A Group Decision Making Model for Integrating Heterogeneous Information

Guangxu Li, Gang Kou, and Yi Peng

***Abstract*—This paper proposes a group decision-making (GDM) method for integrating heterogeneous information. To avoid information loss, instead of transforming heterogeneous information into a single form, the proposed method integrates heterogeneous information using a weighted-power average operator. The consensus degree between the individual-decision matrix and the group-decision matrix is then calculated based on the deviation degree. In addition, the feedback mechanism with the iterative algorithm is used to adjust the individual decision matrix, which does not reach the consensus. Furthermore, a ranking formula with heterogeneous technique for order preference by similarity to an ideal solution is adopted to select the best alternative. A numerical example of supplier selection is introduced to validate the proposed model and compare it with other similar GDM models. The results illustrate that the proposed method cannot only avoid information loss, but also effectively integrate heterogeneous information in heterogeneous GDM environment.**

***Index Terms*—Group consensus, group-decision making (GDM), heterogeneous information, information fusion.**

I. Introduction

DECISION-making problems are important in decision science and have wide applications [1], [2]. Three steps were proposed to solve decision-making problems under linguistic information [3]. The developments of Computing with Words (CW) methodology in decision-making was reviewed in [4]. Peng *et al.* [5] proposed a multicriteria convex quadratic programming model to analyze credit data. Taha and Panchal [6] proposed a decision-making method in energy systems to study the effects of stakeholders' preferences with multiple technologies and uncertain preferences. Kou *et al.* [7] developed some multicriteria decision-making methods to evaluate the clustering algorithms for financial risk analysis. A multiattribute decision-making method as a practical and effective way was presented to meet the different assessment requirements of decision makers was presented [8]. A dynamic fuzzy multicriteria decision-making method was further proposed for performance evaluation [9]. However, these decision-making methods cannot satisfy the increasing complexity of evaluation processes.

In many circumstances, decision problems involve more than one decision maker and are studied extensively under the subject area of group decision-making (GDM). GDM processes are characterized by choosing the best option or opinions from a set of alternatives. Since individuals have different decision preferences, a principal problem in GDM is how to integrate the individual decision preferences into the group preference. Some theories and methods of information fusion have been reviewed and developed based on the aggregating operators in GDM [10]–[16]. However, in real situations, the decision makers may come from different fields and have different characteristics, which usually cause them to show diverging opinions. Therefore, reaching a maximum degree of consensus among decision makers is an important research topic in GDM.

Consensus processes in GDM are defined as iterative and dynamic group-discussion processes that help experts or decision makers to bring their opinions closer [17]–[21]. A high degree of group consensus is desirable when individual opinions are integrated into group opinions. Thus, the group-consensus process refers to the procedure of acquiring the maximum level of consensus concerning solution alternatives among the decision makers. Many researchers have studied the consensus methods of GDM. For example, Ben-Arieh and Chen [22] proposed two consensus models based on the individual expert opinions and the group-aggregated opinions in linguistic GDM. Ben-Arieh *et al.* [23] described the importance of group consensus and proposed minimum-cost consensus models. An automatic approach was proposed to obtain the group consensus under a crisp environment [24]. Dong *et al.* [25] analyzed the internal relationships among some ordered weighted-averaging operators and proposed a consensus operator under the continuous linguistic model. The consensus indices were proposed to determine the degree of consensus in AHP models [26]. Cabrerizo *et al.* [27] proposed the information granularity being regarded as an important and useful asset supporting the goal to reach consensus in GDM and improved the level of consensus. Ureña *et al.* [28] presented an open source framework fully developed in *R* to carry out consensus guided in

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GDM, and the system includes tools to visualize the evolution of the GDM process. An automatic consensus approach was presented for achieving group opinion satisfaction, which allows the decision makers to modify their decision matrices under the intuitionistic fuzzy environment [29]. A consensus reaching process was proposed to integrate experts' weights generated dynamically and its applications were given in managing noncooperative behaviors [30]. Some other consensus models have also been proposed in GDM [31]–[38]. These studies could obtain the maximum degree of consensus in homogeneous GDM. However, real-life decision-making problems are complex, the attributes can be quantitative or qualitative, and the values of the attributes can be given using different numerical types such as real numbers, interval numbers, triangular fuzzy numbers, and trapezoidal fuzzy numbers. A complex GDM problem often contains heterogeneous information and the previous studies cannot deal with heterogeneous information. In this paper, a GDM model for integrating heterogeneous information is proposed. In addition, an algorithm for obtaining the consensus solution is introduced based on the degree of deviation between an individual heterogeneous decision-making matrix and a GDM matrix.

Heterogeneous GDM problems can be defined in three frameworks [39]. The first heterogeneous GDM framework relates to different preference formats. Decision makers express their opinions by different preference relations such as preference orderings, utility functions, multiplicative preference relations, and fuzzy preference relations [40]–[47]. The second heterogeneous GDM framework appears when each expert has different levels of knowledge and background related to the problem [48], [49], or when the experts have different linguistic label sets to assess preferences, i.e., the multigranular and unbalanced linguistic contexts [50], [51]. The third framework is focused on the heterogeneous expressions of the experts, which are used to express or provide their particular preferences for the attributes of each alternative. It provides information about attributes, which consists of not only crisp or uncertain information, but also interval numbers, fuzzy numbers, and linguistic data. For example, Chou *et al.* [52] proposed a fuzzy simple additive weighting system to aggregate the fuzzy weights under GDM, but the consensus of the experts was not considered in the aggregation state. Chen and Chen [53] proposed a new information-aggregation algorithm to combine the fuzzy consensus opinions of the experts into a heterogeneous GDM, but this method did not provide the feedback mechanism necessary to adjust the inconsistent attributes if the GDM process failed to achieve consensus. Das *et al.* [54] developed the extended Bonferroni mean (EBM) operator with Atanassov's intuitionistic fuzzy sets to integrate the heterogeneously interrelated criteria, but the EBM operator cannot capture the sophisticated nuances that the user wants to reflect in the aggregated value. A fuzzy evaluation method was proposed to deal with uncertainty and manage heterogeneous information [55]. The information was transformed into a common format, which may have caused some of it to be lost in the evaluation process. A systematic method was developed to solve the heterogeneous GDM with information based on attribute ratings, including linguistic labels, real numbers, interval numbers, and fuzzy numbers, and provided a comparison analysis between the proposed approach and fuzzy TOPSIS [56]. However, this method did not consider the consensus process of the opinions among the experts. Two processes are required before obtaining the final solution in GDM [40]: 1) consensus process and 2) selection process.

Building on previous research results, this paper aims to integrate the heterogeneous information within a new consensus method under the third heterogeneous framework. The proposed consensus method not only avoids the problem of information loss, but also takes account of the feedback mechanism necessary to adjust the inconsistent attributes if the GDM process fails to achieve consensus. To avoid information loss, the heterogeneous information should not be transformed into a single form. The power-average (PA) operator, which was proposed by Yager [55], is used to integrate the individual-opinions matrix into a group-opinions matrix, because tit can capture the sophisticated nuances that the user wants to reflect in the aggregated value [57]. PA operator is a nonlinear aggregation operator, it cannot only reflect the relationship between the input data, but also measure the similarity of these data. In the information fusion process, the decision makers hope to retain their original views. Based on its characteristic, we can use PA operator to integrate the heterogeneous information. After the aggregation process, the consensus process is presented. If every individual opinion reaches a consensus, the ranking should be continued. Otherwise, a simple and intuitive feedback mechanism is used to adjust the inconsistent attributes until the individual opinions reach a consensus. In the consensus process, a degree of deviation is used to measure the degree of consensus between each individual decision matrix and integration group-decision matrix, and the intermediate-value method is applied to adjust the inconsistent attributes. Finally, the technique for order preference by similarity to an ideal solution (TOPSIS) [58] is used to rank the heterogeneous group-decision matrices when consensus is reached.

The remainder of this paper is organized as follows. Section II introduces definitions and notations for fuzzy sets and the PA operator. The consensus process, the feedback mechanism, and the selection process in heterogeneous GDM are presented in Section III. Section IV illustrates the GDM process of the proposed method using a numerical example. Section V presents a comparison analysis with other integration methods for GDM and Section VI concludes this paper.

II. Definitions and Notations

In the section, some basic definitions and properties of fuzzy numbers and PA operator are reviewed. The basic definitions and notations below will be used throughout this paper until otherwise stated.

Definition 1 [59]: A triangular fuzzy number $\tilde{A}$ is given by $\tilde{A} = (a, b, c)$, $0 \leq a \leq b \leq c$ if the membership function

$\mu_{\tilde{A}} : R \to [0, 1]$ is defined as follows:

$$\mu_{\tilde{A}} = \begin{cases} (x-a)/(b-a), & a \le x < b \\ 1, & x = b \\ (c-x)/(c-b), & b < x \le c. \end{cases} \tag{1}$$

Definition 2 [59]: A trapezoidal fuzzy number $\tilde{A}$ is given by $\tilde{A} = (a, b, c, d)$, $0 \le a \le b \le c \le d$ if the membership function $\mu_{\tilde{A}} : R \to [0, 1]$ is defined as follows:

$$\mu_{\tilde{A}} = \begin{cases} (x-a)/(b-a), & a \le x \le b \\ 1, & b \le x \le c \\ (d-x)/(d-c), & c \le x \le d \\ 0, & \text{others}. \end{cases} \tag{2}$$

Property 1 [59]: Given two fuzzy numbers $\tilde{A} = (a_1, a_2, \ldots, a_n)$, $\tilde{B} = (b_1, b_2, \ldots, b_n)$ and a positive real number λ, some of the main operations of the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be expressed as follows.

1) $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)$.
2) $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, \ldots, a_n b_n)$.
3) $\lambda\tilde{A} = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$.
4) $\tilde{A}/\tilde{B} = (a_1/b_n, a_2/b_{n-1}, \ldots, a_n/b_1)$.
5) (Euclidean distance) $d(\tilde{A}, \tilde{B}) = \sqrt{\sum_{i=1}^{n} (a_i - b_i)^2}$.

Definition 3 [57]: Let $a_1, \ldots, a_n$ be a collection of arguments; the PA operator $\mathrm{PA}(a_1, \ldots, a_n)$ is defined as

$$\mathrm{PA}(a_1, \ldots, a_n) = \sum_{i=1}^{n} (1 + T(a_i)) a_i \Big/ \sum_{i=1}^{n} (1 + T(a_i)) \tag{3}$$

where

$$T(a_i) = \sum_{\substack{j=1 \\ j \ne i}}^{n} \mathrm{Sup}\big(a_i, a_j\big) \tag{4}$$

where $\mathrm{Sup}(a, b)$ denotes the support between a and b. Three properties of the support are also given as: 1) $\mathrm{Sup}(a, b) \in [0, 1]$; 2) $\mathrm{Sup}(a, b) = \mathrm{Sup}(b, a)$; and 3) $\mathrm{Sup}(a, b) \ge \mathrm{Sup}(x, y)$ if $|a - b| < |x - y|$. Obviously, $\mathrm{Sup}(a, b)$ is essentially a similarity index. The more similar, closer the two values, and the more they support each other.

Definition 4: Based on Definition 3, the weighted power average (WPA) operator $\mathrm{WPA}(a_1, \ldots, a_n)$ is defined as

$$\mathrm{WPA}(a_1, \ldots, a_n) = \sum_{i=1}^{n} (1 + T(a_i)) a_i w_i \Big/ \sum_{i=1}^{n} w_i (1 + T(a_i)) \tag{5}$$

where

$$T(a_i) = \sum_{\substack{j=1 \\ j \ne i}}^{n} w_j \mathrm{Sup}\big(a_i, a_j\big). \tag{6}$$

Based on [12], the support $\mathrm{Sup}(a_i, a_j)$ can be calculated as follows:

$$\mathrm{Sup}\big(a_i, a_j\big) = 1 - \frac{d\big(a_i, a_j\big)}{\sum_{\substack{j=1 \\ j \ne i}}^{n} d\big(a_i, a_j\big)}. \tag{7}$$

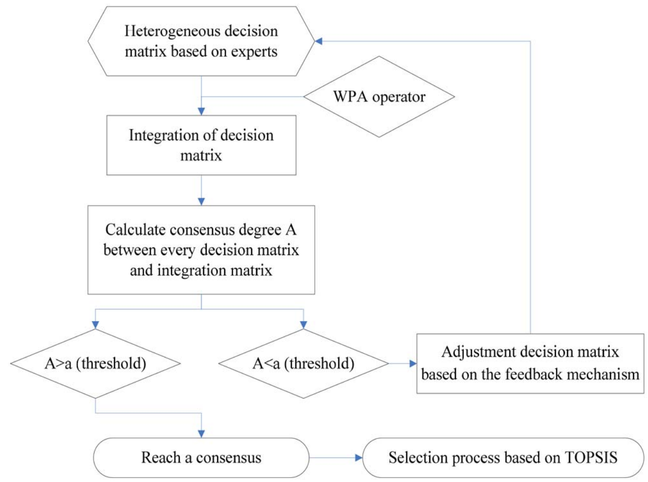

Fig. 1. Heterogeneous GDM processes.

III. HETEROGENEOUS GDM

Assume that $E = \{e_1, e_2, \ldots, e_k\}$ is a group of experts, $x = \{x_1, x_2, \ldots, x_m\}$ is a set of alternatives, and $C = \{c_1, c_2, \ldots, c_n\}$ is a set of evaluation attributes. The decision matrix can be denoted as $V^k = (x_{ij}^k)_{m \times n}$, where x_{ij}^k is the assessed value given by the expert e_k for the attribute c_j of the alternative x_i. In this paper, the assessed value x_{ij}^k is considered based on four different forms of information: 1) real numbers (S_1); 2) interval numbers (S_2); 3) triangular fuzzy numbers (S_3); and 4) trapezoidal fuzzy numbers (S_4). S_i denotes the set of the assessed values, and $S_i \cap S_j = \varnothing (i \ne j)$, where $\varnothing$ is an empty set. Based on the information in this paper, the GDM processes are represented in Fig. 1.

In GDM, the attributes may be given by benefit or cost criteria. In order to eliminate the difference in the attribute index on the dimension, for any S_i, the normalized processes are presented as follows:

$$\tilde{x}_{ij}^k = \begin{cases} x_{ij}^k \Big/ \sum_{i=1}^{m} x_{ij}^k, & \forall j \in I_1 \\ \Big(1/x_{ij}^k\Big) \Big/ \Big(\sum_{i=1}^{m} \Big(1/x_{ij}^k\Big)\Big), & \forall j \in I_2 \end{cases} \tag{8}$$

where I_1 is associated with a set of benefit criteria and I_2 is associated with a set of cost criteria.

We denote the set of the normalized matrix by $\tilde{V} = (\tilde{V}^1, \tilde{V}^2, \ldots, \tilde{V}^k)$, where $\tilde{V}^l = (\tilde{x}_{ij}^l)_{m \times n}$. Then, the group-decision matrix $G\tilde{V} = (g\tilde{x}_{ij})_{m \times n}$ can be integrated by a WPA operator.

Based on Fig. 1, after the integration process, three stages are developed for GDM: 1) consensus process; 2) feedback mechanism; and 3) selection process.

A. Consensus Process

In GDM, achieving consensus is often considered to be a satisfactory result. Therefore, the experts are required to participate in a discussion to reach a consensus solution. In the consensus process, an algorithm for obtaining the consensus solution is introduced based on the degree of deviation between an individual decision-making matrix and a GDM matrix. Meanwhile, the predefined consensus-acceptability threshold is also given.

Definition 5: Let $\tilde{V}^l = (\tilde{x}^l_{ij})_{m\times n}$ be the normalized individual decision-making matrix and $G\tilde{V} = (g\tilde{x}_{ij})_{m\times n}$ be the GDM matrix derived by the PA operator; then, the degree of deviation between $\tilde{V}^l$ and $G\tilde{V}$ is defined as follows:

$$D\left(\tilde{V}^l, G\tilde{V}\right) = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} d\left(\tilde{x}^l_{ij}, g\tilde{x}_{ij}\right). \tag{9}$$

Based on Definition 5, the degree of consensus between the individual decision-making matrix and the GDM matrix is defined as follows.

Definition 6: Let $\tilde{V}^l = (\tilde{x}^l_{ij})_{m\times n}$ be the normalized individual decision-making matrix $G\tilde{V} = (g\tilde{x}_{ij})_{m\times n}$ and be the GDM matrix; then, the degree of consensus between $\tilde{V}^l$ and $G\tilde{V}$ is given by

$$\mathrm{CD}\left(\tilde{V}^l, G\tilde{V}\right) = \frac{1}{1 + D\left(\tilde{V}^l, G\tilde{V}\right)}. \tag{10}$$

The degree of consensus $\mathrm{CD}(\tilde{V}^l, G\tilde{V})$ has the following properties: 1) $0 \le \mathrm{CD}(\tilde{V}^l, G\tilde{V}) \le 1$ and $\mathrm{CD}(\tilde{V}^l, G\tilde{V}) = 0$ if and only if $\tilde{V}^l$ and $G\tilde{V}$ are completely dissimilar; 2) $\mathrm{CD}(\tilde{V}^l, G\tilde{V}) = \mathrm{CD}(G\tilde{V}, \tilde{V}^l)$; and 3) $\mathrm{CD}(\tilde{V}^l, G\tilde{V}) = 1$ if and only if $\tilde{V}^l$ and $G\tilde{V}$ are completely similar ($\tilde{V}^l = G\tilde{V}$).

In heterogeneous GDM, because the experts have different levels of knowledge and different backgrounds, complete similarity between the individual decision-making matrix and the GDM matrix is impossible. Therefore, the threshold α of acceptable consensus degree is used to determine whether every expert reaches a consensus. Selection of the consensus threshold α is very important to the result of the decision-making process. However, there is no unified approach for choosing a consensus threshold α. Based on [31], when decision-making is very important, the consensus threshold α can be chosen to have a high value such as $\alpha = 0.9$ or a larger value. In the other case, when decision time is more urgent and the experts need to quickly select the best alternative, the consensus threshold α can be chosen as a lower value, such as $\alpha = 0.8$ or a smaller value. Moreover, if the consensus degree $\mathrm{CD}(\tilde{V}^l, G\tilde{V}) \ge \alpha$, it shows that the decision-making process has reached a consensus; conversely, the feedback mechanism is applied to adjust the decision-making matrix until a consensus is achieved.

B. Feedback Mechanism

In order to reach the consensus, an objective method based on an intermediate value is proposed to modify the experts' opinions. The processes are carried out as follows.

Step 1: Calculate the intermediate value matrix $I\tilde{V}^u = (I\tilde{x}^u_{ij})_{m\times n}$ between the GDM matrix $G\tilde{V} = (g\tilde{x}_{ij})_{m\times n}$ and the nonconsensus matrix $\tilde{V}^u = (\tilde{x}^u_{ij})_{m\times n}$.

Here

$$I\tilde{x}^u_{ij} = \frac{1}{2}g\tilde{x}_{ij} + \frac{1}{2}\tilde{x}^u_{ij}. \tag{11}$$

Step 2: Let $I\tilde{V}^u$ be the modified expert opinion decision-making matrix, and go back to the consensus process to determine the new GDM matrix $NG\tilde{V}$ and the new degree of consensus $\mathrm{NCD}(I\tilde{V}^l, NG\tilde{V})$. If the degree of consensus $\mathrm{NCD}(I\tilde{V}^l, NG\tilde{V}) \ge \alpha$, then the decision-making process has reached a consensus; conversely, the process should return to step 1 in the feedback mechanism.

The modified decision-making matrix is moved closer to the GDM matrix, so the iterative process improves the degree of consensus. When the decision-making process reaches consensus, the selection process is given based on heterogeneous TOPSIS.

C. Selection Process Based on Heterogeneous TOPSIS

Let $G\tilde{V} = (g\tilde{x}_{ij})_{m\times n}$ be the GDM consensus matrix, the heterogeneous TOPSIS is given as follows.

Step 1: Select the heterogeneous positive ideal solution (HPIS) $g\tilde{x}^+$ and the heterogeneous negative ideal solution (HNIS) $g\tilde{x}^-$, where

$$g\tilde{x}^{s_1+} = \max_i g\tilde{x}_{ij},\ g\tilde{x}^{s_1-} = \min_i g\tilde{x}_{ij},\quad \text{for } g\tilde{x}_{ij} \in S_1 \tag{12}$$

$$g\tilde{x}^{s_2+} = \left[\max_i g\tilde{x}^l_{ij}, \max_i g\tilde{x}^r_{ij}\right]$$
$$g\tilde{x}^{s_2-} = \left[\min_i g\tilde{x}^l_{ij}, \min_i g\tilde{x}^r_{ij}\right],\ \text{for}\quad g\tilde{x}_{ij} \in S_2 \tag{13}$$

$$g\tilde{x}^{s_3+} = \left(\max_i g\tilde{x}^l_{ij}, \max_i g\tilde{x}^m_{ij}, \max_i g\tilde{x}^r_{ij}\right)$$
$$g\tilde{x}^{s_3-} = \left(\min_i g\tilde{x}^l_{ij}, \min_i g\tilde{x}^m_{ij}, \min_i g\tilde{x}^r_{ij}\right),\quad \text{for}\quad g\tilde{x}_{ij} \in S_3 \tag{14}$$

$$g\tilde{x}^{s_4+} = \left(\max_i g\tilde{x}^l_{ij}, \max_i g\tilde{x}^{m_1}_{ij}, \max_i g\tilde{x}^{m_2}_{ij}, \max_i g\tilde{x}^r_{ij}\right)$$
$$g\tilde{x}^{s_4-} = \left(\min_i g\tilde{x}^l_{ij}, \min_i g\tilde{x}^{m_1}_{ij}, \min_i g\tilde{x}^{m_2}_{ij}, \min_i g\tilde{x}^r_{ij}\right),$$
$$\text{for}\quad g\tilde{x}_{ij} \in S_4. \tag{15}$$

Step 2: Calculate the distance D^+_i between each alternative and the HPIS and D^-_i between each alternative and the HNIS, where

$$D^+_i = \sum_{j=1}^{n} d\left(g\tilde{x}_{ij}, g\tilde{x}^+_j\right),\quad i = 1, 2, \ldots, m \tag{16}$$

$$D^-_i = \sum_{j=1}^{n} d\left(g\tilde{x}_{ij}, g\tilde{x}^-_j\right),\quad i = 1, 2, \ldots, m. \tag{17}$$

Step 3: Calculate the degree of similarity between the ideal solutions

$$\tilde{S}_i = \frac{D^-_i}{D^+_i + D^-_i},\quad i = 1, 2, \ldots, m. \tag{18}$$

Step 4: Rank the order according to $\tilde{S}_i$ in descending order, and select the best decision-making alternative.

IV. Numerical Example

In this section, a numerical example of supplier selection adapted from [60] is considered to illustrate the proposed method. Assume that a car company needs to select a suitable supplier to purchase some automobile parts. Three experts

TABLE I
DECISION MATRIX V^1 GIVEN BY e_1

	A_1	A_2	A_3	A_4	A_5
C_1	(3,4,5,6)	(6,7,8,9)	(5,6,7,8)	(1,2,3,4)	(2,3,4,5)
C_2	(70,90,92)	(30,80,90)	(50,60,85)	(75,80,95)	(80,85,95)
C_3	[4,10]	[7,9]	[4,9]	[6,10]	[2,8]
C_4	[65,88]	[87,90]	[45,58]	[70,90]	[92,95]
C_5	119	110	120	118	100

TABLE II
DECISION MATRIX V^2 GIVEN BY e_2

	A_1	A_2	A_3	A_4	A_5
C_1	(5,6,7,8)	(2,3,4,5)	(3,4,5,6)	(1,2,3,4)	(6,7,8,9)
C_2	(80,85,95)	(50,60,85)	(30,80,90)	(75,80,95)	(70,90,92)
C_3	[4,7]	[5,8]	[3,6]	[7,9]	[8,10]
C_4	[75,88]	[87,90]	[45,58]	[66,87]	[89,95]
C_5	120	118	115	108	119

TABLE III
DECISION MATRIX V^3 GIVEN BY e_3

	A_1	A_2	A_3	A_4	A_5
C_1	(5,6,7,8)	(3,5,6,7)	(4,5,6,7)	(4,5,8,9)	(1,4,6,7)
C_2	(72,80,95)	(50,60,85)	(74,80,85)	(65,70,81)	(82,84,92)
C_3	[6,8]	[6,8]	[7,10]	[5,7]	[3,6]
C_4	[75,89]	[82,90]	[78,86]	[66,78]	[65,90]
C_5	111	116	110	120	105

e_1, e_2, and e_3 constitute the component-purchase group. The weights of the experts are given by $(\omega_1, \omega_2, \omega_3) = (0.3, 0.3, 0.4)^T$. There are five automobile parts suppliers A_1, A_2, A_3, A_4, and A_5. Five attributes: 1) C_1 (quality of the product); 2) C_2 (the level of technology); 3) C_3 (flexibility); 4) C_4 (delivery time); and 5) C_5 (price), are considered. The attributes C_1, C_2, C_3, and C_4 are benefit criteria and C_5 is cost criterion. Because the decision-making environment is rather complex, there are several types of attribute values, including trapezoidal fuzzy numbers, triangular fuzzy numbers, interval numbers, and real numbers. After assessing and analyzing the five suppliers, the results of the attribute assessments by the three experts are shown in Tables I–III.

When the heterogeneous GDM is applied to solving the supplier-selection problem, the specific evaluation processes are shown as follows.

Step 1: Normalize the decision-making matrix using (8) and the operations of the fuzzy numbers, and then, obtain the normalized decision-making matrix listed in Tables IV–VI.

Step 2: Integrate the normalized decision-making matrices using the WPA operator, (6), and (7), and then, obtain the GDM matrix $G\tilde{V}$ as listed in Table VII.

Step 3: Calculate the degree of consensus between each normalized decision-making matrix and the GDM matrix using (9) and (10); we have

$$\mathrm{CD}\left(\tilde{V}^1, G\tilde{V}\right) = 0.945, \quad \mathrm{CD}\left(\tilde{V}^2, G\tilde{V}\right) = 0.950$$
$$\mathrm{CD}\left(\tilde{V}^2, G\tilde{V}\right) = 0.959.$$

Because the supplier is important for the car company [31], the consensus threshold α can be chosen to have a high value; in this decision-making processes, we choose $\alpha = 0.955$.

Based on the results of step 3, we find that the first and second experts do not achieve consensus. Therefore, the feedback mechanism is applied to adjust the initial normalized decision matrix in step 4.

Step 4 (First Iteration): Calculate the intermediate-value matrix $I\tilde{V}^u = (I\tilde{x}_{ij}^u)_{m\times n}$ between the group-decision matrix and the nonconsensus matrix according to (11) and calculate the new GDM $NG\tilde{V}$ by the WPA operator, (6) and (7); we obtain $I\tilde{V}^1$, $I\tilde{V}^2$, and $NG\tilde{V}$, as listed in Tables VIII–X.

Step 5: Calculate the degree of consensus between each adjusted intermediate-value matrix $I\tilde{V}$ and the new GDM matrix $NG\tilde{V}$ using (9) and (10); we obtain

$$\mathrm{CD}\left(I\tilde{V}^1, NG\tilde{V}\right) = 0.966, \mathrm{CD}\left(I\tilde{V}^2, NG\tilde{V}\right) = 0.969.$$

Because $\mathrm{CD}(I\tilde{V}^1, NG\tilde{V}) > \alpha$ and $\mathrm{CD}(I\tilde{V}^2, NG\tilde{V}) > \alpha$, based on the feedback mechanism (steps 4 and 5), the first and second experts achieve consensus with the third expert. Thus, all of the experts have reached consensus, so the selection process is given based on heterogeneous TOPSIS as follows.

Step 6: Based on the new GDM consensus matrix $NG\tilde{V}$, select the HPIS $g\tilde{x}^+$ and HNIS $g\tilde{x}^-$ by (12)–(15); we obtain

HPIS:

$$g\tilde{x}^+ = ((0.128, 0.184, 0.277, 0.442), (0.176, 0.223, 0.291) [0.146, 0.339], [0.198, 0.254], 0.212)^T.$$

HNIS:

$$g\tilde{x}^- = ((0.070, 0.114, 0.232, 0.388), (0.101, 0.169, 0.267), [0.099, 0.289], [0.145, 0.199], 0.195)^T.$$

TABLE IV
NORMALIZED DECISION MATRIX $\tilde{V}^1$ GIVEN BY e_1

	A_1	A_2	A_3	A_4	A_5
C_1	(0.094,0.148,0.227,0.353)	(0.188,0.259,0.364,0.529)	(0.156,0.222,0.318,0.471)	(0.031,0.074,0.136,0.235)	(0.063,0.111,0.182,0.294)
C_2	(0.153,0.228,0.302)	(0.066,0.203,0.295)	(0.109,0.152,0.279)	(0.164,0.203,0.311)	(0.175,0.215,0.311)
C_3	[0.087,0.435]	[0.152,0.391]	[0.087,0.391]	[0.130,0.435]	[0.043,0.348]
C_4	[0.154,0.245]	[0.207,0.251]	[0.107,0.162]	[0.166,0.251]	[0.219,0.265]
C_5	0.190	0.205	0.188	0.191	0.226

TABLE V
NORMALIZED DECISION MATRIX $\tilde{V}^2$ GIVEN BY e_2

	A_1	A_2	A_3	A_4	A_5
C_1	(0.156,0.222,0.318,0.471)	(0.063,0.111,0.182,0.294)	(0.094,0.148,0.227,0.353)	(0.031,0.074,0.136,0.235)	(0.188,0.259,0.364,0.529)
C_2	(0.175,0.215,0.311)	(0.109,0.152,0.279)	(0.066,0.203,0.295)	(0.164,0.203,0.311)	(0.153,0.228,0.302)
C_3	[0.100,0.259]	[0.125,0.296]	[0.075,0.222]	[0.175,0.333]	[0.200,0.370]
C_4	[0.179,0.243]	[0.208,0.249]	[0.108,0.160]	[0.158,0.240]	[0.213,0.262]
C_5	0.193	0.196	0.201	0.215	0.195

TABLE VI
NORMALIZED DECISION MATRIX $\tilde{V}^3$ GIVEN BY e_3

	A_1	A_2	A_3	A_4	A_5
C_1	(0.132,0.182,0.280,0.471)	(0.079,0.152,0.240,0.412)	(0.105,0.152,0.240,0.418)	(0.105,0.152,0.320,0.529)	(0.026,0.121,0.240,0.412)
C_2	(0.164,0.214,0.277)	(0.114,0.160,0.248)	(0.169,0.214,0.248)	(0.148,0.187,0.236)	(0.187,0.225,0.268)
C_3	[0.154,0.296]	[0.154,0.296]	[0.179,0.370]	[0.128,0.259]	[0.077,0.222]
C_4	[0.173,0.243]	[0.189,0.246]	[0.180,0.235]	[0.152,0.213]	[0.150,0.246]
C_5	0.202	0.193	0.204	0.187	0.214

TABLE VII
GDM MATRIX $G\tilde{V}$

	A_1	A_2	A_3	A_4	A_5
C_1	(0.128,0.184,0.276,0.435)	(0.107,0.172,0.260,0.411)	(0.117,0.172,0.260,0.411)	(0.061,0.105,0.210,0.353)	(0.086,0.160,0.260,0.411)
C_2	(0.164,0.219,0.295)	(0.098,0.170,0.272)	(0.119,0.192,0.272)	(0.158,0.196,0.281)	(0.173,0.223,0.291)
C_3	[0.117,0.327]	[0.144,0.325]	[0.119,0.332]	[0.143,0.335]	[0.104,0.306]
C_4	[0.169,0.244]	[0.200,0.248]	[0.136,0.191]	[0.158,0.233]	[0.190,0.257]
C_5	0.196	0.198	0.198	0.197	0.212

TABLE VIII
INTERMEDIATE-VALUE MATRIX $I\tilde{V}^1$

	A_1	A_2	A_3	A_4	A_5
C_1	(0.111,0.166,0.252,0.394)	(0.147,0.216,0.312,0.470)	(0.137,0.197,0.289,0.441)	(0.046,0.090,0.173,0.294)	(0.074,0.136,0.221,0.353)
C_2	(0.159,0.223,0.298)	(0.082,0.187,0.283)	(0.114,0.172,0.275)	(0.161,0.199,0.296)	(0.174,0.219,0.301)
C_3	[0.102,0.381]	[0.148,0.358]	[0.103,0.362]	[0.137,0.385]	[0.074,0.327]
C_4	[0.162,0.244]	[0.203,0.249]	[0.122,0.176]	[0.162,0.242]	[0.204,0.261]
C_5	0.193	0.202	0.193	0.194	0.219

Step 7: Calculate distances D_i^+andD_i^-; by (16) and (17), we have

$$D_1^+ = 0.087,\ D_2^+ = 0.186,\ D_3^+ = 0.205$$
$$D_4^+ = 0.240,\ D_5^+ = 0.162$$
$$D_1^- = 0.291,\ D_2^- = 0.201,\ D_3^- = 0.182$$
$$D_4^- = 0.144,\ D_5^- = 0.229.$$

Step 8: Calculate the degree of similarity $\tilde{S}_i$ of the ideal solution

$$\tilde{S}_1 = 0.770, \tilde{S}_2 = 0.520, \tilde{S}_3 = 0.470, \tilde{S}_4 = 0.375, \tilde{S}_5 = 0.585.$$

Step 9: Rank the order according to $\tilde{S}_i$ in descending order

$$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4.$$

TABLE IX
INTERMEDIATE-VALUE MATRIX $I\tilde{V}^2$

	A_1	A_2	A_3	A_4	A_5
C_1	(0.142,0.203,0.297,0.453)	(0.085,0.142,0.221,0.353)	(0.105,0.160,0.244,0.382)	(0.046,0.090,0.173,0.294)	(0.137,0.210,0.312,0.470)
C_2	(0.170,0.217,0.303)	(0.104,0.161,0.275)	(0.093,0.197,0.283)	(0.161,0.199,0.296)	(0.163,0.225,0.297)
C_3	[0.108,0.293]	[0.135,0.311]	[0.097,0.277]	[0.159,0.334]	[0.152,0.338]
C_4	[0.174,0.243]	[0.204,0.248]	[0.122,0.175]	[0.158,0.237]	[0.201,0.259]
C_5	0.194	0.197	0.200	0.206	0.203

TABLE X
NEW GDM MATRIX $NG\tilde{V}$

	A_1	A_2	A_3	A_4	A_5
C_1	(0.128,0.184,0.277,0.442)	(0.101,0.168,0.256,0.411)	(0.115,0.168,0.256,0.412)	(0.070,0.114,0.232,0.388)	(0.075,0.152,0.256,0.411)
C_2	(0.164,0.218,0.291)	(0.101,0.169,0.267)	(0.129,0.196,0.267)	(0.156,0.195,0.272)	(0.176,0.223,0.287)
C_3	[0.124,0.321]	[0.146,0.319]	[0.131,0.339]	[0.140,0.320]	[0.099,0.289]
C_4	[0.170,0.244]	[0.198,0.248]	[0.145,0.199]	[0.157,0.229]	[0.182,0.254]
C_5	0.196	0.197	0.199	0.195	0.212

TABLE XI
COMPARISON OF THE PROPOSED MODEL WITH [24]

	The proposed method	The consensus method in [24]
numbers of non-consensual matrix	2	2
times of iterations	1	1
Evaluation results	(0.770,0.520,0.470,0.375,0.585)	(0.790,0.559,0.582,0.448,0.605)
Ranking results	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	$A_1 \succ A_5 \succ A_3 \succ A_2 \succ A_4$

TABLE XII
TRANSFORMED MATRIX $\tilde{V}_t^1$

	A_1	A_2	A_3	A_4	A_5
C_1	0.206	0.345	0.292	0.119	0.162
C_2	0.228	0.188	0.180	0.226	0.234
C_3	0.261	0.272	0.239	0.283	0.196
C_4	0.200	0.229	0.134	0.208	0.242
C_5	0.190	0.205	0.188	0.191	0.226

TABLE XIII
TRANSFORMED MATRIX $\tilde{V}_t^2$

	A_1	A_2	A_3	A_4	A_5
C_1	0.292	0.162	0.206	0.119	0.335
C_2	0.234	0.180	0.188	0.226	0.228
C_3	0.180	0.211	0.149	0.254	0.285
C_4	0.211	0.228	0.134	0.199	0.238
C_5	0.193	0.196	0.201	0.215	0.195

TABLE XIV
TRANSFORMED MATRIX $\tilde{V}_t^3$

	A_1	A_2	A_3	A_4	A_5
C_1	0.266	0.221	0.227	0.277	0.200
C_2	0.218	0.174	0.210	0.191	0.227
C_3	0.225	0.225	0.275	0.194	0.150
C_4	0.208	0.218	0.208	0.183	0.198
C_5	0.202	0.193	0.204	0.187	0.214

TABLE XV
NEW GDM $G\tilde{V}_t$

	A_1	A_2	A_3	A_4	A_5
C_1	0.256	0.237	0.240	0.182	0.229
C_2	0.226	0.180	0.194	0.212	0.229
C_3	0.222	0.235	0.226	0.239	0.204
C_4	0.207	0.224	0.163	0.195	0.223
C_5	0.196	0.198	0.198	0.197	0.212

Therefore, the best supplier from whom to purchase automobile parts is A_1.

To illustrate the effectiveness of the proposed method, comparison analyses are shown in Section V.

V. COMPARISON ANALYSES

In this section, comparison analysis was performed to validate the effectiveness of the proposed method.

First, the proposed model was compared with a related consensus model developed in [24]. In the consensus process for the supplier selection example from the previous section, the threshold of acceptable similarity was set as 0.047 for the comparison model. In the selection process, the heterogeneous TOPSIS was applied to the comparison model to keep consistency of processing information. The comparative results were shown in Table XI. It can be seen that the two models generated the same top-ranked suppliers. The numbers of nonconsensual matrices and iteration times are also the same. Moreover, the proposed method avoids the problem of information loss and can calculate the heterogeneous information in the selection process.

To further validate the superiority of the proposed method, it was compared with the methods in which heterogeneous

TABLE XVI
TRANSFORMED MATRIX $\tilde{V}_t^1$

	A_1	A_2	A_3	A_4	A_5
C_1	(0.094,0.148,0.227,0.353)	(0.188,0.259,0.364,0.529)	(0.156,0.222,0.318,0.471)	(0.031,0.074,0.136,0.235)	(0.063,0.111,0.182,0.294)
C_2	(0.153,0.228,0.228,0.302)	(0.066,0.203,0.203,0.295)	(0.109,0.152,0.152,0.279)	(0.164,0.203,0.203,0.311)	(0.175,0.215,0.215,0.311)
C_3	(0.087,0.087,0.435,0.435)	(0.152,0.152,0.391,0.391)	(0.087,0.087,0.391,0.391)	(0.130,0.130,0.435,0.435)	(0.043,0.043,0.348,0.348)
C_4	(0.154,0.154,0.245,0.245)	(0.207,0.207,0.251,0.251)	(0.107,0.107,0.162,0.162)	(0.166,0.166,0.251,0.251)	(0.219,0.219,0.265,0.265)
C_5	(0.190,0.190,0.190,0.190)	(0.205,0.205,0.205,0.205)	(0.188,0.188,0.188,0.188)	(0.191,0.191,0.191,0.191)	(0.226,0.226,0.226,0.226)

TABLE XVII
TRANSFORMED MATRIX $\tilde{V}_t^2$

	A_1	A_2	A_3	A_4	A_5
C_1	(0.156,0.222,0.318,0.471)	(0.063,0.111,0.182,0.294)	(0.094,0.148,0.227,0.353)	(0.031,0.074,0.136,0.235)	(0.188,0.259,0.364,0.529)
C_2	(0.175,0.215,0.215,0.311)	(0.109,0.152,0.152,0.279)	(0.066,0.203,0.203,0.295)	(0.164,0.203,0.203,0.311)	(0.153,0.228,0.228,0.302)
C_3	(0.100,0.100,0.259,0.259)	(0.125,0.125,0.296,0.296)	(0.075,0.075,0.222,0.222)	(0.175,0.175,0.333,0.333)	(0.200,0.200,0.370,0.370)
C_4	(0.179,0.179,0.243,0.243)	(0.208,0.208,0.249,0.249)	(0.108,0.108,0.160,0.160)	(0.158,0.158,0.240,0.240)	(0.213,0.213,0.262,0.262)
C_5	(0.193,0.193,0.193,0.193)	(0.196,0.196,0.196,0.196)	(0.201,0.201,0.201,0.201)	(0.215,0.215,0.215,0.215)	(0.195,0.195,0.195,0.195)

TABLE XVIII
TRANSFORMED MATRIX $\tilde{V}_t^3$

	A_1	A_2	A_3	A_4	A_5
C_1	(0.132,0.182,0.280,0.471)	(0.079,0.152,0.240,0.412)	(0.105,0.152,0.240,0.418)	(0.105,0.152,0.320,0.529)	(0.026,0.121,0.240,0.412)
C_2	(0.164,0.214,0.214,0.277)	(0.114,0.160,0.160,0.248)	(0.169,0.214,0.214,0.248)	(0.148,0.187,0.187,0.236)	(0.187,0.225,0.225,0.268)
C_3	(0.154,0.154,0.296,0.296)	(0.154,0.154,0.296,0.296)	(0.179,0.179,0.370,0.370)	(0.128,0.128,0.259,0.259)	(0.077,0.077,0.222,0.222)
C_4	(0.173,0.173,0.243,0.243)	(0.189,0.189,0.246,0.246)	(0.180,0.180,0.235,0.235)	(0.152,0.152,0.213,0.213)	(0.150,0.150,0.246,0.246)
C_5	(0.202,0.202,0.202,0.202)	(0.193,0.193,0.193,0.193)	(0.204,0.204,0.204,0.204)	(0.187,0.187,0.187,0.187)	(0.214,0.214,0.214,0.214)

TABLE XIX
NEW GDM MATRIX $G\tilde{V}_t$

	A_1	A_2	A_3	A_4	A_5
C_1	(0.128,0.184,0.276,0.435)	(0.107,0.172,0.260,0.412)	(0.117,0.172,0.260,0.412)	(0.061,0.105,0.210,0.353)	(0.086,0.160,0.260,0.412)
C_2	(0.164,0.218,0.218,0.295)	(0.098,0.171,0.171,0.271)	(0.120,0.192,0.192,0.271)	(0.158,0.196,0.196,0.281)	(0.173,0.223,0.223,0.291)
C_3	(0.118,0.118,0.327,0.327)	(0.145,0.145,0.325,0.325)	(0.120,0.120,0.332,0.332)	(0.143,0.143,0.334,0.334)	(0.104,0.104,0.304,0.304)
C_4	(0.169,0.169,0.244,0.244)	(0.200,0.200,0.248,0.248)	(0.136,0.136,0.191,0.191)	(0.158,0.158,0.233,0.233)	(0.189,0.189,0.256,0.256)
C_5	(0.196,0.196,0.196,0.196)	(0.198,0.198,0.198,0.198)	(0.198,0.198,0.198,0.198)	(0.197,0.197,0.197,0.197)	(0.212,0.212,0.212,0.212)

information is transformed into a single form before continuing to GDM. For simplicity, we denote the comparative analysis methods as THI-1 and THI-2.

1) *THI-1 Method:* Transform the heterogeneous information into crisp numbers based on the mean method; the transformed matrices $\tilde{V}_t^1$, $\tilde{V}_t^2$, and $\tilde{V}_t^3$ and the new GDM matrix $G\tilde{V}_t$ are given in Tables XII–XV.
2) *THI-2 Method:* Because crisp numbers, interval numbers, and triangular fuzzy numbers can be seen in the special forms of trapezoidal fuzzy numbers, we can transform the heterogeneous information into trapezoidal fuzzy numbers; the transformed matrices $\tilde{V}_t^1$, $\tilde{V}_t^2$, and $\tilde{V}_t^3$ and the new GDM matrix $G\tilde{V}_t$ are given in Tables XVI–IX.

TABLE XX
COMPARISON OF THE PROPOSED MODEL WITH THI METHODS

	The proposed method	THI-1 method	THI-2 method
numbers of non-consensual matrix	2	0	3
times of iterations	1	0	1
Evaluation results	(0.770,0.520,0.47,0.375,0.585)	(0.772,0.617,0.463,0.409,0.679)	(0.741,0.604,0.409,0.418,0.709)
Ranking results	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	$A_1 \succ A_5 \succ A_2 \succ A_3 \succ A_4$	$A_1 \succ A_5 \succ A_2 \succ A_4 \succ A_3$

Based on Tables XII–XIX and the GDM processes, the comparison results of the three methods were listed in Table XX.

Table XX shows that all three methods generated the same top-two ranked suppliers. These results illustrate the effectiveness of the proposed method. Although the proposed method retains some accurate information, the numbers of the nonconsensual matrix are smaller than those of THI-2. The THI-1 method has fewer numbers of nonconsensual matrices and iterations, but it loses information when it transforms heterogeneous data into crisp numbers in its decision-making processes. Moreover, Table XX indicated that the differences among the alternatives are more obvious using the proposed model than the other two methods. These results will be helpful for experts to choose the best supplier.

VI. CONCLUSION

This paper proposes a GDM method with heterogeneous fuzzy information, which finds the most reasonable decision-making alternatives. In the proposed method, the heterogeneous data are not transformed into a single form, but are directly integrated by a WPA operator. The consensus processes and the feedback mechanism with the iterative algorithm are used to adjust the individual-decision matrix if it does not reach consensus. Furthermore, a ranking formula with heterogeneous TOPSIS is adopted to select the best alternative. A numerical example of supplier selection indicates that the proposed model is effective and practical. The comparisons of the proposed model with other related models were conducted to show the advantage of the proposed model. The ranking results generated by the proposed model have larger differences among the alternatives than the other three models, which indicate that the proposed method can be helpful during the supplier selection process.

One of the future research directions is to solve large scale problem in GDM, such as the consistency of information in large-scale GDM and the application of GDM method in social media. Moreover, the dynamic GDM method with the heterogeneous information is another future research direction.

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