Optimization of Systemic Stability of Directed Network Using Genetic Algorithm

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***Abstract*— Flow dynamics in directed network can lead to cascade failures from node and link removal, and this is used as a paradigm for systemic risks in financial systems where the flow is a money flow. In order to reduce systemic risk, we analyze the network topology and find ways of rewiring to ensure that the time for the first node failure can be maximized. The analysis is numerical using genetic algorithm to evolve a network by rewiring towards one with higher systemic stability. The results show that a network can become more systemic stable if the incoming flow of all the nodes becomes more similar to that of the outgoing flow. For financial network, the way to reduce the risk of cascade bankruptcies is to share the systemic risk in the form of the fluctuation of capital value transfer by all banks. Our simple model of directed network shows that one way to improve the systemic stability of a network is to rewire it towards a perfect Watts-Strogatz network.**

Keywords—systemic stability, financial network, flow dynamics, directed network, genetic algorithm

I. INTRODUCTION

Network science has emerged as a very important area for multidisciplinary research with a plethora of applications in many fields in the last two decades. It has been used for modelling of social systems [1], [2], food webs [3], brain network [4], epidemic contagion [5], traffic simulations [6] and internet [7], where the nodes and links that form a graph becomes the simplest model in the description of complex systems for the investigation of their underlying properties. In this paper, we like to address the application of network science in computational finance. Mantegna [8] and Gao [9] used correlation of daily rate of return between different stocks to describe the structure with an undirected graph in the financial market. More sophisticated models that employ directed graph have also been used in describing the flow of information between companies, in the context of their time series listed in the stock markets [10]. The temporal pattern of cash flow, through payments, contracts, liabilities and assets transferal in this kind of directed network is generally very complex, as there is no law of conservation concerning the cash flow. There is however the limitation of flow capacity in a financial network and the formation of source and sink are signatures of disasters. In particular, when there is news or rumors that quickly spread in the social network, the effect of exogenous shocks and their propagation will generally have important impact on the cash flow in the financial network, leading sometimes to disaster such as the financial tsunami in 2008. The dynamics of flow in a given network therefore can lead to domino effect and the study of the failure of the system is a topic of systemic risk [11]–[13].

Systemic risks in financial network is intrinsically linked with some form of financial crises triggered by some sudden failure of a particular node or link [14]. The systemic risks in banking systems is of great social importance and has been studied with different kinds of models [15]–[18], while the containment of bankruptcy of a particular financial institute and the associated monitoring of the network to avoid disaster is also of great research interest in academia. In this paper, we investigate the relation between the topology of the financial network to its resistance of network disintegration caused by external shocks. In the investigation of R. Albert, H. Jeong, and A.-L. Barabási on the problem of error and attack tolerance of complex networks, they found that scale-free networks have a higher tolerance to random node removal than homogeneous networks [19] [20]. These results focus on the abrupt removal of subgraphs due to some external factors such as cyber-attacks for internets. Sornette et al, on the other hand, studied empirically both external impacts and self-organizing effects on complex networks [21]. We like to generalize these works to directed network, without limiting ourselves to scale free networks.

In a very simplified model of the financial system, each bank is represented by a node in a directed network and possesses a randomly predetermined amount of capital. Varying amounts of money are transferred among these banks at every time unit along the direction of edges. After some time, the total value of assets for some bank might be non-positive and we declare the bank bankrupted. The bankrupted nodes are subsequently removed from the network along with their links. Thus, the bankrupted bank will affect the money flow of its neighbors and the financial damage can propagate in the network since these removed banks are the sources and sinks of capital of their neighbors. Therefore, a node removal may lead to the collapse of other banks in the neighborhood, leading to a cascade of node failure. Over time, the network may be disconnected. When this happens, we say that there is a collapse of the entire financial system. It is of our interest to analyze different network topologies and optimize the susceptibility of directed networks with capital flow to node failure. This study therefore is an investigation on the relation between systemic risk and the topology of the financial network. In general, there can be astronomically large number

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of ways to form a connected directed network with N nodes and L links, for large N and L. This implies that our problem is computationally complex, in that we must search in a large solution space of directed networks to find one that has the highest systemic stability, even after we have defined measures to characterize the network failure.

For optimization problem with a very large solution space, genetic algorithm (GA) has been used in many disciplines with amazing success. The algorithm searches for locally optimal solutions by mimicking the process of natural selection in the biological world [22]. In GA, solutions with higher fitness in a population of candidate solutions are selected as survivors. This is similar to the phenomena in which species with higher resilience of external environmental pressure has a higher chance of survival. An optimal solution can be found after generations of populations have passed. GA has been employed in solving different problems such as spacecraft antenna optimization [23], knapsack problems [24], portfolio management [25], optimal strategies in game theory [26], and gene network [27]. We will apply GA to find the directed network with high systemic stability. In this paper, the different types of networks are introduced in section II. Two models of GA are compared in section III. Analysis of the results is in section IV, followed by the discussion.

II. Systemic Stability of Directed Networks

We first describe three most common classes of networks, the Watts and Strogatz (WS) network, the Erdős–Rényi (ER) network, and the Barabási–Albert (BA) network [28]. We then generalize these networks to directed networks with N nodes and L links. WS networks are networks with short average path lengths and high clustering coefficients, with $N \gg L \gg \ln(N) \gg 1$. A regular ring lattice is constructed with R layers. An example of a regular ring lattice is shown in Fig. 1. In the network, the nodes are n_0, n_1, …, n_{N-1}. For each node n_i, the neighbors of that node are $n_{(i-r) \bmod N}$, $n_{(i-r+1) \bmod N}$, $\ldots n_{(i+r) \bmod N}$. Therefore the number of links is simply $L=NR$. After the construction of the ring lattice, each link of the lattice has a probability β of being rewired. Notice that self-loops and multiple links between two nodes are not allowed during rewiring [29].

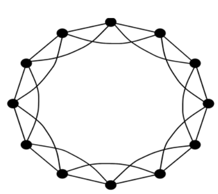

Fig. 1. A regular ring lattice with R=2 and N=12 in the construction of WS network.

ER networks are network where undirected links are distributed randomly. In an ER network with N nodes, there are $N(N-1)/2$ possible slots where a link can be placed to connects two different nodes. To construct an ER network, L links are being placed onto the slots one by one randomly to connect N nodes such that each slot can only contain one link at maximum [30]. There exists algorithm that ensures the ER network constructed is a connected one. BA networks are networks in which their degree distribution follows an inverse power law. To construct a BA network, a small graph is formed as the ancestor network, such as a star network with k+1 nodes for some k, in which one of the nodes has k neighbors, and this is the center of the star with the other nodes of degree one, being only connected to the center node. Based on this initial star network, new nodes are added to the network as offspring nodes. The offspring node is connected to nodes in the existing network with a probability proportional to the degree of the nodes the offspring is connecting. The probability p_i that a link will be formed between the node i and the new node is $p_i = \frac{K_i}{\sum_j K_j}$, where K_i is the degree of node i [31].

Next, we construct a directed network from the undirected network by assigning a direction to each link in the network. Each node has K_{in} incoming links and K_{out} outgoing links. There are many features to characterize the systemic stability of a directed network. One of the easy measures of systemic stability is to consider the time needed for some nodes to be removed, while there is continuous flow along links. Another measure is the time needed for the directed network to evolve to a disconnected network after sufficient number of node removal. Here a directed network is considered disconnected if at least one pair of nodes has no route connecting them. We see from this discussion the systemic stability for directed network necessitates the description of the dynamics of flow. We now illustrate our idea on the flow dynamic in the context of financial system. We consider a bank as a node and the network as an interbank complex system. The flow between nodes is a flow of money. For simplicity, our model assumes that each bank is initially endowed with the same capital value C=1 (so that C_1, C_2, …, C_N are set to 1). Each bank also has K neighbors, with $K=K_{out}+K_{in}$. If node i and node j are connected by an arrow pointing from i to j, then we say that node j is the neighboring bank of i that borrows money from i, (and i lends money to j) so that the arrow from i to j is an outgoing arrow from i and an incoming arrow for j. Among K neighbors of i, there are K_{out} of them being the borrowers from node i. The direction of the arrow signifies the flow of money from bank i to its K_{out} neighbors. Similarly, the neighbors with arrows pointing towards bank i are the banks that lend money to bank i and there are K_{in} of them. The interesting feature that we like to address is the way that one or more nodes start to run out of money due to over-lending (too much capital being transferred from these nodes to their neighbors), and too little borrowing, so that inflow of money cannot cover the outflow. The common outcome for these banks is bankruptcy, and we remove the bankrupted banks from the financial system, along with their links to their neighbors. Under this simplified model of flow dynamics of money, we further assume that at each time t >0, a given bank i (node i) will lend a "value of transfer" $V_{out(i,t)}$ to its K_{out} neighbors. This "value of transfer" $V_{out(i,t)}$ is a random positive number in the range (0, P_{max}] ,where $0<P_{max}\ll 1$ is a model parameter for the money flow. Our model further assumes that the value of transfer is divided equally among the K_{out} neighbors, so that each receives $V_{out(i,t)}/K_{out}$ from node i at time t. Meanwhile node i also receives some inflow of money from its K_{in} neighbors, though each K_{in} neighbor can contribute differently as they have different value of V_{out}, which after all is another random number in the range (0, P_{max}]. Therefore in a value transfer event, N random numbers of $V_{out(i,t)}$ were being generated at time t, and each node will distribute their corresponding value to their neighbors at the same time. The value of C_1, C_2, …, C_N will therefore be updated. After some time, some nodes may

have a negative value of assets. At that time, the node will be removed from the directed network, along with all the links connecting to that node. When progressively more nodes are being removed, the directed network may either become disconnected, or it may continue shrinking without being disconnected in the end.

In this paper, we analyze three classes of directed networks: ER, WS, and BA. An ensemble of 1000 networks in each class is generated, so that each member of the ensemble is a network with N=100 and P_{max}=0.01, and β=0.15 for WS directed networks, L=400 for ER and WS directed networks, and L=384 for BA directed networks. For each directed network in each ensemble, value transfer events were being done repeatedly until the directed network shrinks to one node or become disconnected. We are interested in three key measurements on the evolving directed network. The first one is the t_1 time when the first node is being removed as its capital value is less than zero. This measurement signals the time that the first bank is bankrupted. The second measurement is the time t_d when the directed network becomes disconnected. This signals the time that the bankruptcy has spread across the entire financial system, similar to a global tsunami. This time t_d (which is bigger than t_1) defines a time period $T = t_d - t_1 > 0$ which the financial system can exercise emergency measures to prevent global disaster. The third measurement is the size N_d when the directed network becomes disconnected at time t_d. This measurement provides a scale for the quantification of the disaster. All three quantities (t_1, t_d, N_d) were measured for each network in the simulation of value transfer and we observe that the correlation between t_1 the other two measurements is quite small, with correlation coefficients all less than 0.02. On the other hand, t_d is inversely related to the size of disaster, $t_d \propto 1/N_d$, with a correlation coefficient of 0.9277. Our results on three classes of networks are shown in Table 1. We see that WS directed networks have a much higher chance to become disconnected when value transfers were being done many times, and BA directed networks with same N and L will be relatively stable.

III. Network Optimization by Genetic Algorithm

From the point of view of maintaining systemic stability of financial network, the obvious way to prevent disaster due to cascade bankruptcies is to increase the time t_1 that the first bankruptcy happens. There are many approaches to extend t_1, for example, by various means of rewiring the network so that the node with very small value will receive more value transfer to keep it alive. In this work, we bypass the discussion of these rewiring mechanisms since they are more related to the mathematical analysis of the topology of the network. Here we use an evolutionary approach to seek for the network with highest systemic stability. We start with a population of networks, and evaluate their first time to bankruptcy, t_1. By introducing a measure of fitness, we then perform change to the topology of the network through rewiring.

Generally, in a genetic algorithm, a few random individuals are being selected to form the initial population which will then be subjected to the Darwinian principle of the survival of the fittest to evolve this population. In this process, a critical measure of fitness must be defined so that the survivability of the individuals can be measured, with the weakest being eliminated. Between generations, the individuals with the highest fitness values are allowed to mutate and create a new generation of individuals. The process continues and the fitness of the best individuals will increase over the generations. We can use a stopping criterion of this evolution by imposing a maximum number **G** of generations based on our available computational resource, thereby defining our optimal solution as the fittest individual, since the maximum fitness of the population will only increase monotonically with generation. In the context of network optimization, the fitness of the network is its systemic stability. In the context of maximizing the time t_1 that the first bankruptcy occurs, we can convert the measurement of t_1 that a bank actually goes bankrupt to another easier measurement of the minimum value of capital carried by these N nodes (banks) at a given time t: $C_{Min}(G)=\underset{i=1,..N}{Min}\{C_1,....,C_N\}=C_{j^*}(G)$. Let's call this special node with minimum capital node j^*. (There can be several nodes with the same minimum value, in that case we simply choose one of them randomly). Note that this minimum value C_{Min} as well as j^* are dependent on the generation number G that we stop evolving the population. Now, for a given network, which is an individual in the population at time t, the larger is its C_{Min}, the less likely that the first bankruptcy will occur. Therefore, in a population of M individuals (networks), we can rank their likelihood to bankruptcy by their values of C_{Min}. In another word, instead of using t_1 as the fitness of the network, we can use C_{Min} with much more convenience and ease. The fitness of a network in our GA optimization is measure by its C_{Min}: a fitter network has a larger value of C_{Min}.

Table 1. Mean and standard deviation of t_1, t_d and N_d for the 3 different types of networks, (average of 1000 samples).

Mean and SD	*ER directed networks*	*WS directed networks*	*BA directed networks*
Mean (SD) t_1	2340 (147)	2540 (252)	2120 (54)
Mean (SD) t_d	90500 (61900)	47200 (41600)	69200 (43700)
Mean (SD) N_d	9.00 (2.98)	19.3 (8.26)	3.3 (4.97)

We describe our genetic algorithm with a flow chart shown in Fig.2. We use M=100 directed networks initially at G=0. We then perform t=200 value transfer events to each directed network in a simulation. Thus, each node will experience a value transfer 200 times. The value of the minimum capital value node in the network after the 200 value transfers was defined as C_{Min}. In each generation, 10 directed network with the highest fitness values were are selected to be the parents while being promoted to the next generation. Each parent can undergo genetic operations to produce nine directed networks called the offsprings. The genetic operation used to produce these nine children from a parent is through random rewiring from the parent network for 1, 2, …, up to 10 times. In each rewiring, the head and the tail of the link could move freely to connect two nodes that are not originally connected by another link with the same direction. In this way, the new population still contains 100 directed networks, 10 of them are the old parents, each producing 9 children, so that there are 90 new directed networks making up the new population. Now, the process of randomly initiating the starting population and allowing it to evolve for 150 generations is being repeated for 100 times. The overall fitness value of the directed network is therefore the average of $\boldsymbol{C_{Min}}$ in the 100 simulations. Due to the limits of the computation time, only 150 generations have been generated.

In principle, if we do not use genetic algorithm for network optimization, we should start with a network where no node is a pure source or sink, so that both $\boldsymbol{K_{out}}$ and $\boldsymbol{K_{in}}$ for any node initially are positive. This is for the obvious financial reason that when a bank is a pure source, ($0<\boldsymbol{K}=\boldsymbol{K_{out}}$ and $\boldsymbol{K_{in}}=0$), this bank will soon have its initial capital reduce to 0 when the money transfer between banks operates for some finite time, and this bank will go bankrupt. Similarly, there is no node with ($0<\boldsymbol{K}=\boldsymbol{K_{in}}$ and $\boldsymbol{K_{out}}=0$), as this is a bank that receives an increase of capital without any cost, which is also unrealistic. Since a network that contains pure source or sink will quickly lead to systemic instability, in that some node will quickly have its value reduced to zero, the formulation of using genetic algorithm will also quickly eliminate this network from the population. Therefore, the use of genetic algorithm by default can automatically filter out networks with pure source or sink as the minimum capital node will quickly emerge, leading to a small value of fitness and be eliminated. We see that genetic algorithm provides this immediate advantage in our search for network with high systemic stability. In the next section, we will discuss the topological properties of the fittest network obtained from our application of genetic algorithm

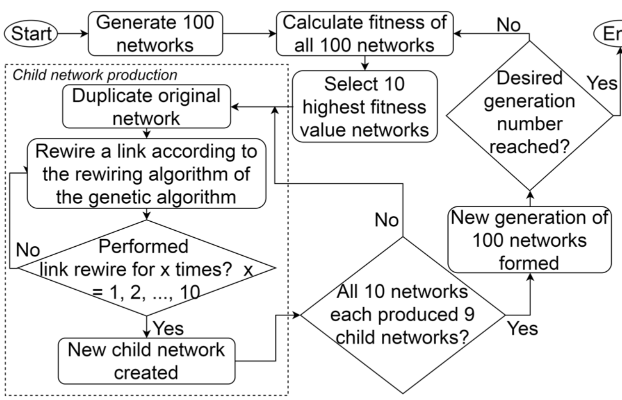

Fig. 2. The algorithm of the genetic algorithm

IV Most Stable Network obtained from Genetic Algorithm

We apply genetic algorithm with $\boldsymbol{G}=150$ generations to find the fittest ER directed networks with $\boldsymbol{N}=100$ and $\boldsymbol{L}=500$. Each node in the network has an out-degree $\boldsymbol{K_{out}}$ and an in-degree $\boldsymbol{K_{in}}$. We define $K_{diff}=K_{in}-K_{out}$ and analyze the standard deviation $\boldsymbol{\sigma_d}$ of $\boldsymbol{K_{diff}}$ of all the nodes in a network. For the fittest network, $\boldsymbol{\sigma_d}$ has essentially zero correlation with the following quantities: generation number, average fitness of the best fitness networks, average fitness of the ensemble of 100 networks at $\boldsymbol{G}$ generation. These observations suggest that there are some other properties of the network that are more correlated to the fitness of the networks. From our ensemble of results obtained in these 100 simulations, we observe that the node $\boldsymbol{j^*}$ with the lowest value $\boldsymbol{C_{Min}}$ in one simulation can be different in another simulation, and the set $\boldsymbol{S}$ of these nodes $\boldsymbol{j^*}$ can be large. In another word, these nodes $\boldsymbol{j^*}$ are not unique and in general they can be very different. Since we are interested in the fittest networks in our simulation, we therefore focus our attention on the topology of the set $\boldsymbol{S}$ and hope that the elements of $\boldsymbol{S}$ have some characteristic topological properties. We now consider the local topology around the node where value transfer takes place. Given a node $\boldsymbol{i}$ (the target node in Fig.3) with $\boldsymbol{K_{out}}$ neighbors ($\boldsymbol{out_1}, \ldots, \boldsymbol{out_{K_{out}}}$), and $\boldsymbol{K_{in}}$ neighbors ($\boldsymbol{in_1}, \ldots, \boldsymbol{in_{K_{in}}}$).

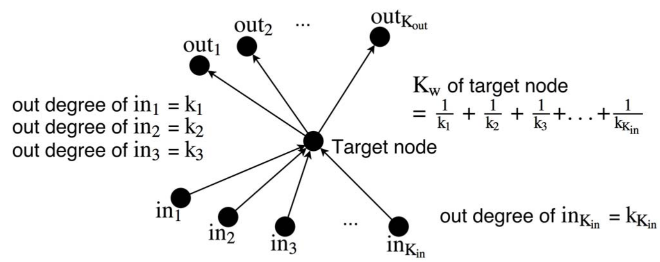

Fig. 3. A diagram illustrating the definition of $\boldsymbol{K_w}$.

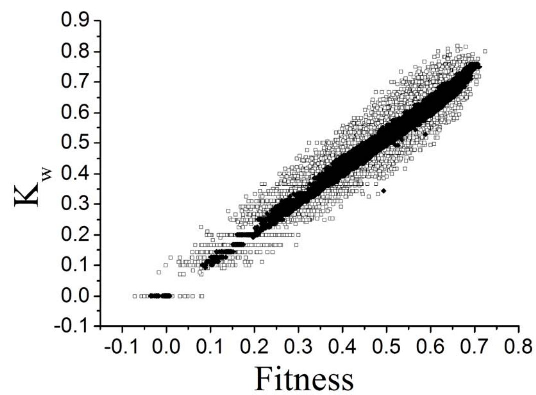

Fig.4. $\boldsymbol{K_w}$ of the lowest capital value nodes $\boldsymbol{j^*}$ (□) and $\boldsymbol{K_w}$ of the lowest $\boldsymbol{K_w}$ nodes (◆) of the entire network versus the fitness of the networks in different generations. Note that $\boldsymbol{K_w}$ of the lowest $\boldsymbol{K_w}$ nodes (◆) versus the fitness has a higher correlation.

Let's denote the out degree of these $\boldsymbol{K_{in}}$ neighbors as $\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3}, \ldots, \boldsymbol{k_{K_{in}}}$. We first define the neighboring topologies of those neighboring nodes $\{(\boldsymbol{out_1}, \ldots, \boldsymbol{out_{K_{out}}}), (\boldsymbol{in_1}, \ldots, \boldsymbol{in_{K_{in}}})\}$ of the target node $\boldsymbol{i}$ and investigate the relation of local topology with the fitness. We then define the weighted in-degree $\boldsymbol{K_w(i)}$ of the target node $\boldsymbol{i}$ by $K_w(i) \equiv \sum_{j=1}^{K_{in}} \left(1/k_j\right)$ and the standard deviation $\boldsymbol{\sigma_w}$ of $\boldsymbol{K_w}$ defined over all the nodes in the network. Now, we analyze the properties of the neighbor nodes with links pointing to the target nodes, which are the set $\boldsymbol{I}=(\boldsymbol{in_1}, \ldots \boldsymbol{in_{K_{in}}})$ shown in Fig.3. We investigate the average values of ($\boldsymbol{K_w}$, $\boldsymbol{K_{out}}$) of the set $\boldsymbol{I}$ of the target node, which can be any randomly chosen node $\boldsymbol{i}$ or specifically the lowest $\boldsymbol{K_w}$ nodes, and check if there is any correlation with the fitness value of the network. We find that for the nodes $\boldsymbol{j^*}$ with the lowest value $\boldsymbol{C_{Min}}$, their value $\boldsymbol{K_w(j^*)}$ versus the fitness have a positive correlation with correlation coefficient of around 0.9495 in Fig.4, while $\boldsymbol{\sigma_w}$ and the fitness somewhat do not have a strong correlation. Since these nodes $\boldsymbol{j^*}$ with the lowest value

C_{Min} of each simulation are not always also the lowest K_w node of the network, the correlation coefficient of lowest capital value nodes j^* (points (□) has lower correlation than (◆) in Fig.4). We also analyzed the correlation between the fitness value of the network for the following four quantities:
(1) average K_{out} of the neighbors of the lowest K_w node, which are the average of the numbers $\{k_1, k_2, k_3, \ldots, k_{K_{in}}\}$ in Fig.3,
(2) average K_w of the neighbors of the lowest K_w node,
(3) average value of K_w of the neighbors of any random node,
(4) average value of K_{out} of the neighbors of any random node.
We find no significant correlation *except* (1). These analysis on the statistical properties of the local topological structures around the lowest K_w node inspire us to design an improved genetic algorithm, using the correlation between fitness of the network and the local topology. In this improved genetic algorithm, the 10 best networks out of 100 networks are being promoted to the next generation, and each of the 10 networks produces 9 extra child networks by random rewiring, which is similar to that of the original genetic algorithm. However, instead of doing a random rewiring, we perform a non-random rewiring as follow. If node ω is the node with the highest weighted in-degree, and node α is the node with the lowest weighted in-degree. For each rewiring, we then remove a link from a random node pointing to ω, and create a link from another random node pointing to α. This new rewiring process can be tested for its efficiency. For the initial set of networks in our genetic algorithm (G=0), we construct 100 "perfect directed WS networks", which are WS networks with zero rewiring probability, as shown in Fig.5(a) and the local topology of this perfect directed WS network in Fig.5(b)

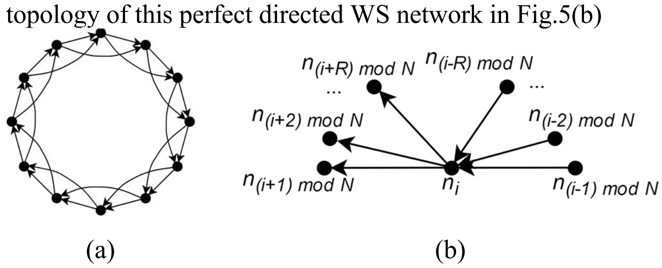

Fig. 5. (a) An example of a perfect directed WS network. (b) The topology of a node n_i in a perfect directed WS network that contains node $n_1, n_2, \ldots, n_N$.

Based on the statistical analysis of the local topologies, we are inspired to introduce modifications to the original genetic algorithm by incorporating these perfect directed WS networks into our evolutionary computation. To test the effectiveness of the improved genetic algorithm, we use WS networks with N=100, L=500, and β=0.15. It turns out that the results of these WS networks are similar to that of the results of the ER networks. Now, we set G=150 for the stopping criterion for the evolution using the improved genetic algorithm. The fitness of the best network increases much more rapidly with the improved genetic algorithm. The maximum fitness value of each generation increased quite rapidly at the first few generations, and it gradually reached a steady value at later generations, as shown in Fig.6(a). We also observe in Fig.6(a) that the fitness of WS networks is higher than ER networks. In Table 1, it was shown that t_1 of WS networks are generally longer than that of ER networks. Since the fitness of both genetic algorithms measures the value of the minimum value nodes after the simulations, a longer t_1 would mean a higher fitness. At the zeroth generation in Fig.6(a), the structures of WS and ER networks have not been altered yet, and it was shown that the fitness of WS networks are generally higher than that of ER, which is consistent with the results of Table 1. The simulation results in Fig.6(a) indicate that the fitness increases monotonically, with the improved GA much faster, reaching a level near the fitness of perfect WS networks after about 20 generations. This shows that the improved genetic algorithm provides an efficient way of rewiring to make the networks more stable.

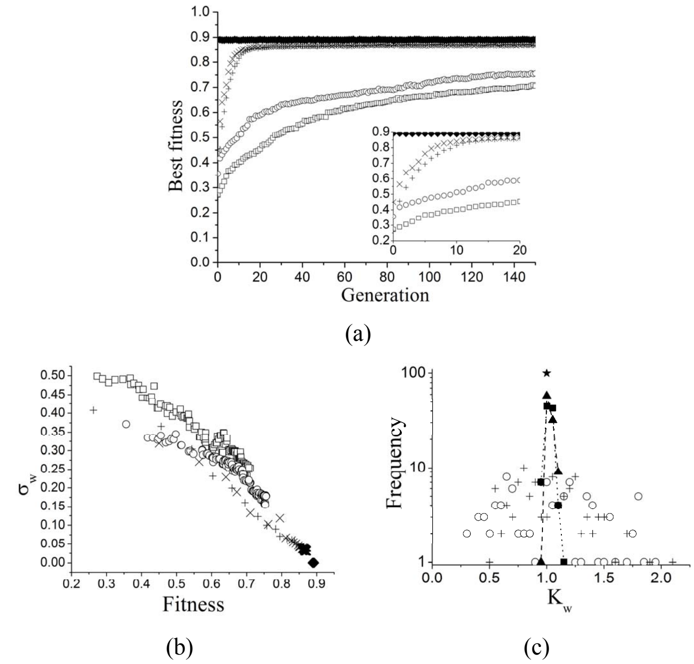

Fig.6 (a) The maximum fitness versus the generation number. (b) Standard deviation σ_w versus fitness of the best fitness networks in different generations. Symbols: ER networks of the original GA (□), WS networks of the original GA(○), ER networks of the improved GA(+), WS networks of the improved GA (╳), and perfect WS networks of the original GA (◆). (c) Approximated distribution of K_w for the best networks of different types networks: ER networks of the improved GA at generation 0 (○), WS networks of the improved GA at generation 0 (+), best ER network of the improved GA at generation 150 (▲), best WS network of the improved GA at generation 150 (■), and perfect WS networks of the original GA (★).

In this application of evolutionary computation, we see that the K_w distribution of the best networks in the latest generations in the improved genetic algorithm for both ER and WS network are close to the K_w distribution of perfect WS networks. Thus, we try to start with a population of directed WS network, we find that the fitness of the fittest network of each generation remains about the same value for all generations. In Fig.6(b), the standard deviation σ_w of the perfect WS networks is 0, and when σ_w versus fitness of the other networks are being plotted, they all show that σ_w converges to zero when the fitness increases. This means that after a large number of generations, the networks in those later generations will have topology very similar to a perfect WS network. Furthermore, the standard deviation σ_w of the best network versus the fitness of both genetic algorithms consistently shows a negative correlation. Since the fitness of the best network increases with the generation number, while σ_w decreases with the generation number, it is reasonable to conjecture that the best network found after many generation of evolution will have very small σ_w, meaning that the fittest network may be a network where the weighted in-degree K_w of all the nodes in the network are the same. Consequently, we can propose that a directed WS network, in which the links are either all pointing in the clockwise direction or all pointing in the anticlockwise direction as shown in Fig.5, will be the optimal design for a directed network with maximal systemic stability. From the data shown in Fig.6(a) and 6(b), we see that the improved genetic algorithm generally increases the fitness of the networks in a much shorter time than the original genetic algorithm. The effectiveness of this improved optimization scheme can be traced back to the design of our dynamics, where capital transfer events among different nodes are determined randomly. Based on these numerical results, we expect the homogenization of network in terms of the weighted degrees of nodes should in general prolong the time elapsed before the occurrence of the first bankruptcy in the network. This is clearly supported by the probability distribution of the K_w values at different times, and for different networks, as shown in Fig.6(c).

IV. Discussion

We find that systemic stability can be increased by increasing the fitness of the network, measured by C_{Min}, which is positively correlated with the minimum K_w of the network (Fig.4.) The use of genetic algorithm provides a simple optimization procedure to obtain a stable network, with the genetic operator being the rewiring of links. As shown in Fig.6, the fitness of the fittest network increases monotonically with generation number, and the K_w distribution become narrower. In realistic application to a given network of N nodes and L links, this approach may not be possible, but the numerical results indicate that the homogenization towards the perfect WS network is the way to improve the systemic stability. Although we cannot prove this statement analytically, numerical experiments and statistical analysis of the results support this conjecture. Physically this means that a network can become more systemic stable if the incoming flow of all the nodes becomes more similar to that of the outgoing flow. For financial network, the way to reduce the risk of cascade bankruptcies is to share the systemic risk by all banks.

V. Acknowledgement

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References

[1] J. Scott, *Social network analysis*. Sage, 2012.
[2] R. Crane and D. Sornette, “Robust dynamic classes revealed by measuring the response function of a social system,” *Proc. Natl. Acad. Sci.*, vol. 105, no. 41, pp. 15649–15653, 2008.
[3] J. A. Dunne, R. J. Williams, and N. D. Martinez, “Food-web structure and network theory: the role of connectance and size,” *Proc. Natl. Acad. Sci.*, vol. 99, no. 20, pp. 12917–12922, 2002.
[4] M. Rubinov and O. Sporns, “Complex network measures of brain connectivity: uses and interpretations,” *Neuroimage*, vol. 52, no. 3, pp. 1059–1069, 2010.
[5] W. Cai, L. Chen, F. Ghanbarnejad, and P. Grassberger, “Avalanche outbreaks emerging in cooperative contagions,” *Nat. Phys.*, vol. 11, no. 11, pp. 936–940, 2015.
[6] M. Rieser and K. Nagel, “Network breakdown ‘at the edge of chaos’ in multi-agent traffic simulations,” *Eur. Phys. J. B*, vol. 63, no. 3, pp. 321–327, 2008.
[7] K. L. Calvert, M. B. Doar, and E. W. Zegura, “Modeling internet topology,” *Commun. Mag. IEEE*, vol. 35, no. 6, pp. 160–163, 1997.
[8] R. N. Mantegna, “Hierarchical structure in financial markets,” *Eur. Phys. J. B-Condens. Matter Complex Syst.*, vol. 11, no. 1, pp. 193–197, 1999.
[9] Y.-C. Gao, Y. Zeng, and S.-M. Cai, “Influence network in the Chinese stock market,” *J. Stat. Mech. Theory Exp.*, vol. 2015, no. 3, p. P03017, 2015.
[10] O. Kwon and J.-S. Yang, “Information flow between stock indices,” *EPL Europhys. Lett.*, vol. 82, no. 6, p. 68003, 2008.
[11] M. Eboli, “Financial Applications of Flow Network Theory,” in *Advanced Dynamic Modeling of Economic and Social Systems*, Springer, 2013, pp. 21–29.
[12] M. Eboli, “A flow network analysis of direct balance-sheet contagion in financial networks,” Kiel Working Paper, 2013.
[13] M. Eboli, “Simulations of Financial Contagion in Interbank Networks: Some Methodological Issues,” in *Propagation Phenomena in Real World Networks*, D. Król, D. Fay, and B. Gabryś, Eds. Springer International Publishing, 2015, pp. 311–327.
[14] F. Schweitzer, G. Fagiolo, D. Sornette, F. Vega-Redondo, A. Vespignani, and D. R. White, “Economic networks: The new challenges,” *science*, vol. 325, no. 5939, p. 422, 2009.
[15] J. P. Gleeson, T. R. Hurd, S. Melnik, and A. Hackett, “Systemic risk in banking networks without Monte Carlo simulation,” in *Advances in Network Analysis and its Applications*, Springer, 2012, pp. 27–56.
[16] A. G. Haldane and R. M. May, “Systemic risk in banking ecosystems,” *Nature*, vol. 469, no. 7330, pp. 351–355, Jan. 2011.
[17] Z. Feinstein, “It’s a Trap: Emperor Palpatine’s Poison Pill,” *ArXiv Prepr. ArXiv151109054*, 2015.
[18] L. Eisenberg and T. H. Noe, “Systemic risk in financial systems,” *Manag. Sci.*, vol. 47, no. 2, pp. 236–249, 2001.
[19] R. Albert, H. Jeong, and A.-L. Barabási, “Error and attack tolerance of complex networks,” *nature*, vol. 406, no. 6794, pp. 378–382, 2000.
[20] R. Cohen, K. Erez, D. Ben-Avraham, and S. Havlin, “Resilience of the Internet to random breakdowns,” *Phys. Rev. Lett.*, vol. 85, no. 21, p. 4626, 2000.
[21] D. Sornette, F. Deschâtres, T. Gilbert, and Y. Ageon, “Endogenous versus exogenous shocks in complex networks: An empirical test using book sale rankings,” *Phys. Rev. Lett.*, vol. 93, no. 22, p. 228701, 2004.
[22] M. Mitchell, *An introduction to genetic algorithms*. MIT press, 1998.
[23] G. S. Hornby, A. Globus, D. S. Linden, and J. D. Lohn, “Automated antenna design with evolutionary algorithms,” in *AIAA Space*, 2006, pp. 19–21.
[24] K. Y. Szeto and J. Zhang, “Adaptive genetic algorithm and quasi-parallel genetic algorithm: application to knapsack problem,” in *Large-Scale Scientific Computing*, Springer, 2006, pp. 189–196.
[25] M. Benbouziane, “Portfolio selection using genetic algorithm,” no. Journal Article, 2012.
[26] D. Wu and K. Y. Szeto, “Applications of genetic algorithm on optimal sequence for Parrondo games,” in *ECTA 2014-Proceedings of the International Conference on Evolutionary Computation Theory and Applications*, 2014, p. 30.
[27] P. François, “Evolving phenotypic networks in silico,” in *Seminars in cell & developmental biology*, 2014, vol. 35, pp. 90–97.
[28] R. Albert and A.-L. Barabási, “Statistical mechanics of complex networks,” *Rev. Mod. Phys.*, vol. 74, no. 1, p. 47, 2002.
[29] P. Erdos and A. Rényi, “On the evolution of random graphs,” *Bull Inst Intern. Stat.*, vol. 38, no. 4, pp. 343–347, 1961.
[30] D. J. Watts and S. H. Strogatz, “Collective dynamics of ‘small-world’networks,” *nature*, vol. 393, no. 6684, pp. 440–442, 1998.
[31] A.-L. Barabási and R. Albert, “Emergence of scaling in random networks,” *science*, vol. 286, no. 5439, pp. 509–512, 1999.