Book Reviews

In this section, the IEEE Control Systems Society publishes reviews of books in the control field and related areas. Readers are invited to send comments on these reviews for possible publication in the Technical Notes and Correspondence section of this TRANSACTIONS. The S-CS does not necessarily endorse the opinions of the reviewers.

R. L. KASHYAP
Associate Editor—Book Reviews
School Elec. Eng.
Purdue Univ.
Lafayette, Ind. 47907

Introduction to Linear and Nonlinear Programming—D. G. Luenberger (Reading, Mass.: Addison-Wesley, 1973). *Reviewed by E. Polak.*

Elijah Polak received the B.E.E. degree from the University of Melbourne, Melbourne, Australia, in 1956, and the M.S. and Ph.D. degrees in electrical engineering from the University of California, Berkeley, in 1959 and 1961, respectively. He is Professor of Electrical Engineering and Computer Sciences, University of California, Berkeley. His general interests lie in the areas of system theory and computational aspects of optimization.

David Luenberger's latest book is very well organized and very readable. It is unquestionably one of the best introductory texts to linear and nonlinear programming, currently available. It shares with the books of Zangwill [1] and Polak [2] the very important feature of presenting optimization algorithms in the context of a unifying theory, dealing with convergence and rate of convergence. The book is at the advanced senior or beginning graduate level. The overall choice of material included in Luenberger's book is excellent, except that a number of interesting original results developed recently by Luenberger and his students are somewhat overemphasized.

The book is divided into three, essentially equal parts: Part I—Linear Programming, Part II—Unconstrained Problems, and Part III—Constrained Minimization. Part I does an excellent job of presenting the simplex algorithm in tableau form, the simplex algorithm in matrix form, the revised simplex method, and LU decomposition in the simplex method, as well as duality, sensitivity, the dual simplex method, and a primal-dual method. In addition, there is a rather interesting chapter on the reduction of linear inequalities. The difficulties caused by degeneracies are not dealt with in detail, but are left for the reader to resolve through three exercises, with substantial hints. This seems appropriate in an introductory text.

The second part of the book begins with a chapter on optimality conditions, convex and concave functions, the concept of rate of convergence, and Zangwill's global convergence theorem A [1]. The choice of Zangwill's convergence theorem A rather than one of the more general theorems by Polyak [3] or Polak [2] or Zangwill [1] is somewhat unfortunate, because many modern algorithms do not satisfy the conditions of Zangwill's theorem A. These include all algorithms using the Armijo step size rule [2], Zoutendijk's methods of feasible directions, and Rosen's gradient projection method. As a result, for example, Luenberger finds himself confined to algorithms using the exact minimization along the line step size rule, which has been pretty much abandoned in most algorithms in favor of the Armijo step size rule. Ironically, Polyak's and Polak's convergence theorems are no more complicated than Zangwill's theorem A and no harder to apply. One must assume that these results were not too widely known at the time Luenberger wrote his book.

To compensate the reader for some of the deficiencies of the material on convergence, theory and on step size choice in the gradient and Newton methods, Luenberger has included a number of very nice rate of convergence results and has written the most illuminating description of conjugate gradient and variable metric methods that this reviewer has ever seen. Rigorous results are given for quadratic costs only, but this is about all one can do at an introductory level. Further Luenberger shows how extremely sensitive the variable metric method can be to the precision of the step length calculation, and why this is so, and how things can be remedied by means of a self-scaling device. The section on methods for finding the minimum of a function of a single variable is excellent, except for a minor amount of terminological confusion: the secant method is called the method of "false position", and the cubic interpolation procedure is the one that *should* be used rather than the one most people are using.

The last part of the book opens up with a more or less standard chapter on optimality conditions and sensitivity. Next comes a chapter containing a very poor section on methods of feasible directions, excellent but excessively long sections dealing with a form of the gradient projection method, and absolutely unique sections on the reduced gradient method. Rate of convergence results are included throughout. Then comes a chapter on penalty function and barrier methods which is up to date and standard in content, except for expressions of doubt as to whether it really makes sense to advance through a sequence of increased penalties or to use extrapolation. Both of these are basic principles of penalty function practice in nonlinear programming. The book ends with a chapter on cutting plane methods and primal-dual type methods. Unfortunately most of the interesting results in this area have been developed after Luenberger's book was completed.

The book also contains three appendices dealing with the prerequisite mathematical material, as well as a reasonable number of exercises.

All in all, one has to congratulate David Luenberger on having written a very lucid and original introduction to optimization algorithms.

This work was supported by the National Science Foundation Grant GK-10656X3.

REFERENCES

[1] W. I. Zangwill, *Nonlinear Programming: A Unified Approach.* Englewood Cliffs, N. J.: Prentice-Hall, 1969.
[2] E. Polak, *Computational Methods in Optimization.* Prentice-Hall, N. Y.: Academic, 1971.
[3] B. T. Polyak, "Gradient methods for the minimization of functionals," *Zh. Vychisl. Mat. Mat. Fiz.* vol. 3, no. 4, pp. 643–653, 1963.