are the offered maximum FTR quantity (MW) and the bid price ($/MW). The numbers in the 4th column (RC) is the reduced cost or marginal price of the FTR variable available from the optimal solution. The FTR obAB has a reduced cost of -$2. The 5th column shows the bid constraint prices (CP) or shadow prices (value of dual variable) of the binding bid limit constraint.

TABLE V
RESULT OF CASE1 – METHOD 1

	Bids		Cost/price		FTR Awards	
Product	MW	$/MW	RC	CP	Amount	price
obAB	100	10	-2	0	0	12
obCA	140	1	0	8	140	-7
opAB	1000	12	0	0	71.685	12
opAC	1000	7	0	0	282.315	7

It shows that the CP for FTR (obCA) = $8, other CPs = 0. The numbers in the 6th column are the cleared FTRs in MW available from the solution. The 7th column gives the clearing FTR price of different FTRs, following the pricing rules.

a) FTR Cleared

obAB = 0 (not cleared), OBCA=140 MW (Fully cleared, bound at the upper bid limit), opAB= 71.685MW, and opAC = 282.315 MW; opAB and opAC are partially cleared and therefore marginal.

b) Determination of FTR clearing price

(1) Method 1

FTR obAB is not at all cleared so as per our price relationship, so FTR clearing price = 10 - (-2) = $12 /MW, where $-2 is the reduced cost of the variable obAB, and $10 is the offered bid price.

FTR obCA is fully cleared therefore the price =1- 8= -$7/MW, where $8 is the shadow price corresponding to max bid limit constraint for obCA (λ^{+}_{obCA}) and $1 is the offered bid price. FTRs opAB and opAC, each is partially cleared, so the prices are opAB = bid price = $12/MW, and opAC = bid price = $7 / MW.

(2) Method 2

For case 1, only two link LKAB and LKAC have non-zero shadow price, all other links have zero shadow price (not binding). The price of FTR, i = obAB can be found as below.

$$\sum_{k} (SO_{i,k}.\tau_k) = SO(obAB, LKAB) * \tau_{LKAB}$$

$$+ SO(obAB, LKAC) * \tau_{LKAC} = 0.7333*15.334 + .2667*2.8338$$

$$= \$12.00$$

Similarly, the prices of other FTRs are calculated and shown in table-6. Notice that the shadow price of link LKAB is $15.334. Price difference between nodes A and B is $15.334*SO (obAB, LKAB) = 11.24 in congestion cost and price difference between A and C is $15.334*SO (obAB, LKAC) = $0.76, total $12.0; which is the clearing price of the FTR obAB. This means that if any market participant dispatches 1 MW from A to B, they will be charged $11.24 and at the same time, they will be paid the same amount for holding the FTR from A to B.

TABLE VI
FTR PRICES FOR CASE1 - METHOD 2

FTR	SO/SP(*, LKAB)	Shadow price of LKAB	SO/SP(*, LKAC)	Shadow price of LKAC	FTR Price
obAB	0.7333	15.334	0.2667	2.8338	12
obCA	-0.3333	15.334	-0.6667	2.8338	-7
opAB	0.7333	15.334	0.2667	2.8338	12
opAC	0.3333	15.334	0.6667	2.8338	7

(*) Stands for appropriate FTR

If they do not execute the additional 1 MW, they will get $11.24. Similar reasoning holds for obligation FTR from A to C. Clearing prices for obligation FTRs, can be derived directly from the nodal prices. In a lossless network, the FTR price simply means the difference of LMPs in congestion cost, between the two ends of the FTR path.

The price for option FTR cannot be calculated from nodal prices. Clearing price of option FTRs is the algebraic sum of the product of SP matrix elements, which contains either zero or positive numbers, and the prices of the binding link constraints, as shown earlier.

Another point to note here is that all elements in the SP matrix for option FTRs are non-negative, and all τ's are also non-negative, so price of option FTRs cannot be negative. It will be either zero or positive.

3) Interpretation of Link Flows

The flow in the different links (k) are the sum of the link flows due to all cleared obligation as well as option FTRs. The table-7 shows the flows (MW) in the different links. The flows are given by these relationships as shown earlier in the model section.

Note that the flows due to option FTRs are always non-negative, because all negative elements in the shift factor matrix for option FTR are replaced by zero, and cleared FTRs are non-negative. The flows for either obligation FTRs or option FTRs are not restricted. Total flow in the links are limited to their capacity limits. Hence flow in LKAB is 146 MW due to option FTRs, but a flow of -46.662 MW due to obAB, made the total flow limited to its limit of 100 MW.

TABLE VII.
LINK FLOWS – CASE 1

	Flow due to option FTRs	Flow due to obligation FTRs	Total flow
LKAB	146.662	-46.662	100
LKBA	0	46.662	46
LKBC	94,096	-46.662	48
LKCB	19.118	46.662	65
LKAC	207.33	-93	114
LKCA	0	93	93

Thus LKAB flow limit constraint is binding with a constraint price of $15.334/MW. In this problem, LKAC flow limit constraint is also binding at its limit of 114 MW, with a constraint price of $2.8338/MW.

Revenue Adequacy

Total congestion Rent $= \sum_k \tau_k * Flow_k^{\max}$

$$= \tau_{LKAB} * Flow^{\max}_{LKAB} + \tau_{LKAC} * Flow^{\max}_{LKAC} = 15.334 * 100$$
$$+ 2.8338 * 114 = \$1533.4 + \$323.05 = \$1856.45$$

Payments to the FTR holders
obCA will be paid =-$7*140 = -$980 (obCA Received)
opAB will be paid = $12*71.685 = $860.22
opAC will be paid = $7*282.315 = $1976.205
Total = $1856.425
Thus congestion rent collected by the SO (FTR Manager) equals the payment to the FTR holders.

Summary of Results and Conclusions

A number of cases with obligation and option FTRs with different directions and bid prices as shown in table 1 are studied in order to test and pre-audit a new FTR software.The detailed results could not be included in the paper due to space limitation. It is observed from these results that:

1. Two methods of price calculation are established. Two pricing rules are used to calculate the FTR clearing prices. Both methods gave the same price, as expected.
2. The bids are cleared depending on bid prices, bid limits, link capacities and network topology and parameters (Shift factors). In general, it can be seen, referring to pricing rule-1, that, FTRs are cleared whenever their buying bid prices are either equal to or greater than the clearing prices. If bid price is less than clearing price, the FTR is not cleared. Similarly, for selling bids, FTRs are cleared when its bid price is less than or equal to its auction clearing price.
3. Clearing price of option FTRs is the algebraic sum of the product of SP matrix elements, which contains either zero or positive numbers, and the prices of the binding link constraints are positive, as shown in pricing rule 2. Thus clearing price of option FTR will be either zero or positive, never negative.
4. Following pricing rule 2, if all the link shadow prices are zero, the clearing price of all obligation and option FTRs are zero. See case 7.
5. The clearing price of option FTR is always greater than or equal to obligation FTR price, for the same path as shown in case 5.
6. In a lossless network, the clearing price for obligation FTR can be calculated from nodal prices. It simply means the difference of LMPs in congestion cost, between the two ends of the FTR path. But the clearing price for option FTR cannot be calculated from nodal prices.
7. It is shown in case 3, that clearing price of obAB equals negative of price of obBA. It is also shown analytically. This is not true for option FTRs, because the price of option FTRs can never be negative.
8. In case of tied bids, the awarded FTRs have been pro-rated.
9. In case of negative clearing price for obligation FTRs, the FTR holder is paid for providing counter flow.
10. It has also been observed that the congestion cost, shadow prices of binding link flow constraints times the link capacities, can cover exactly the payment to the FTR holders, clearing price times FTR cleared. It is a standard result.
11. The above test results have covered standard cases as well as other cases in order to check the clearing price formation and their properties. The results are compared with a reputed FTR provider's results. It is seen that all our above results exactly matched with the providers'. This development and case studies are helpful for in-house testing and pre-auditing a new FTR program from vendors.

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DISCLAIMER

Transpower, or NZ Electricity Authority, Victoria University of Wellington, or Waikato Institute of Technology is not responsible for any error or omission. The authors are solely responsible for such error.

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Appendix –A: System data and Shift Factor Matrix

Circuit	X=1-ckt	X=2-ckts
A-B	0.0002	0.0001
B-C	0.0003	0.00015
A-C	0.00025	0.000125

Shift Factor Matrix for obligation FTRs (SO)

	LKAB	LKBA	LKBC	LKCB	LKAC	LKCA
OBAB	.7333	-.7333	-.2667	.2667	.2667	-.2667
OBBA	-.7333	.7333	.2667	-.2667	-0.2667	.2667
OBBC	-0.4	0.4	0.6	-0.6	0.4	-0.4
OBCB	0.4	-0.4	-0.6	0.6	-0.4	0.4
OBAC	0.333	-0.333	0.3333	-0.333	0.6667	-0.667
OBCA	-.3333	0.333	-.3333	0.3333	-.6666	.6667

Note that the SO matrix is used for obligation FTRs. The SO matrix should be augmented replacing all negative entries with zero for using in the option FTRs as SP matrix