Fundamental Design Consideration of Sampling Circuit

Keita Kurihara, Kensuke Kobayashi †, Masafumi Uemori, Miho Arai, Haruo Kobayashi *
Electronics and Informatics Division, Gunma University, 1-5-1 Tenjin-cho Kiryu 376-8515 Japan
† Technical Consultant, Penang, Malaysia *e-mail: k_haruo@el.gunma-u.ac.jp

Abstract – This paper discusses a theoretical issue of a sampling circuit to maximize SNR while keeping its bandwidth constant, in order to realize a wideband large dynamic range sampling circuit for communication system and measuring instrument applications. We consider two time-constants τ_1, τ_2 in the sampling circuit, where τ_1 is a product of (signal source internal impedance + sampling switch on-resistance) and (hold capacitance), and τ_2 is a sampling time window (aperture time). We have derived that $\tau_2 = 1.50\tau_1$ is the condition to maximize SNR keeping the bandwidth constant. We will call it as a strobe sampling circuit when $\tau_2 = 1.50\tau_1$ is realized.

Keywords: Sampling Oscilloscope, Impulse Sampling, ADC, Track/Hold Circuit, SNR, Bandwidth, GB product

I. INTRODUCTION

Recently much attention is being paid to high frequency signals in communication systems and electronic measurement systems, where wideband sample/hold circuits with large dynamic range are required [1]-[6]. This paper investigates a sampling method to realize a wideband and large dynamic sampling circuit; we consider here the sampling condition which maximizes SNR while keeping its bandwidth constant. We will show that the optimal condition lies in the middle of track/hold and impulse sampling methods, and we will call it as a strobe sampling method. We will also generalize sampling theory to unify track/hold and impulse sampling as well as strobe sampling methods.

II. SAMPLE/HOLD CIRCUIT

2.1 Sample/Hold Circuit and Operation

Let us consider a sample/hold (S/H) circuit which consists of a switch (SW) and a hold capacitor (Fig.1). When the switch is on, a hold capacitor is charged according to the input voltage V_{in} (sample mode). Then the switch turns off and its instantaneous input voltage is held in the capacitor (hold mode).

A S/H circuit in front of an ADC in SoC usually uses an input buffer A1 and an output buffer A2 (Fig.1). However a wideband sampling oscilloscope usually does not use the input buffer because a wideband input buffer is difficult to implement. In this paper we consider a S/H circuit without the input buffer.

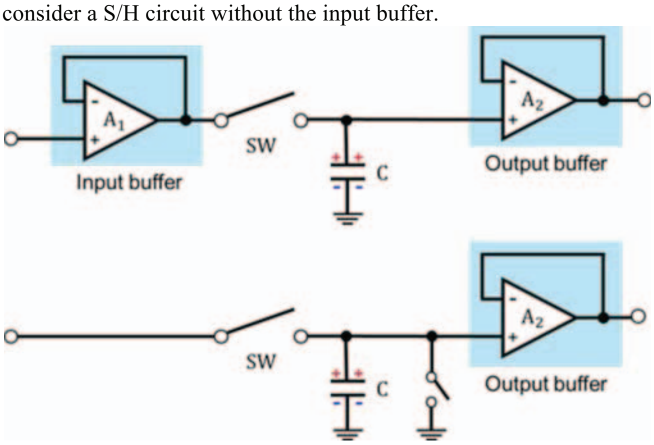

FIGURE 1: S/H CIRCUITS WITH INPUT BUFFER (TOP) AND WITHOUT IT (BOTTOM).

2.2 Noise in S/H Circuit

The thermal noise power of the S/H circuit in hold mode is given by

$$V_{n.out}^2 = \int_0^\infty \frac{4k_BTR_{off}}{1+(2\pi f)^2R_{off}^2C^2}df = \frac{k_BT}{C}.$$

When the S/H circuit uses a small C for wideband applications, the thermal noise power increases.

2.3 Two Time Constants τ_1, τ_2 in S/H Circuit

We consider two time constants in the S/H circuit. The first one is τ_1 which is defined as τ_1=RC, where $R = R_{SG} + R_{ON}$, R_{SG} is the signal source internal impedance and R_{ON} is the switch ON-resistance. The second one is the aperture time τ_2 [2,6]. Hereafter we assume that R = 50Ω.

III. PROBLEM FORMULATION OF SNR AND BANDWIDTH RELATIONSHIP IN S/H CIRCUIT

3.1 Track/Hold Circuit (in case $\tau_1 \ll \tau_2$)

We will call a S/H circuit as a track/hold (T/H) circuit when $\tau_1 \ll \tau_2$ is satisfied, and it is often used in front of an ADC in SoC. We consider its step response (Fig.2), and to obtain N-bit accuracy, the following has to be satisfied.

$$1-\left(1-e^{-\tau_2/\tau_1}\right) = e^{-\tau_2/\tau_1} < 1/2^{N+1}.$$

Thus

$$\tau_2/\tau_1 > (N+1)\cdot ln2.$$

Since the aperture time τ_2 is long enough compared to the RC time constant τ_1 in the S/H circuit, the hold capacitor is charged upto almost V_{in}. We consider its step response; then its output signal amplitude S ≒ 1 and its output noise rms value is $N = \sqrt{k_BT/C}$. Thus its SNR (= V_{signal}/V_{noise}) is given by

$$SNR_1 = \sqrt{\frac{C}{k_B\,T}} \propto \sqrt{C}.$$

Also its bandwidth ω_{BW1} is given by

$$\omega_{BW1} = \frac{1}{\tau_1} \propto \frac{1}{C}.$$

We see that the relationship between SNR and bandwidth ω_{BW1} is a trade-off. Also its transfer function is given by

$$H_1(j\omega) = \frac{1}{1+j\tau_1\omega}. \quad (1)$$

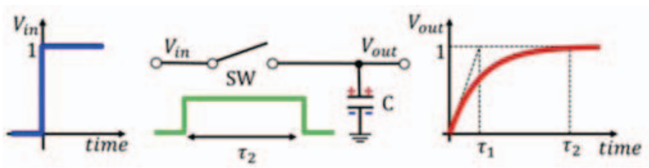

FIGURE 2: INPUT AND OUTPUT WAVEFORMS OF T/H CIRCUIT.

3.2 Impulse Sampling Circuit (in case $\tau_1 \gg \tau_2$)

We will call a S/H circuit as an impulse sampling circuit when $\tau_1 \gg \tau_2$ is satisfied, and it is often used in a wideband sampling oscilloscope. Its aperture time τ_2 is designed as a short time to realize the wideband signal capture and for its hold capacitor not to affect the signal source. Since the aperture time τ_2 is very short, the

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hold capacitor cannot be charged upto V_{in} (Fig.3). We consider its step response; then its output signal amplitude $S \propto 1/C$ and its output noise rms value is $N = \sqrt{k_B T/C}$. Thus its SNR ($= V_{signal}/V_{noise}$) is given by

$$SNR_2 \propto \frac{1/C}{\sqrt{k_B T/C}} \propto \frac{1}{\sqrt{C}}.$$

Also its transfer function is given by

$$H_2(j\omega) = \frac{\tau_2}{\tau_1} sinc\left(\frac{\tau_2 \omega}{2}\right) \cdot e^{-j\frac{\tau_2 \omega}{2}}. \quad (2)$$

Its bandwidth ω_{BW2} depends on only τ_2, and as τ_2 becomes er, ω_{BW2} increases:

$$sinc\left(\frac{\tau_2 \cdot \omega_{BW2}}{2}\right) = \frac{1}{\sqrt{2}}.$$

$$\omega_{BW2} \approx \frac{2.78}{\tau_2}.$$

On the other hand, the signal gain is proportional to τ_2. Hence its relationship between SNR and bandwidth is a trade-off. Furthermore very short pulse (τ_2) generation for wideband is very difficult.

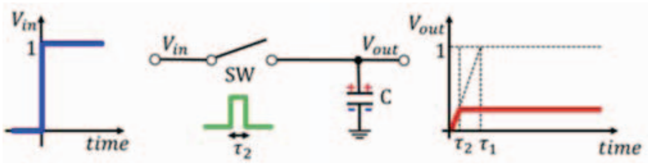


FIGURE 3: INPUT AND OUTPUT WAVEFORMS OF IMPULSE SAMPLING CIRCUIT.

3.3 Problem Formulation

"Consider to design a wideband S/H circuit with its bandwidth of ω_{BW}. Then obtain τ_1 and τ_2 to maximize its SNR while keeping its bandwidth ω_{BW} constant."

This is a nonlinear optimization problem, and it is difficult to obtain its solution analytically; we have solved it with derivation of theoretical equations and their numerical calculation as well as SPICE simulation. We will show that the solution lies between the T/H sampling and the impulse sampling, and call it as a strobe sampling. We will denote its solution as τ_{1opt} and τ_{2opt}.

IV. UNIFIED S/H CIRCUIT THEORY

In this section, we will derive S/H circuit theory which unifies T/H circuit and impulse sampling circuit theories.

In the S/H circuit with τ_1 and τ_2, the step response $s(t)$ (Fig.4) and the impulse response $h(t)$ are expressed as follows:

$$s(t) = \begin{cases} 0 & (t < 0) \\ 1 - e^{-\frac{t}{\tau_1}} & (0 \leq t < \tau_2) \\ 1 - e^{-\frac{\tau_2}{\tau_1}} & (\tau_2 \leq t) \end{cases} \quad (3)$$

$$h(t) = \begin{cases} 0 & (t < 0) \\ 1/\tau_1 \cdot e^{-\frac{t}{\tau_1}} & (0 \leq t < \tau_2) \\ 0 & (\tau_2 \leq t) \end{cases}$$

Their derivation is based on the time-equivalent sampling concept for taking care of sampling operation (Fig.5). The transfer function $H_3(s)$ is obtained by Laplace transform of $h(t)$:

$$H_3(s) = \frac{1}{1+\tau_1 s}\left\{1 - e^{-\frac{\tau_2}{\tau_1}(1+\tau_1 s)}\right\}. \quad (4)$$

Eq.(4) yields to eq.(1) in case $\tau_1 \ll \tau_2$ (T/H circuit case), and also it approaches eq.(2) in case $\tau_1 \gg \tau_2$ (impulse sampling case). Also we obtain gain characteristics by letting $s = j\omega$ (Fig.6):

$$|H_3(j\omega)| = \left|\frac{1}{1+j\omega\tau_1}\left\{1 - e^{-\frac{\tau_2}{\tau_1}(1+j\omega\tau_1)}\right\}\right|$$

$$= \sqrt{\frac{1}{1+\tau_1^2\omega^2}} \times \sqrt{\left(1 - e^{-\frac{\tau_2}{\tau_1}} cos(\omega\tau_2)\right)^2 + \left(e^{-\frac{\tau_2}{\tau_1}} sin(\omega\tau_2)\right)^2}.$$

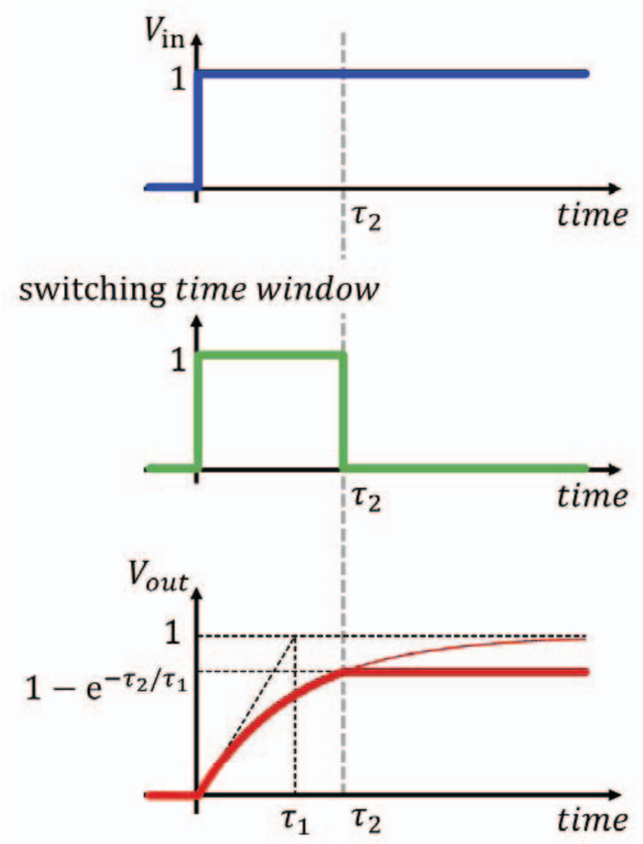


FIGURE 4: STEP RESPONSE OF S/H CIRCUIT

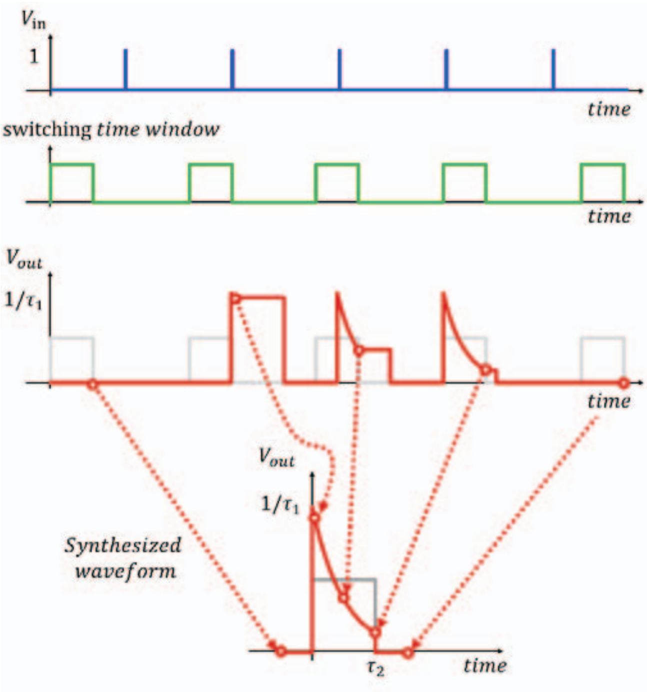


FIGURE 5: IMPULSE RESPONSE OF S/H CIRCUIT OBTAINED BY EQUIVALENT-TIME SAMPLING.

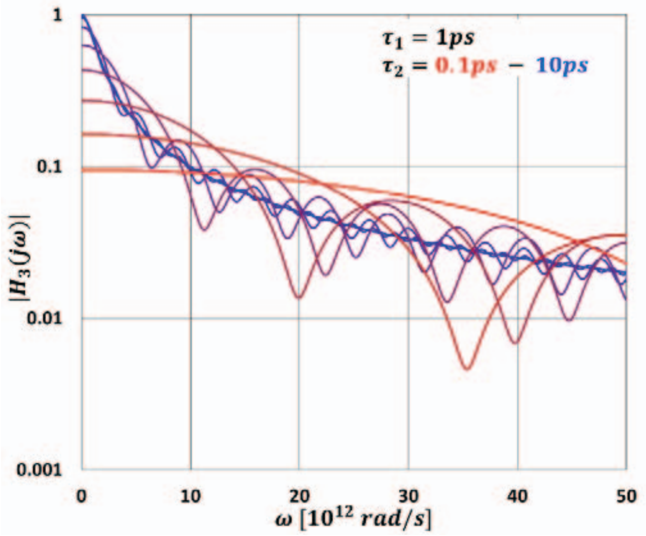


FIGURE 6: GAIN CHARACTERISTICS OF S/H CIRCUIT.

Next we will derive SNR for the S/H circuit. Its output noise is given by

$$N_{rms} = \sqrt{\frac{k_B T}{C}}.$$

Based on eq.(3), its output signal amplitude for the step input is given by

$$S = 1 - e^{-\frac{\tau_2}{\tau_1}}.$$

Thus SNR($=$ S/N$_{rms}$) of the S/H circuit is given by

$$SNR_3 = \frac{1 - e^{-\frac{\tau_2}{\tau_1}}}{\sqrt{k_B\, T/C}} = \sqrt{\frac{\tau_1}{k_B T R}}\left(1 - e^{-\frac{\tau_2}{\tau_1}}\right). \qquad (5)$$

Next we will consider the bandwidth ω_{BW3} of the S/H circuit, which is rigorously defined by

$$|H_3(j\omega_{BW3})| = \frac{1}{\sqrt{2}}|H_3(j0)|. \qquad (6)$$

This equation is difficult to solve analytically and hence we have solved it with numerical simulation and obtained ω_{BW3}.

Now we will consider GB product dependence on τ_2 in the S/H circuit (Fig.7). They show that the GP product is 2.8 times larger in the impulse sampling mode than the T/H mode, which is also confirmed from the following theoretical calculation:

$$\frac{GB\ Product_2}{GB\ Product_1} = \frac{DC\ Gain_2 \cdot \omega_{BW2}}{DC\ Gain_1 \cdot \omega_{BW1}}$$
$$\approx \frac{(\tau_2/\tau_1\,) \cdot (2.78/\tau_2\,)}{(1) \cdot (1/\tau_1\,)} = 2.78. \qquad (7)$$

In T/H circuit, its DC Gain is constant, and its bandwidth is fixed with τ_1 and does not depends on τ_2. In impulse sampling circuit, its DC Gain is proportional to τ_2, and its bandwidth is inversely proportional to τ_2. Hence, GB product in each mode is constant as in Fig.7. Therefore, we can interpret that S/H circuit operates as impulse sampling mode when $\tau_2/\tau_1 < 10$, while as T/H mode when $\tau_2/\tau_1 > 5$,

Next, we will consider SNR dependence on τ_1 in the S/H circuit as in Fig.8 ($k_B = 1.38\times10^{-23}$, T $= 300$). It shows that the unified S/H circuit has maximum SNR in case $\tau_2/\tau_1 = 1.26$. The same results can be obtained from the theoretical calculation:

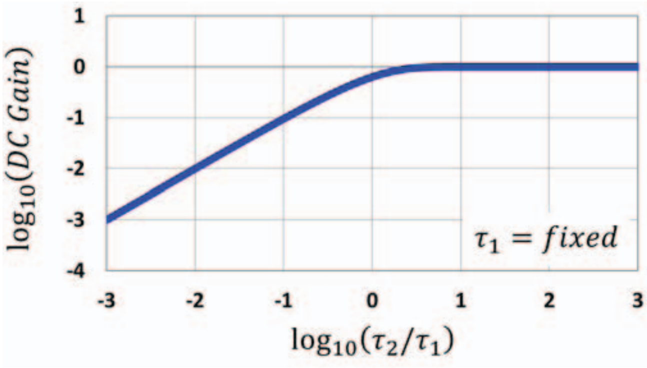
$$\frac{\partial}{\partial\tau_1} SNR_3 = 0.$$

Thus

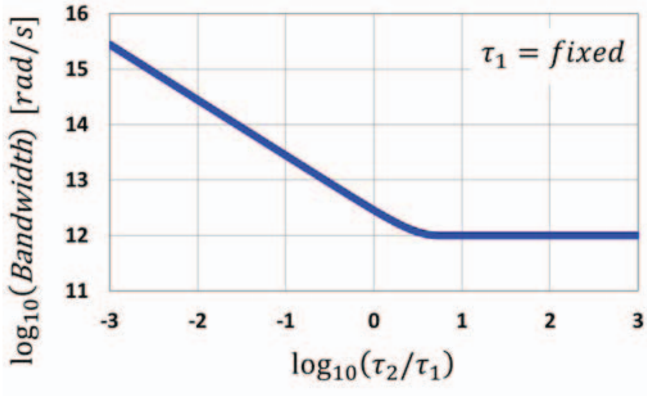
$$2\frac{\tau_2}{\tau_1} + 1 = e^{\frac{\tau_2}{\tau_1}}.$$

Therefore, we have the following:

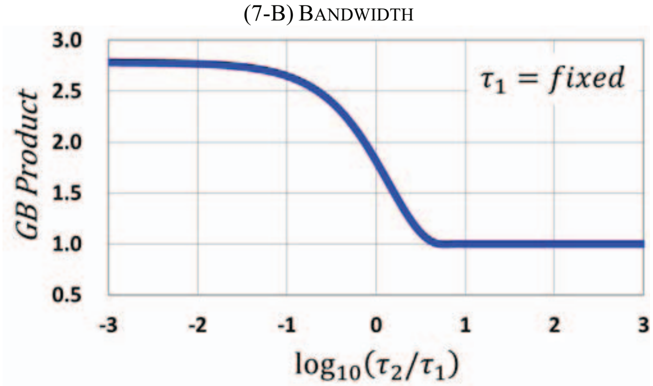
$$\frac{\tau_2}{\tau_1} \approx 0,\ \ 1.26. \qquad (8)$$



(7-A) DC GAIN



(7-B) BANDWIDTH



(7-C) GB PRODUCT

FIGURE 7: GB PRODUCT DEPENDENCE ON τ_2 ($\tau_1 = 10^{-12}$).

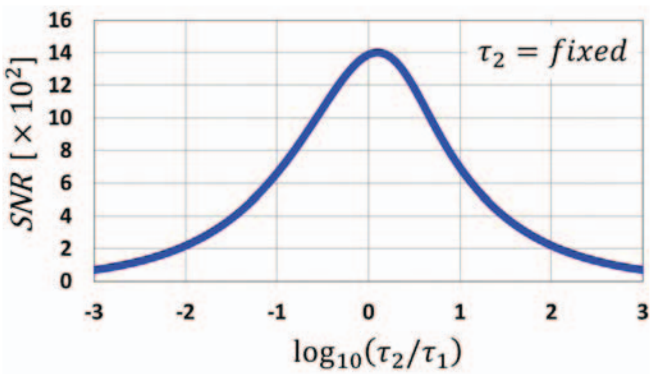


FIGURE 8: SNR DEPENDENCE ON τ_1 ($\tau_2 = 10^{-12}$).

V. Condition of Maximum SNR Under Constant Bandwidth

In this section we will derive τ_1, τ_2 in the S/H circuit to maximize SNR while keeping its bandwidth constant.

We assume that the S/H circuit is the first order system, then the bandwidth ω_{BW3} can be obtained from the rise time t_{r10-90} (10%-90%) in the step response as follows:

$$\omega_{BW3} \approx \frac{2.20}{t_{r10-90}}. \tag{9}$$

We have confirmed that eq.(6), (9) are (almost) the same, and hereafter we will use eq.(9).

In the S/H circuit, its rising time points (10%, 90%) are given by

$$\begin{cases} -t_{10\%} = \tau_1 \, ln\left\{1 - 0.1\left(1 - e^{-\frac{\tau_2}{\tau_1}}\right)\right\} \\ -t_{90\%} = \tau_1 \, ln\left\{1 - 0.9\left(1 - e^{-\frac{\tau_2}{\tau_1}}\right)\right\} \end{cases}.$$

Thus the rise time from 10% to 90% is given by

$$\begin{aligned} t_{r10-90} &= t_{90\%} - t_{10\%} \\ &= \tau_1 \, ln \frac{1 - 0.1\left(1 - e^{-\frac{\tau_2}{\tau_1}}\right)}{1 - 0.9\left(1 - e^{-\frac{\tau_2}{\tau_1}}\right)}. \end{aligned} \tag{10}$$

It follows from eq.(6) that the rise time from 10% to 90% is constant when the bandwidth ω_{BW3} is constant, and hence

$$\tau_2 = -\tau_1 \, ln\left\{1 - \frac{10\left(1 - e^{\frac{t_{r10-90}}{\tau_1}}\right)}{1 - 9e^{\frac{t_{r10-90}}{\tau_1}}}\right\}. \tag{11}$$

Thus τ_2 is a function of τ_1 under constant bandwidth ω_{BW3}, and SNR is given by

$$SNR = 10\sqrt{\frac{1}{k_B T R}}\sqrt{\tau_1}\,\frac{1 - e^{\frac{t_{r10-90}}{\tau_1}}}{1 - 9e^{\frac{t_{r10-90}}{\tau_1}}}. \tag{12}$$

Eq.(12) gives SNR as a function of τ_1 under constant bandwidth, and using numerical calculation we obtained τ_{1opt} which maximize SNR, and also obtained τ_{2opt} using τ_{1opt} and eq.(11).

Then we have that regardless of the bandwidth ω_{BW3} (which is restricted to be constant), SNR becomes maximum with the following condition (Fig.9):

$$\tau_{1opt} : \tau_{2opt} = 1.00 : 1.50. \tag{13}$$

We have verified the result of eq.(13) using SPICE simulation (Fig.10).

VI. Discussion

Fig.11 shows τ_2/τ_1 versus normalized SNR, and we see that SNR degrades in impulse sampling ($\tau_2/\tau_1 \ll 1$) which is used in a sampling oscilloscope. The T/H circuit (without the input buffer) is not suitable for wideband sampling because the hold capacitor is exposed to the signal source and the signal reflection becomes problem. Thus we propose here a strobe sampling with $\tau_2 \approx 1.50\tau_1$ for the wideband and large dynamic range S/H circuit.

VII. Conclusion

We have derived the bandwidth and SNR relationships in the S/H circuit and obtained the condition to maximize SNR while keeping the bandwidth constant, and then proposed a new sampling method called as a strobe sampling. The strobe sampling lies between the T/H sampling and the impulse sampling, and its SNR is much better than the impulse sampling and more than comparable to the T/H circuit. We have also shown that the GP product in impulse sampling mode is 2.8 times larger than the T/H mode.

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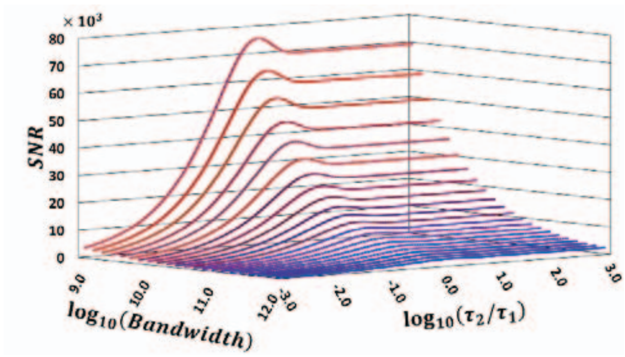

Figure 9: SNR vs. $\log_{10}(\tau_2/\tau_1)$ vs. bandwidth in S/H circuit.

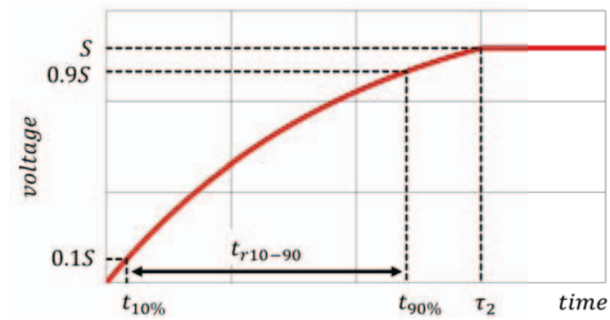

Figure 10: Step response obtained by SPICE simulation.

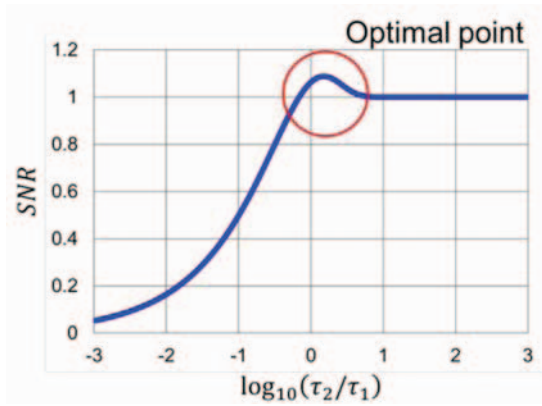

Figure 11: Normalized SNR vs. $\log_{10}(\tau_2/\tau_1)$ in S/H Circuit.