History of A-C Wave Form, Its Determination and Standardization

FREDERICK BEDELL
FELLOW AIEE

Synopsis: With the birth of the transformer and the first distribution of alternating currents, wave form assumed interest, and methods were developed for its determination, chiefly the point-by-point method of instantaneous contact, mechanical oscillograph, cathode-ray oscillograph, and the oscilloscope with stabilized time axis. The point-by-point method, by which were made the first major contributions, is now practically superseded by oscillograph and oscilloscope, each finding increasing use in its field.

With the determination of wave form accomplished, demand arose for its standardization corresponding to expanding applications. No single standard being suited to all applications, different standards have been developed in different fields, as in power, communication, and insulation. While it is desirable that standards, once set up, remain fairly stable, they should be subject to review and occasional change to keep in step with technological advances. Minor revision in communication is in progress. Although standards in other fields do not appear ideal, no immediate revision is recommended. Forty references are appended.

Alternating Current in the Late 80's

THE distribution of alternating current as we now know it began in 1885–86. Previous to that time, alternators were used to operate arc lamps in lighthouses, each machine operating one large arc lamp, but there was no distribution. Late in 1885, William Stanley made his first constant potential transformer, and early in 1886, using six transformers for operation in parallel distributed current at Great Barrington, Mass., to a score of customers at a distance of 4,000 feet. The secondary voltage was 100 volts. Current was supplied from a Siemens alternator, designed for 12 amperes 500 volts, imported for the purpose. At the time of this first installation, it was pointed out by electricians that "if a high potential primary circuit of 500 volts or more were used to distribute electricity throughout a community, there was a grave fire and life danger." A few years later a bill was introduced[1] but not passed in the Virginia legislature to limit a-c pressure to 200 volts, alternating current being considered (by those opposing its use) to be more deadly than direct.

Stanley's installation at Great Barrington was put in regular service in March 1886, and operated until summer, when an attendant dropped a screw driver into the alternator and ruined it. Meanwhile, on April 6, 1886, George Westinghouse, accompanied by W. L. Church, F. L. Pope, W. C. Kerr, and others, had seen the system in operation and determined to actively enter the a-c field. The manufacture of a new type of alternator designed by Stanley, the radial-pole type thereafter generally used, was undertaken in Pittsburgh. The story is well told by Stanley[2] himself with interesting side lights with credit to others.

The following winter the first commercial installation was made at the station of the Buffalo Electric Company. The question of the determination of a-c wave form then arose as an engineering problem. Before considering wave form, we should note the meagerness of knowledge then possessed concerning the behavior of alternating currents.

In the late 80's many misconceptions impeded progress. Series *versus* parallel operation of transformers was much discussed without adequate understanding. In 1886, at the very time that the parallel operation of transformers was successfully accomplished as we have just seen, a distinguished cantor lecturer held such a connection to be impracticable, maintaining that a separate lead to each transformer would be necessary, or each transformer should have a special regulating apparatus.

Referring to constant current operation with transformer primary windings in series, the chief of an electrotechnical testing station explained:[3] "When no secondary current is flowing, the electromotive force in the primary and secondary coils is a maximum. (Quite correct.) We have consequently this disproportion that *the smaller the output of the apparatus the greater the energy consumed.* With the secondary circuit open, and a constant exciting current, the energy used could be as much as ten times as great as under full load." Power factor was overlooked or unknown.

The proponents of the series connection of transformers operated with constant primary current reversed the argument with the same misconception. They held that inasmuch as primary electromotive force decreased with increase of secondary load, power input also decreased (primary current being constant), with the happy result that *as more power was taken out, less power was put in*—surely a condition to be desired. Here again, power factor was neglected or unknown. Yet, without meters for measuring power, power factor or phase, such misconceptions should be expected; they merely reflected the state of the art at the time.

The United States Patent Office had shown a like misconception when in 1883 it refused[4] to grant a patent on a transformer on the ground that it would be impossible to get a larger current out of the secondary winding than was supplied to the primary winding, but in 1886 a patent was allowed for this very thing.

In the Early 90's

Many of the misconceptions of the 80's trailed along into the early 90's. In the field of alternating currents there was a growing collection of isolated facts, some understood and others not, but there was no broad foundation on which to stand. This was the situation, as recalled by the author in 1890 and 1891, when he first became interested in alternating currents and in wave form. The open-magnetic-circuit transformer was still discussed. The merits of the "nonpolar" transformer with its closed magnetic circuit was not yet fully recognized. The capacity "effect" with current before the electromotive force that produced it (the effect before the cause) and the Ferranti "effect," with electromotive force received at the far end of a cable greater than the electromotive force applied, were found baffling.

Two "systems" of a-c distribution were then in use, apparently well-established: the "high-voltage" *constant potential* (Westinghouse) system operating transformers with primary circuits of 1,000 volts and 2,000 volts, with secondary circuits wound for 50 volts and 100 volts for supplying incandescent lamps in parallel;

Paper 42-152, recommended by the AIEE committee on basic sciences for presentation at the AIEE Pacific Coast convention, Vancouver, British Columbia, Canada, September 9–11, 1942. Manuscript submitted June 17, 1942; made available for printing July 21, 1942.

FREDERICK BEDELL is professor emeritus of physics, Cornell University, Ithaca, N. Y., and consulting physicist, R. C. Burt Laboratory, Pasadena, Calif.

The author is indebted to A. C. Crehore, his first co-worker, and to later co-workers to some of whom reference is made; to L. F. Blume of Pittsfield, O. E. Buckley and H. S. Osborne of New York, L. W. Chubb and R. D. Evans of Pittsburgh, who furnished material and references; and to the staff of the California Institute of Technology.

and the *constant current* system, about 10 or 15 amperes for supplying arc lamps in series. Transformers were limited to small sizes, the dictum from high sources that transformers larger than two kilowatts would not be economical being generally accepted. Higher voltages and the use of a-c motors were in the offing. The attempted use of iron wire for a-c transmission has been reported.

It was in 1890–91 that P. N. and L. L. Nunn installed the first commercial transmission of alternating current for power, to operate a 3,000-volt synchronous motor at the Gold King Mine at an altitude of 11,500 feet near Telluride, Colo., from water power 3,000 feet lower at a distance of three miles. The situation at that time was vividly described by P. N. Nunn in an address (mimeographed) before AIEE Los Angeles section, May 8, 1934, on "Early Experiences in the Power Industry," from which the following is quoted:

> "In spite of its shortcomings, alternating current seemed the most feasible. A pair of conventional alternators were installed, one as generator, the other as motor, identical to assure identical 'wave forms,' *whatever that might be.*
>
> "In 1890 alternating current was just plain *freak;* it did not follow Ohm's law and 'clogged' itself in its circuits. Bedell and Crehore had doped out its laws and demonstrated their concepts in 100 pages of solid calculus.
>
> "Wattmeters had not been developed, nor had the term 'power factor' been adopted into the vernacular."

Evidently wave form, power factor, and the clogging effect of impedance were dimly discernible. The need of a standard wave form, "whatever that might be," was thus early recognized. From this first power transmission it was a far cry to the 287,500-volt transmission with its magnificent equipment at Boulder Dam today. The increase from 3,000 volts came slowly with gradual increases in demand for more and more power from greater distances, with notable jumps to 33,000, 40,000, and 60,000 volts. With high voltages and long distances arose problems of insulation and interference, in which questions of wave form play such important part.

Going back to horse-and-buggy days, with constant-current and constant-potential systems both in use, the question arose as to whether incandescent lamps and arc lamps could be supplied from the same system. Early in 1891 a promoter, an ardent believer in the future of the constant-current system,* sought the aid of A. C. Crehore and the author in the development of a constant-current to constant-potential transformer for operating incandescent lamps from such a system. Easier said than done, but with the enthusiasm of youth, the problem was tackled. In this connection the development of the principles governing the flow of alternating currents was undertaken, while at the same time experiments were started on a high-voltage constant-current arc-light circuit, the only supply available.

From the experimental work much was learned, although it was never completed. As at Great Barrington it was learned that a short circuit by a screw-driver could wreck a constant-potential generator; so the lesson was now learned that an open circuit caused by the slipping of a connector from a mercury cup could wreck a constant-current system and cause a general blackout! A mercury-cup contact is ill-suited for high-voltage experiment. Permission for experiment was withdrawn. The work, thus summarily ended, was never renewed; meanwhile a better solution had been found by Elihu Thomson—the "tub" transformer with floating secondary winding delivering constant current from a constant-potential primary winding. Constant-potential primary distribution had become firmly established.

The theoretical work, on the other hand, the development of the principles governing the flow of alternating current, gave better results, opening a fruitful and ever widening field with direct bearing on a-c wave form. This work, prepared initially without thought of publication, formed the basis for the first paper by Bedell and Crehore[5] presented before the Institute at its annual convention (then general meeting) in Chicago just 50 years ago. In this the principles governing current flow in transient as well as in steady conditions were first fully developed. The sequel published in book[6] form later in the same year defined the limitation to telephony (page 201) due to change in wave form of a complex wave along a line[6a] with distributed capacity and the modification produced therein by self-induction, successfully accomplished later by M. I. Pupin with the use of loading coils. In it were included the development of vector methods for solving a-c problems and the extensive use of circle diagrams, now so common.

The first use was here[5,6] made of $j=\sqrt{-1}$ in a-c analysis. Its use in astronomy and other fields had long been known, the symbol $\sqrt{-1}$ as a sign of perpendicularity appearing in a memoir to the Royal Society of Arts and Letters of Denmark by Caspar Wessel[7] in 1797. It is to A. E. Kennelly,[8] however, that credit[9] should be given for bringing out its full significance in the application of complex quantities to a-c problems, and to C. P. Steinmetz for so ably extending its usefulness.

Vector methods and circle diagrams for solving a-c problems were not infrequently criticized, when they were first developed, as being dependent upon the so-called "sine assumption," whereas in fact electromotive forces produced by a-c generators *are not true sine waves.* To this criticism with its part truth there was no categorical answer. As a circle is defined by three points, a circle diagram permits the determination or predetermination of the performance of a-c machines and systems with a minimum amount of observation and calculation. Experimental diagrams[10] were early found to check** closely with theory. The wide use of the circle diagram today is evidence of its value.

The assumption of equivalent sine waves has likewise proved adequate[11] for many cases. For other cases more elaborate methods,[12] sometimes involving vectors in more than two dimensions, have been developed. These methods, also, have been criticized on the ground that electromotive forces produced by most a-c generators *are practically sine waves;* so that these more elaborate methods are unnecessary. Again a part truth! Both assumptions, sine wave and nonsine wave are open to criticism, but each has its field of usefulness. What allowable limits should be set to the departure from a sine wave remains to be determined.

Wave-Form Determination

H. J. Ryan[13] in 1889 made an extensive study of wave forms of a closed-magnetic-circuit transformer under different operating conditions, using a synchronous contact maker to obtain instantaneous readings point by point. In this way complete wave forms of currents and voltages and their phase relations were obtained, showing definitely the behavior of such a transformer. By this method studies were made of generator wave form by Tobey and Walbridge[14] and the wave

---

* Well into the present century engineers of eminence believed that constant current would be the ultimate in power transmission.

** It is interesting to note that such a close check, even though not complete, could be made without wattmeter, phase, or power-factor meter and with limited instruments for current and voltage measurements. Primary and secondary voltages were determined by reading, with telescope and scale, the elongation of two fine German-silver wires; primary and secondary currents, similarly, with two coarse wires. Very competent were the three observers named in the reference, with later careers of note.

forms of an open-magnetic-circuit transformer by Bedell, Miller, and Wagner,[15] using a liquid-jet contact maker, with some modifications in methods of measurements and the use of capacitors to improve power factor. The open-magnetic-circuit transformer with its large magnetizing current and low power factor could not survive.

The use of a synchronous commutator[16] in place of a contact maker eliminated error caused by duration of contact and made possible the direct determination of flux. The mechanical plotting of points on a synchronous drum was introduced by E. B. Rosa[17] to eliminate tedious plotting by hand. All methods employing synchronously driven mechanism, however, had their day. Besides being cumbersome and inconvenient, they were limited to commercial frequencies and at best gave only points rather than continuous curves.

Meanwhile, a parallel development, starting with the optical study of the excursions of a telephone diaphragm, had led to the oscillograph,[17a] perfected by Blondel, Duddell, and others, employing a suspended element light enough to follow closely the rapid changes in a quantity under observation. There is no need for expanding on the wide and continued usefulness of the oscillograph in the determination of wave form. Point-by-point methods were thus outmoded. The moving element of an oscillograph, however light it may be, has *some* inertia which, though practically negligible for many purposes, limits its ability to follow very rapid changes in the quantity under observation. A vibrator with no weight at all would be most desirable.

It had long been known that a cathode-ray beam would be deflected by a magnetic or electric field. Ryan[18] grasped at this fact and, with a special cathode-ray tube made for him by Mueller Uri, constructed and used the first cathode-ray oscillograph, an oscillograph in which the moving "part" had no weight and could accurately follow the changes in whatever quantity was under observation. A new field was thus opened.

In the cathode-ray oscillograph the spot of light caused by the cathode ray impinging on a fluorescent screen became a curve, by persistence of vision, when the cathode-ray beam was deflected simultaneously by two fields at right angles, one field being set up by the variable under observation, and the other by a known variable of reference. Various Lissajous figures were thus produced. In studying a-c wave form, Ryan used a known sine wave for reference, the resultant curve being a smooth ellipse in case the wave under observation was also a true sine wave. Departure from an ellipse indicated departure of the unknown a-c wave from a sine wave. The observed curve was then laboriously replotted with time as an axis, as there was then no means for obtaining this directly.

The apparatus used by Ryan was cumbersome. The tube required 5,000–10,000 volts accelerating potential, obtained from a motor-driven Wimshurst machine. Furthermore, it required the maintenance of low vacuum, obtained from elaborate vacuum apparatus with frequent attention. These inconveniences were overcome in the low-voltage hot-cathode tube of Johnson,[19] with an accelerating potential of only 300–500 volts and greater sensitivity. The tube contained a small amount of gas and required no re-evacuation during its life. The general use of the cathode-ray tube for determining wave form thus became possible.

To obtain a linear time axis, as now commonly used, required a saw-toothed wave for the wave of reference, instead of the sine wave used by Ryan. Various means for developing such a wave were advanced, some mechanical, as from a synchronously driven rheostat, and some electrical from various types of circuit. Mechanical means were cumbersome and limited in frequency range and never came into general use. Electrical means for obtaining a saw-toothed wave, properly synchronized and stabilized, led to the oscilloscope[20,21] with linear time axis so widely used today.[22,23] To maintain curves stationary, a stable linear sweep circuit is essential. With the oscilloscope practically no energy is drawn from the circuit under test.

With wave form determined, many methods and machines have been developed for its analysis, when plotted either in rectangular or polar[24] co-ordinates, into its harmonic components. Early attempts were made to determine the separate harmonic components in an a-c wave by direct electrical measurement. Some success was obtained by resonance[25] and by other means, as by passing currents of various harmonic frequency through one coil of a split dynamometer, the current to be analyzed through the other. The results, however, were meager, as the harmonic components without amplification, were too small to give significant measurement. Amplification, however, has made possible the development of many successful analyzers that give the components of an a-c wave directly by electrical measurement.[26–29]

Wave-Form Standardization

With the expanding use of alternating currents, the need for the standardization of wave form arose, and the adoption of some factor or factors that would indicate quantitively the degree of departure from a sine wave. Form factor, the ratio of effective to average value, although useful in connection with transformer loss, had no general significance, widely different wave shapes having the same form factor.[30] Other factors were from time to time proposed for this purpose, including distortion factor, peak factor, harmonic factor, curve factor, and deviation, and in some cases, after discussion, were sanctioned by the standards committee. Each factor had its own[31] significance as the numerical measure of the departure of an irregular wave from a pure sine wave, varying each in its own way with variation of amplitudes, phase, and frequencies of the harmonic components of the wave. Each, therefore, had special usefulness for special purposes. Whether a single factor could be found, sufficiently satisfactory for all purposes, was a question.

It was generally agreed that a sine wave of electromotive force at generator terminals or on a transmission line is best for most purposes, and that methods for prescribing allowable departure therefrom should be determined. In 1915 the standards committee, through a subcommittee* on wave form, undertook a study of the subject to ascertain what standard or standards could be specified that would be most suitable in characteristics and practical in application, avoiding tedious analysis and cut-and-try methods as far as possible.

ADMITTANCE STANDARDS

As the troubles caused by a departure from a sine wave depend in many cases upon the frequency of the harmonic or harmonics present, the assignment of penalties to different harmonics according to their frequencies appeared to be an obvious way to make the penalty fit the crime. For doing this, an admittance type of wave-form standard[32] appeared well suited, with the possibility of assigning different allowable weights or penalties to harmonics of different frequencies according to their behavior or misbehavior. The admittance of a circuit is readily measured, being proportional to

---

* Membership of the subcommittee: F. Bedell, chairman; L. W. Chubb, F. M. Farmer, H. S. Osborne, and L. T. Robinson.

current. The admittance of a circuit with capacitance (C) increases, and in a circuit with inductance (L) decreases, with the frequencies of any harmonics in the applied electromotive force. With L and C both in the circuit, the admittance reaches a resonant peak at a particular frequency, the broadness of resonance being controllable by the resistance (R). Values of L and C can, accordingly, be selected for resonance at a particular resonant frequency, giving maximum penalty to a harmonic of that frequency, the admittance and penalty tapering off on each side, more or less rapidly, according to the value of R.

The shape of the tapering slopes on the two sides can be controlled to a certain extent by employing a composite, instead of a simple circuit, with R, L, and C in the admittance standard. Desirable penalties can thus be assigned to different harmonic frequencies according to the degree of crime. The possibility of better weighting thus obtained led to the development of a composite circuit, instead of a simple circuit as an admittance standard, despite the advantage that the latter could be readily duplicated with common laboratory equipment.

The degree of crime, however, and hence the penalty to be assigned, is different in different fields of application, as in power transmission and machinery, in communication or insulation. It soon developed that penalties could not be uniformly prescribed in all fields, and no one universal wave-form standard, however desirable on account of simplicity, would prove generally acceptable. Special standards thus appeared to be necessary to meet practical conditions in each case.

In Inductive Co-ordination

The admittance type of wave-form standard was found to be particularly suitable in the inductive co-ordination field. After extensive studies of induction problems involving power and telephone systems, a *telephone interference factor*, TIF, was proposed[33] in 1919 and a TIF meter, of the admittance type, for measuring power-system wave shape in terms of its influence on telephone circuit noise. Definite weightings were determined, based on the interfering effects of different frequencies, depending in part on the telephonic equipment in use and in part on the characteristics of the human ear. In 1935 changes in telephonic equipment and new studies led to new weightings,[34] and the name telephone interference factor was changed to telephone influence factor, as more appropriate. In 1941 weightings[35] were again revised, with corresponding changes in the admittance net work of the TIF meter. Wave-form standards as applied to inductive co-ordination problems are thus being well cared for.

In the Power Field

In the power field, deviation and distortion factors are both in use, but not extensively. Deviation is included in some specifications, and acceptance tests are made to see that the specification is met. Distortion factor* is more rarely encountered, being sometimes used by designers for calculating performance of machines. Good wave shape is a matter of evolution, attained by experience. With the increase in size of machines good design for wave form becomes less difficult.

By American Standards Association[36] definition, 1.217–10.95.420:

> "The *deviation factor* of a wave is the ratio of the maximum difference between corresponding ordinates of the wave and of the equivalent sine wave to the maximum ordinate of the equivalent sine wave when the waves are superposed in such a way as to make this maximum difference as small as possible."
>
> "The deviation factor of the open-circuit terminal voltage wave of synchronous machines shall not exceed ten per cent unless otherwise specified." (Rule 3.220)

By ASA definition, 1.218–10.95.430:

> "The *distortion factor* of a voltage wave is the ratio of the effective value of the residue after the elimination of the fundamental to the effective value of the original wave."

To determine deviation, as defined, requires a curve of wave form and the use of cut-and-try methods. Deviation is not directly measurable. In contrast with the admittance type of standard, it takes no account of the frequencies of the harmonic components, a characteristic which may be an advantage or a disadvantage according to the use that is made of it. If there were simple means for its determination, deviation would serve well as a general standard, but no such means are available.

Distortion factor, also, takes no recognition of frequencies, although giving total harmonic content. It can, however, be determined by analytical processes, and indirectly from measurement without requiring a curve of wave form, advantages in its favor.

In Dielectric Tests

In dielectric tests, by ASA rule 2.122:

> "The wave shape of the test voltage shall be of acceptable commercial standards: that is, it shall come within the deviation specified as allowable in paragraph 3.220. The test shall be made with a voltage having a crest equal to $\sqrt{2}$ times the test voltage specified."

More recent AIEE Standards[37] specify further in paragraph 3 of appendix:

> "With the test specimen in circuit, the crest factor (ratio of maximum to mean effective) of the test voltage shall not differ by more than five per cent from that of a sinusoidal wave *over the upper half of the voltage range*."

Crest voltages and crest factor may be determined by the use of a synchronous commutator[38] or rectifying tube, specification for a crest voltmeter being given in paragraph 4-80, AIEE Standards.[37]

In a Standards subcommittee report,[39] the use of crest deviation factor (the departure of the crest factor from 1.414 calculated in percentage) is suggested, and it is recommended that, as deviation requires a trace of wave form, it be not used for specifying wave form in dielectric power-factor measurements, and that distortion factor be adopted in its stead.

Summary

Since 1890, when alternating current was considered just plain "freak," not obeying Ohm's law and "clogging itself in its circuits," our knowledge (with or without calculus) has become greatly advanced and the importance of wave form, "whatever that may be," so well recognized as to require most careful standardization.

A fuller discussion of wave-form standardization would here be out of place. Standards must always be subject to revision with technological development, such revision being made only after a strong need has developed. This is not the occasion to suggest revision.

The author has outlined the history of wave form, without venturing a prediction as to the future. The paper merely runs a thread through a maze of material with no pretention to completeness, the story being more fully told in the references and their extensive bibliographies.

References

1. Alternating Current Versus Direct Current, L. B. Stillwell. Electrical Engineering, volume 53, May 1934, pages 708–11.

2. Alternating Current in America, William Stanley. *Journal* of the Franklin Institute, volume 173, 1912, pages 561–80.

3. History of the Transformer (book, translated from the German), F. Uppenborn. E. and F. N. Spon, 1889.

4. Dynamoelectric Machinery (book), S. P. Thompson. E. and F. N. Spon, 1892. Fourth edition, page 727.

---

* Distortion factor is frequently used in the communication field in rating high-quality program and broadcasting equipment.

5. Derivation and Discussion of the General Solution for the Current Flowing in a Circuit Containing Resistance Self-Induction and Capacity With Any Impressed Electromotive Force, **F. Bedell, A. C. Crehore.** AIEE Transactions, volume 9, 1892, pages 303–74.

6. Alternating Currents: an Analytical and Graphical Treatment for Students and Engineers (book), **F. Bedell, A. C. Crehore.** W. J. Johnston Company, 1892, pages 1–325.

6a. Electrical Papers (book), **O. Heaviside.** Electrician Printing and Publishing Company, 1892.

7. Revolving Vectors (book), **G. W. Patterson.** The Macmillan Company, 1911. Page 1.

8. Impedance, **A. E. Kennelly.** AIEE Transactions, volume 10, 1893, pages 175–216.

9. Letter to the editor, **E. W. Rice, Jr.** *General Electric Review*, volume 27, 1924, page 132.

10. Transformer Diagrams Experimentally Determined, **F. Bedell,** assisted by **A. W. Berresford, W. M. Craft, B. Gherardi, Jr.** *Proceedings* International Electrical Congress in Chicago (book), August 1893. AIEE, 1894. Pages 234–51.

11. Reactance, **C. P. Steinmetz, F. Bedell.** AIEE Transactions, volume 11, 1894, pages 640–8.

12. Nonharmonic Alternating Currents (with 51 references), **F. Bedell.** AIEE Transactions, volume 46, 1927, pages 648–58.

13. Transformers, **H. J. Ryan.** AIEE Transactions, volume 7, 1889, pages 1–19.

14. Investigation of Stanley A-C Arc Dynamo, **W. B. Tobey, G. H. Walbridge.** AIEE Transactions, volume 7, 1890, pages 367–78.

15. Hedgehog Transformer and Condensers, **F. Bedell, K. B. Miller, G. F. Wagner.** AIEE Transactions, volume 10, 1893, pages 497–527.

16. The Use of the Synchronous Commutator in A-C Measurements, **F. Bedell.** *Journal* of the Franklin Institute, volume 176, 1913, pages 385–404.

17. An Electric Curve Tracer, **E. B. Rosa.** *Physical Review*, volume 6, 1898, pages 17–42.

17a. Oscillographs and Their Tests, **A. E. Kennelly, R. N. Hunter, A. A. Prior** (with extensive bibliography). AIEE Transactions, volume 39, 1920, pages 443–87.

18. The Cathode-Ray A-C Wave Indicator, **H. J. Ryan.** AIEE Transactions, volume 22, 1904, pages 539–48.

19. A Low-Voltage Cathode-Ray, Oscillograph, **J. B. Johnson.** *Journal* of the Optical Society of America, volume 6, 1922, pages 701–12.

20. The Oscilloscope: a Stabilized Cathode-Ray Oscillograph With Linear Time Axis, **F. Bedell, H. J. Reich.** AIEE Transactions, volume 46, 1927, pages 546–9.

21. Stabilized Oscilloscope With Amplified Stabilization, **F. Bedell, J. G. Kuhn.** *Review of Scientific Instruments*, volume 1, 1930, pages 227–36.

22. The Cathode-Ray Oscillograph, **J. B. Johnson.** *Journal* of the Franklin Institute, volume 212, 1931, pages 687–715.

23. Theory and Applications of Electron Tubes (book), **H. J. Reich.** McGraw-Hill Book Company, Inc., New York, N. Y., 1939, page 592.

24. Analysis of Periodic Waves, **L. W. Chubb.** *Electric Journal*, volume 11, 1914, pages 91–6.

25. Resonance Analysis of Alternating and Polyphase Currents, **M. I. Pupin.** AIEE Transactions, volume 11, 1894, pages 523–50.

26. An Electrical Frequency Analyzer, **R. L. Wegel, C. R. Moore.** AIEE Transactions, volume 43, 1924, pages 457–65.

27. The Empirical Analysis of Complex Electric Waves, **J. W. Horton.** AIEE Transactions, volume 46, 1927, pages 535–40.

28. An Analyzer for the Voice Frequency Range, **C. R. Moore, A. S. Curtis.** *Bell System Technical Journal*, volume 6, 1927, pages 217–29.

29. Analyzer for Complex Electric Waves, **A. G. Landeen.** *Bell System Technical Journal*, volume 6, 1927, pages 230–47.

30. Form Factor and Its Significance, **F. Bedell,** assisted by **R. Bown, H. A. Pidgeon.** AIEE Transactions, volume 34, 1915, pages 1135–42.

31. Distortion Factors, **F. Bedell,** assisted by **R. Bown, C. L. Swisher.** AIEE Transactions, volume 34, 1915, pages 1143–57; discussion, pages 1171–2000.

32. Characteristics of Admittance Type of Wave-Form Standard (a review of work of subcommittee on wave-form standard of the standards committee), **F. Bedell.** AIEE Transactions, volume 35, 1916, pages 1155–70; discussion 1171–86.

33. Review of Work of the Subcommittee on Wave-Shape Standard of the Standards Committee, **H. S. Osborne.** AIEE Transactions, volume 38, part I, 1919, pages 261–88; discussion, pages 291–303.

34. Measurement of Telephone Noise and Power Shape, **J. M. Barstow, P. W. Blye, H. E. Kent.** AIEE Transactions, volume 54, 1935, December section, pages 1307–15.

35. Frequency Weighting for Message Circuit Noise, engineering report 45. Joint Committee on Development and Research, Edison Electric Institute and Bell Telephone System, July 24, 1941, pages 1–12.

36. American Standards for Electrical Rotating Machinery, electrical standards committee. American Standards Association, 1936.

37. AIEE Standard No. 4, Measurement of Test Voltage in Dielectric Tests (revised January 1940, with extensive bibliography). June 1940.

38. Notes on the Synchronous Commutator, **J. B. Whitehead.** AIEE Transactions, volume 29, 1920, pages 407–38.

39. Report on Wave Form in Dielectric Power-Factor Measurements, standards subcommittee report. Electrical Engineering, volume 59, June 1940, pages 255–6.