Nonlinear Models of Digital Filters and Their Application Fields

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***Abstract*—There are many situations in which linear filters perform poorly, as in the presence of signal-dependent or multiplicative noises or processing of signals having non-Gaussian statistics. In these cases it is evident to apply nonlinear filters. The author focuses on the mathematical models of nonlinear filters and their application fields. The example of digital predistorter synthesis on the basis of perceptron network and polynomial is represented for power amplifier linearization.**

I. INTRODUCTION

Linear filters are widely used in signal processing because of their well-known properties and the inherent simplicity which can in fact guarantee satisfactory performance in a number of applications. There are, however, some situations in which linear filters perform poorly, for example, in the presence of signal-dependent or multiplicative noises and non-Gaussian statistics for the signals. In sys1em modeling applications, for example, it may be necessary to deal with the nonlinearities which characterize real-world systems. In an image processing environment, it is known that linear filters are hot able to remove the noise, in particular the impulsive one superimposed on a picture, without blurring the edges. Moreover, it is often necessary to take into account the intrinsic nonlinear behavior of the human visual system or of the optical imaging systems, resulting from the nonlinear relation between the optical intensity and the optical field. For all these reasons, nonlinear filtering has been applied to a number of different applications for many years [1–7].

II. CLASSES OF NONLINEAR FILTERS

The most important nonlinear filters families [1] include

– homomorphic filters, relying on a generalized super position principle;

– morphological filters based on geometrical rather than analytical properties;

– nonlinear mean filters, using nonlinear definitions of means;

– order statistics filters based on ordering properties of the input samples, for instance, median filter;

– polynomial filters, for instance, the truncated Volterra filter;

– neural networks, which attempt to model nonlinear systems using interconnections of simple nonlinear devices called artificial neurons.

III. FIELDS OF NONLINEAR FILTERS APPLICATION

Several applications of nonlinear filters can be found in the literature, involving discrete signals and systems in one or more dimensions [1–7]. An overview of some of them is demonstrated below.

A. Nonlinear System Modeling

Modeling of nonlinear systems is one of the first practical applications of the discrete Volterra series. In some cases, depending on the nature of the nonlinearities, high-order kernels and/or odd kernels can be necessary [1]. This happens, for example, in the communication field when modeling highly distorted reference channels, nonlinear transmit amplifiers and nonlinear bandpass channels in digital transmission systems.

In satellite communication systems, the amplifiers located in the satellites usually operate at or near the saturation region in order to conserve energy. The saturation nonlinearities of the amplifiers introduce nonlinear distortions in the signals they process. The satellite channel is typically modeled using three distinct components. The path from the earth to the satellite as well as from the satellite to the earth may be modeled as linear dispersive systems. The amplifier characteristics are modeled usually using memoryless nonlinearities. The equalizers at the receiver must be able to compensate for the nonlinear distortions so that the full capacity of the channel can be utilized [5], [6].

B. Echo Cancellation

In modern digital subscriber loop modems, echo cancellation techniques ensure full duplex transmission with adequate channel separation. The purpose of the canceller is to remove the near-end cross talk, or "echo" signal, interfering with the signal coming from the distant transmitter. Since the last signal may be highly attenuated (40–50 dB) and the attenuation of the hybrid can be as low as about 10 dB, the required attenuation of the echo signal is of the order of 50–60 dB to achieve an acceptable signal-to-echo interference ratio [1], [2], [5].

In order to guarantee this level of echo attenuation it is very often necessary to take into account nonlinear distortions deriving in practical systems from transmitted pulse asymmetries, saturation effects in transformers and nonlinearities of data converters. A method for expanding an arbitrary nonlinear function of a number of bits in a series is similar to the Volterra expansion. This expansion involves only

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a finite number of terms including products of couples of bits which correspond to the nonlinear operator [2–5].

The echo canceller is implemented as an adaptive filter whose output must compensate for the actual echo by adding to the transversal realization some nonlinear taps [2–5].

C. Noise Cancellation

Nonlinear noise cancellation or compensation of nonlinear distortions is another possible field of application of nonlinear filtering. In digital communication systems, for example, the channel nonlinearities can very often be modeled as memoryless. Since these effects take place in a network where linear filtering operations are used, the overall effect of the channel on the input signal is a nonlinear mapping with memory which is effectively described by a discrete Volterra series. The Volterra series technique has been, in fact, applied for adaptive equalization of channel nonlinearities and nonlinear intersymbol cancellation [2–5].

The nonlinear canceller adaptively synthesizes a model of the nonlinear interference impulse response which is used to compensate for the channel nonlinearity [2], [5].

D. Image Processing

Image edge detection is a basic tool in nonlinear two-dimensional signal processing having many uses in robotic vision, automatic inspection, image coding and so on [6].

The problem of *eliminating noise in an image* without damaging the edges is one of the typical issues which can be faced by nonlinear techniques. In particular, the case of a picture which has been taken in bad illumination conditions and thus has compressed gray level dynamics and is rather noisy has been considered.

Another possible application of nonlinear filters is in image preprocessing for *texture discrimination*. Following the design procedure, an operator able to discriminate textures having patterns formed by adjacent or separate impulses, according to the given specifications [6].

Differential pulse code modulation and hybrid transform coding are effective methods for reduction of the bit rate in digital transmission of images. Both methods use a predictor and subsequent coding of the prediction error. However, the efficiency of fixed linear predictors is quite limited, as the image signal is far from stationary. To overcome this problem, switched or adaptive linear predictors have been proposed, but their ability to track abrupt spatial and temporal variations remain limited, because of their inherent low-pass characteristics. *A nonlinear predictor*, gathering information from high-order statistical moments and able to possess a sort of high-pass effect, will be more efficient [6].

Skipping frames at the transmitter and interpolating the skipped frames at the receiver is an attractive method of coding image sequences at low bit rates. Since linear interpolation by temporal filtering performs poorly in moving areas, the interpolation algorithm must compensate for the motion of the object in the scene. An alternative approach is based on the use of *a nonlinear interpolation filter*. The arguments to validate are the same used for the prediction case; in fact the problem has been considered as that on finding the frame interpolator as the optimal predictor on a block-by-block basis [6].

E. Predistortion

The harmonic distortions introduced by loudspeakers into the audio signals are caused by nonlinearities in the loudspeaker characteristics. The main causes of the nonlinearities are the nonuniform flux of the permanent magnet and the nonlinear response of the suspensions. Several methods have been devised to characterize and compensate for such distortions. One commonly employed model for loudspeakers yields low-order nonlinearity in the form of a truncated Volterra system model. Compensation for the nonlinear distortions is typically achieved by predistorting the audio signals prior to introducing them to the loudspeakers [7].

Predistortion is a kind of the power amplifier (PA) linearization technique which modifies the input to a power amplifier such that it is complementary to the distortion characteristics of the power amplifier in communication channels. The cascaded response of complementary predistortion and amplifier distortion should therefore result in a linear response. The technique is generally applied at radio frequency, intermediate frequency or baseband. The digital predistortion is generally performed at baseband. The parameters of digital predistorter (DPD) are stored in a look-up table or register table which can be updated with adaptive feedback [7].

IV. Exampel of digital predistortion

DPD yields nonlinear predistortion in order to compensate PA nonlinear distortion or to linearize PA [7].

A structure of an adaptive DPD link to PA in accordance with the feedforward algorithm of DPD learning is shown in Fig. 1 [8].

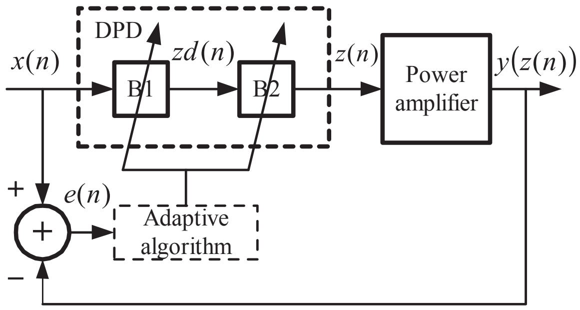

Fig. 1. The structure of the DPD feedforward learning algorithm

Here, DPD is described by the nonlinear operator S composed of two nonlinear operators S_1, S_2, which are characteristic of two cascade connected blocks correspondently. The operators S_1 and S_2 are introduced in the operational equations

$$zd(n) = S_1[x(n)], \quad z(n) = S_2[zd(n)],$$

where n is the normalized discrete time, $x(n)$, $z(n)$ are the DPD input and output signals respectively, $zd(n)$ is the output signal of the block described by the operator S_1 and shown in Fig. 1.

Models of nonlinear operators S_1 and S_2 are constructed by solving the approximation problem

$$\| y(z(n)) - x(n) \| \Rightarrow \min_{n \in [0, N_x]}, \qquad (1)$$

where $y(z(n))$ is the PA output signal, N_x is the number of samples of the input signal $x(n)$. At first we build a model of the operator S_1, followed by a model of the operator S_2. Polynomials and neural networks can be used as nonlinear operator models.

Parameters of cascade DPD nonlinear models are derived from the solution of the approximation problem (1) by iteration procedures under the following equations:

$$z_{k+1}(n) = z_k(n) + e_k(n)\,,\ k \geq 1\,,$$

$$zd_{k+1}(n) = zd_k(n) + e_k(n)\,,\ k \geq 1\,,$$

where k is the iteration number, $e_k(n)$ is the error of PA linearization at the k-th iteration,

$$e_k(n) = x(n) - y(z_k(n))\,.$$

For k=1 let us assume that

$$z_1(n) = zd_1(n) = x(n)\ .$$

PA has a low nonlinearity that is why its model can be represented as the convergent Volterra series. As a result the predistorter is weakly nonlinear. This condition influences the quick convergence of the DPD synthesis iteration procedure.

At the cascade DPD synthesis the decomposition of the approximation problem (1) is carried out, i.e. the approximation problem (1) with a high dimension is divided into two approximation problems with less dimensions solved in DPD blocks consecutively. Under this approach the ill conditionality problem of the approximation problem (1) is solved.

We use polynomial perceptron network (PPN) [9] as models of the nonlinear operators S_1 and S_2. PPN is a single-layer network [9]. This network is characterized by the simplicity of its learning algorithm and high speed of convergence to the solution of the approximation problem.

The PPN model is described by the expression [8], [9]

$$y(n) = f\left(\mathbf{W}^T \mathbf{F}(\mathbf{X}(n))\right), \qquad (2)$$

where f is the nonlinear activation function of the network, $\mathbf{X}(n)$ is the vector of input signals, $\mathbf{X}(n) = [x_1(n), x_2(n), ..., x_Q(n)]^T$, T denotes transposing of the vector, $\mathbf{W}(n)$ is the vector of network weights, $\mathbf{W} = [w_1, w_2, ..., w_G]^T$, $\mathbf{F}(\mathbf{X}(n))$ is the vector containing elements with multidimensional transformation,

$$\mathbf{F}(\mathbf{X}(n)) = \big[1, x_1(n), ..., x_Q(n), ..., x_1^2(n), ..., x_Q^2(n), ...,$$

$$..., x_1(n)x_2(n), ..., x_{Q-1}(n)x_Q(n), ..., x_1^P(n), ..., x_Q^P(n)\big]^T,$$

P is the degree of the function-vector element, $y(n)$ is the output signal of the model (2).

The PNN structure is shown in Fig. 2.

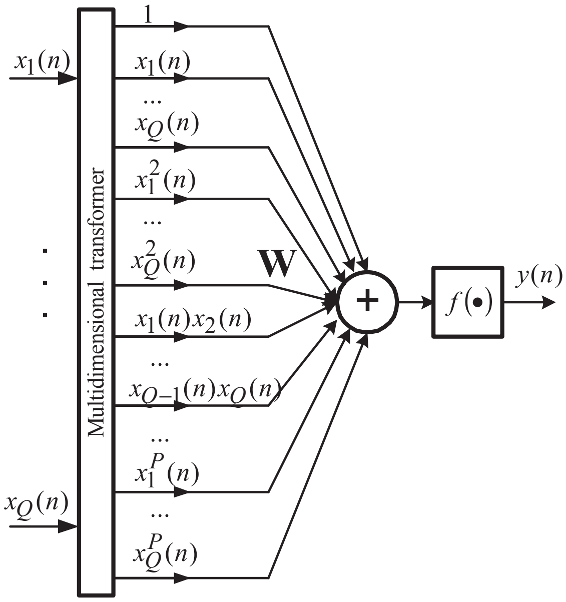

Fig. 2. The PNN structure.

For DPD synthesis let us assume that the activation function is linear in the model (2). We can rewrite (2) as

$$y(n) = \mathbf{W}^T \mathbf{F}(\mathbf{X}(n))\,. \qquad (3)$$

The input signal vector $\mathbf{X}(n)$ is formed on the basis of delay line. Multidimensional transformation $\mathbf{F}$ in the model (3) is performed under conditions of constructing intermodulation spectral components of the DPD output [10].

As a result we obtain the following PPN model [8]

$$y(n) = y_1(n) + y_2(n)\,, \qquad (4)$$

where

$$y_1(n) = \sum_{i=0}^{M} x(n-i) \sum_{k=0}^{(P-1)/2} w_{2k+1}^{(1)} |x(n-i)|^{2k} +$$

$$\sum_{i=1}^{M} x(n-i) \sum_{k=1}^{(P-1)/2} w_{2k+1}^{(2)} |x(n)|^{2k} +$$

$$+ \sum_{i=0}^{M} x^*(n-i)\, x^2(n) \sum_{k=0}^{(P-3)/2} w_{2k+3}^{(3)} |x(n)|^{2k} +$$

$$+\sum_{i=1}^{M} x^*(n)x^2(n-i)\sum_{k=0}^{(P-3)/2} w_{2k+3}^{(4)}\left|x(n-i)\right|^{2k}+$$

$$+\sum_{i=1}^{M} x(n)\sum_{k=1}^{(P-1)/2} w_{2k+1}^{(5)}\left|x(n-i)\right|^{2k}+$$

$$+x^2(n)\sum_{i=0}^{M} x^*(n-i)\sum_{k=0}^{(P-3)/2} w_{2k+3}^{(6)}\left|x(n-i)\right|^{2k}+$$

$$+\left|x(n)\right|^2\sum_{i=0}^{M} x(n-i)\sum_{k=0}^{(P-3)/2} w_{2k+3}^{(7)}\left|x(n-i)\right|^{2k}+$$

$$+\sum_{i=0}^{M}\left(x^*(n)\right)^2 x^3(n-i)\sum_{k=0}^{(P-5)/2} w_{2k+5}^{(8)}\left|x(n-i)\right|^{2k}, \quad (5)$$

* is the sign of complex conjugation, M is the memory length, P is the odd degree of polynomial,

$$y_2(n)=\sum_{i=1}^{M}\left(x^*(n-i)\right)^2 x^3(n)\sum_{k=0}^{(P-5)/2} w_{2k+5}^{(9)}\left|x(n)\right|^{2k}+$$

$$+\sum_{i=1}^{M} x^*(n)x^2(n-i)\sum_{k=1}^{(P-3)/2} w_{2k+3}^{(10)}\left|x(n)\right|^{2k}+$$

$$+\sum_{i=1}^{M}\left(x^*(n-i)\right)^2 x^3(n)\sum_{k=1}^{(P-5)/2} w_{2k+5}^{(11)}\left|x(n-i)\right|^{2k}+$$

$$+\sum_{i=1}^{M}\left(x^*(n)\right)^2 x^3(n-i)\sum_{k=1}^{(P-5)/2} w_{2k+5}^{(12)}\left|x(n)\right|^{2k}+$$

$$+\left|x(n)\right|^2\sum_{i=1}^{M} x(n)\sum_{k=1}^{(P-3)/2} w_{2k+3}^{(13)}\left|x(n-i)\right|^{2k}+$$

$$+\sum_{i=1}^{M}\left|x(n-i)\right|^2 x(n)\sum_{k=1}^{(P-3)/2} w_{2k+3}^{(14)}\left|x(n)\right|^{2k}. \quad (6)$$

The expression (5) describes the radially pruned Volterra model (RPVM). RPVM is the regressive form of the truncated Volterra series. Volterra kernels in RPVM are built on a hypercube grid and radial directions are selected on the basis of the 3rd order kernel [10].

In practice a nonlinear PA exhibits a memory effect that is why a nonlinear predistorter with memory should be used for compensation of PA nonlinearity.

In the represented example, we assume the PA model is Wiener–Hammerstein model composed of a linear time-invariant (LTI) system, followed by a memoryless nonlinearity, which is in turn followed by another LTI system [8]. The LTI blocks before and after the memoryless nonlinearity have the system functions given by

$$H_1(z)=\frac{1+0.5z^{-2}}{1-0.2z^{-1}},\ H_2(z)=\frac{1-0.1z^{-2}}{1-0.4z^{-1}}$$

correspondently. The memoryless nonlinear part of the described PA model is given by

$$w(n)=b_1 v(n)+b_3 v(n)\left|v(n)\right|^2+b_5 v(n)\left|v(n)\right|^4,$$

where $b_1=1.0108+0.0858j$, $b_3=0.0879-0.1583j$, $b_5=-1.0992-0.8891j$, $v(n)$ is the output signal of the LTI system with the function $H_1(z)$.

The input signal for the PA model is the complex envelope of a GSM-signal with four carriers in the frequency baseband with the bandwidth of 20 MHz. The sampling frequency of the GSM-signal complex envelope is 184.32 MHz.

The dependences of the normalized magnitude and phase change of the PA output signal on the PA input signal normalized magnitude are depicted in Fig. 3, *a*, *b*.

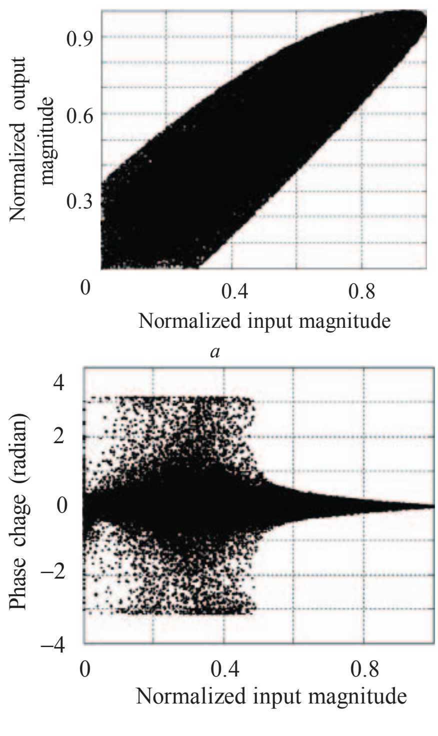

Fig. 3. PA characteristics and signl PSD without DPD.

The adaptive DPD based on RPVN (5) with P=7, PPN (4) with P=7 and the cascade connection of these models are constructed to linearize the above-mentioned PA model. It should be noted, that all the top indexes of the sums in expressions (5) and (6) are (P–1)/2. The memory length of the investigated models is 4 (M=4 in (5), (6)).

The PA linearization error is estimated by normalized mean-square error (NMSE). This error is calculated from the expression

$$\mathrm{NMSE} = 10\log_{10}\left(\frac{\sum_{n=0}^{N_x-1}\left|y(z(n))-x(n)\right|^2}{\sum_{n=0}^{N_x-1}\left|x(n)\right|^2}\right), \text{ dB},$$

where $x(n)$ is the complex-valued input signal of the cascade connection between DPD and PA shown in Fig. 1, N_x is the number of samples in the input signal $x(n)$, N_x=106339, $y(z(n))$ is the PA complex-valued output signal.

NMSE estimated on the 45$^{\text{th}}$ iteration of the DPD adaptive algorithm and the number Q of coefficients in DPD models are presented in Table I.

TABLE I. NMSE AND Q IN DPD MODELS

Model		NMSE	Q
	RPVM (5) at P=7	–72.87	145
	PPN (4) at P=7	–75.50	221
Cascade connection	RPVM (5) at P=7 and RPVM (5) at P=7	–78.14	290
	PPN (4) at P=7 and RPVM (5) at P=7	–79.14	366

As can be seen from Tab. 1, the PPN model provides higher accuracy of PA linearization than RPVM. The use of the cascade DPD structure leads to an increased PA linearization accuracy. The highest accuracy is achieved by the cascade DPD structure with PPN and RPVM.

On the basis of the decomposition of the DPD nonlinear operator approximation problem into two subproblems solved consecutively, it is possible to remove the ill conditionality from the DPD nonlinear operator approximation due to reduction of the approximation problem dimension. This decomposition is realized in a cascade DPD synthesis. DPD block models are built as the polynomial perceptron network and the radially pruned Volterra model. DPD is synthesized for compensation of nonlinear distortion in PA Wiener–Hammerstein model at the complex envelope of a GSM-signal with four carriers. The executed predistortion resultes in the fact that the cascade DPD composed of PPN and RPVM linearizes PA with the highest accuracy.

V. CONCLUSION

The concept of optimum linear filtering has had enormous impact on the recent development of the various techniques to estimate and process stationary time series. The obvious advantage of a linear filter is its simplicity in design and implementation. However, with the minimum mean-square error criterion, the ultimate solution to the optimum filter is in finding the conditional mean which is, in general, a nonlinear function of observed data. In some cases, the performance of a linear filter may be unacceptable. Another important factor in favor of nonlinear filters is the vast capability of modern computers which enables us to overcome the complexity of the nonlinear filtering problem.

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