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RESEARCH ARTICLE

Optimal Modified Performance of MIMO Networked Control Systems With Packet Dropouts and Bandwidth Constraints

LINGLI CHENG, HENG ZHAN, YU SHI, AND XISHENG ZHAN
College of Electrical Engineering and Automation, Hubei Normal University, Huangshi 435002, China

Corresponding author: Lingli Cheng (lcheng@hbnu.edu.cn)

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ABSTRACT In this paper, the optimal modified performance problem of multiple-input multiple-output (MIMO) networked control systems (NCSs) with additional interference is investigated, the additional interference includes channel noise, encoding and decoding, quantization, packet dropouts, and bandwidth constraints. Most existing researches focus on the optimal modified performance of NCSs with communication constraints in a single channel or with a single communication constraint in a dual channel. Based on this, and combined with the actual constraints, this paper studies the optimal modified performance of NCSs with multiple communication constraints in a dual channel (forward and feedback channels). By using frequency domain analysis and various decomposition methods, the explicit expression for optimal modified performance under NCSs stability is obtained. The relationship between the optimal modified performance of the NCSs with the internal and external constraints of the NCSs can be observed by the explicit expression of the optimal modified performance. Finally, the accuracy of the obtained conclusions is verified through several sets of numerical simulations.

INDEX TERMS Bandwidth constraints, channel noise, multiple-input multiple-output networked control systems, optimal modified performance, packet dropouts.

I. INTRODUCTION

With the rapid development of automation technology and unmanned technology, network control technology has also developed rapidly. Networked control systems (NCSs) are widely used in various fields because of its unique advantages [1], [2], [3], [4], [5]. Although various studies on T-S fuzzy systems [6], [7], [8], multi-agent systems [9], [10], [11], and nonlinear systems have been hot topics for researchers in recent years, the popularity of NCSs has not diminished [12], [13], [14], [15].

In [12], the network security problem of nonlinear NCSs based on resilient event-triggered mechanisms was studied. The output feedback H_∞ control problem of event-triggered Markov jump NCSs with signal quantization was discussed in [13]. At the same time, in [14], the dynamic output feedback control problem of NCSs with time-varying delays was investigated. The data-driven event-triggered control of a class of unknown discrete-time NCSs was researched in [15]. It can be seen that the research on NCSs is highly respected by researchers. At the same time, the performance issues in NCSs cannot be ignored.

An improved security-based event-triggered fuzzy control method was proposed in [16] to address the asymptotic stability problem of nonlinear NCSs under DoS attacks. In [17], the stability analysis problem of NCSs with uncertainties and transmission delays was studied. The optimal tracking performance of NCSs with various communication

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constraints was considered in [18]. It is not difficult to find that the research on the stability of NCSs with Lyapunov stability theory is relatively mature, while there is still a slight lack of research on the optimal tracking performance of NCSs [19], [20], [21]. According to [22], the research on the tracking performance of the NCS mainly focuses on the ability of the output signal to track some typical reference input signals after the NCS reaches a steady state. Therefore, the optimal tracking performance of the NCSs with channel constraints can be directly considered. In the research on the optimal tracking performance of NCSs, the optimal modified performance of NCSs deserves more attention from researchers.

In [23], the optimal modified performance issue of multi-input multi-output (MIMO) NCSs under multiple constraints was discussed. The optimal modified performance issue of MIMO NCSs with packet dropouts and bandwidth constraints was investigated in [24]. Meanwhile, in [25], the optimal modified performance of MIMO NCSs with dual channel interference was analyzed. According to [26], it can be inferred that the introduction of modified factors can make the tracking performance of the given plant limited even without the presence of an integrator to track it. Taking into account the above analysis, there are relatively few research findings on the optimal modified performance of NCSs at present, and the research process is also relatively complex. However, it plays an important role in promoting the development of research on the performance of NCSs. Therefore, it is also necessary to study the modified performance of NCSs.

As is well known, the NCSs are closed-loop feedback control systems with closed-loop circuits connected by components such as controlled objects, sensors, controllers, and actuators. Therefore, NCSs inevitably encounter issues such as time delay, packet dropouts, quantization, bandwidth constraints and networked security and other problems [14], [27], [28], [29], [30]. The number of nodes in the systems, bandwidth, network traffic, and transmission protocols can all cause time delay. Packet dropouts occurs when communication networked are subject to external interference or signal fluctuations, causing data packets to fail to be transmitted properly, resulting in loss of feedback data for the systems. Quantization affects the system by quantizing signals into finite-length bit strings before transmission, leading to quantization errors and preventing signals from being infinitely precise. Bandwidth constraints arise due to the limited bandwidth of communication networked, resulting in situations such as single or multiple transmissions during packet delivery. Networked security issues arise due to the openness of networked, making the system susceptible to various attacks and disturbances, leading to security concerns. In summary, these interference factors will affect the performance of the NCSs, so it is particularly important to study these interference factors and formulate corresponding strategies to improve the performance of the NCSs. Most of the existing research on the optimal modified performance of NCSs focuses on the study of constraints in the feedback channel, and the results are not fully applicable to the case where the same interference factors exist in the forward channel. So in this paper, the optimal modified performance of NCSs with multiple interference factors in the dual channels is discussed.

The main contributions of this paper are as follows:

1) A new model of NCSs with communication constraints is established in this paper, and the corresponding research results are obtained, which provides theoretical guidance for subsequent research on the performance of NCSs.
2) The optimal modified performance of NCSs with packet dropouts, bandwidth constraints and other interference factors is investigated.
3) By using the theory of stochastic system stability, Youla parameterization and decomposition method, combined with the set of all two degree of freedom (2)-DOF) controllers that stabilize the systems, the optimal modified performance conditions of the NCSs can be obtained.

The paper is organized as follow. The problem is formulated in Section II. The optimal modified performance of NCSs with packet dropouts, bandwidth constraints and other interference factors is derived in Section III. Some examples are given to illustrate the obtained results in Section IV. The conclusions are presented in Section V.

Notations: The following are the standard symbols involved in this paper. For any matrix U, its transpose and conjugate transpose are represented as U^T and U^H, respectively. := means defined as, $\mathbb{R}\mathcal{H}_\infty$ denotes the matrix of rational and stable functions, $\|\cdot\|$ is the Euclidean norm, $\|\|_F$ represents the Frobenius norm, inf defines the infimum and $\mathbb{E}\{\}$ denotes the mathematical expectation, $Prob\{\heartsuit\}$ stands for the probability of $\heartsuit$, I is the unit matrix. The open right-half plane is defined as $\mathbb{C}_+ := \{s : Re(s) > 0\}$, and the open left-half plane is defined as $\mathbb{C}_- := \{s : Re(s) < 0\}$. The Lebesgue spaces $\mathcal{L}_2$ can be represented as:

$$\langle \mathcal{F}, \mathcal{G}\rangle := \frac{1}{2\pi}\int_{-\infty}^{\infty} tr\left[\mathcal{F}^H(jw)\,\mathcal{G}(jw)\right]dw$$

$\mathcal{H}_2$ and $\mathcal{H}_2^\perp$ are subspaces and form an orthogonal pair of $\mathcal{L}_2$ [31]:

$$\begin{aligned}
\mathcal{H}_2 &:= \{\mathcal{G} : \mathcal{G}(s) \in \mathbb{C}_+,\\
\|\mathcal{G}\|_2^2 &:= \sup_{\sigma>0}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\|\mathcal{G}(\sigma+j\theta)\|_F^2\,d\theta < 0\},\\
\mathcal{H}_2^\perp &:= \{\mathcal{G} : \mathcal{G}(s) \in \mathbb{C}_-,\\
\|\mathcal{G}\|_2^2 &:= \sup_{\sigma<0}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\|\mathcal{G}(\sigma+j\theta)\|_F^2\,d\theta < 0\}.
\end{aligned}$$

II. PROBLEM FORMULATIONS

The framework diagram of the MIMO NCSs studied in this paper is shown in figure 1. It can be seen from figure 1 that codec and noise interference exist in the forward channel, and bandwidth constraint, packet dropouts and quantization exist

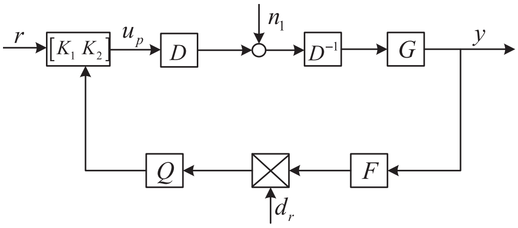

FIGURE 1. The diagram block of NCSs with packet dropouts and bandwidth constraint.

in the feedback channel. In figure 1, r, u_p, n_1, y represent random reference input signal control signal, controller output, channel noise and systems output, respectively, and their Laplace transforms can be expressed as: $\tilde{r}$, $\tilde{u}_p$, $\tilde{n}_1$, and $\tilde{y}$. $[K_1\ \ K_2]$ denotes the 2-DOF controllers, D and D^T stand for encoder and decoder, respectively. G indicates given plant, y expresses output signals, F is bandwidth, which can be simulated using a low-pass Butterworth filters. Q means uniform quantizer, which is stood by $Y_q = y + n$, quantizing noise is described as n_2, and n_1, n_2 are white Gaussian noise, and the white Gaussian noise variance is ϵ_1^2 and ξ_1^2, respectively. For any channel i, the spectral densities of r_i, n_{1i} and n_{2i} are replaced by $\varepsilon_i, \epsilon_i$ and ξ_i, respectively, in same time, it can be defined as $\Sigma = diag\{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n\}$, $\Lambda = diag\{\epsilon_1, \epsilon_2, \cdots, \epsilon_n\}$, $\Xi = diag\left\{\frac{\xi_1}{\sqrt{12}}, \frac{\xi_2}{\sqrt{12}}, \cdots, \frac{\xi_n}{\sqrt{12}}\right\}$, their variances are respectively assumed to be: ε_i^2, ϵ_i^2, $\frac{\xi_i^2}{12}$. The parameter d_r indicates whether data packet dropouts have occurred:

$$d_r = \begin{cases} 0, & \text{if the data packet dropout rates occur}, \\ 1, & \text{if the data packet dropout rates not occur}. \end{cases}$$

with a probability distribution given by:

$$Prob\{d_r = 0\} = \sigma, Prob\{d_r = 1\} = 1 - \sigma,$$

where σ is the probability of packet dropout.

The expressions for the controller input and system output can be obtained by simulating Figure 1 of the NCSs:

$$\begin{aligned} u_p &= K_1 r + K_2(Q + d_r F y), \\ y &= G D^{-1}\left(D u_p + n_1\right). \end{aligned} \tag{1}$$

By solving equations (1) simultaneously and performing simple calculations, the result can be obtained:

$$\begin{aligned} y = G(I - K_2 d_r FG)^{-1} K_1 r + G(I - K_2 d_r FG)^{-1} K_2 Q \\ + G(I - K_2 d_r FG)^{-1} D^{-1} n_1. \end{aligned} \tag{2}$$

The tracking error of the system is defined as $e = r - y$, which can be obtained from known conditions:

$$\begin{aligned} e &= r - y \\ &= \left[I - G(I - K_2 d_r FG)^{-1} K_1\right] r \\ &\quad - G(I - K_2 d_r FG)^{-1} K_2 Q - G(I - K_2 d_r FG)^{-1} D^{-1} n_1 \\ &= A_1 r - A_2 n_1 - A_3 Q, \end{aligned} \tag{3}$$

where $A_1 = I - G(I - K_2 d_r FG)^{-1} K_1$, $A_2 = G(I - K_2 d_r FG)^{-1} D^{-1}$, $A_3 = G(I - K_2 d_r FG)^{-1} K_2$.

According to [32], the modified performance index of NCSs can be defined as:

$$J_\lambda = \mathbb{E}\left\{\left[e^{-\lambda t} e^T(t)\right]\left[e^{-\lambda t} e(t)\right]\right\}, \tag{4}$$

where $\lambda > 0$ means modified factor.

A given plant with time-delay can be described as [32]:

$$G(s) = G_1(s) e^{-\tau s},$$

where $G(s)$ is the transfer function matrix of a given plant, $G_1(s)$ represents a rational transfer function, τ stands for time-delay.

The conclusion obtained from [23] states that the rational transfer function $(1 - \sigma) FG$ can be decomposed into:

$$(1 - \sigma) FG = e^{-\tau s} N M^{-1} = \tilde{M}^{-1} \tilde{N}, \tag{5}$$

where N, M, $\tilde{M}$, $\tilde{N} \in \mathbb{RH}_\infty$, and satisfies the Bezout identity [23]:

$$\begin{bmatrix} \tilde{X} & -\tilde{Y} \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & Y \\ e^{-\tau s} N & X \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}, \tag{6}$$

where $\tilde{X}$, $\tilde{Y}$, X, $Y \in \mathbb{RH}_\infty$, the stabilizing two-parameter compensators $\mathcal{K}$ can be characterized by Youla parameterization [33]:

$$\begin{aligned} \mathcal{K} := \{\, K : K &= [K_1\ K_2] \\ &= \left(\tilde{X} - e^{-\tau s} R \tilde{N}\right)^{-1} \left[\Theta \quad \tilde{Y} - R\tilde{M}\right], \Theta, R \in \mathbb{RH}_\infty \,\}. \end{aligned} \tag{7}$$

It can be obtained from the decomposition method in [31]:

$$N = (1 - \sigma) F L_z N_n, \tilde{M} = \tilde{B}_p \tilde{M}_m, \tag{8}$$

In equations (8), L_z and $\tilde{B}_p$ stand for all pass factors, N_n and $\tilde{M}_m$ mean minimum phase parts. L_z contains all non-minimum phase zeros $z_i (z_i \in \mathbb{C}_+, i = 1, \cdots, n)$ in the given plant, B_p contains all poles $p_j (p_j \in \mathbb{C}_+, j = 1, \cdots, m)$ of the given plant. The coprime factorization of L_z and $\tilde{B}_p$ is as follows:

$$L_z(s) = \prod_{i=1}^{N_z} L_i(s), \quad \tilde{B}_p(s) = \prod_{j=1}^{N_p} \tilde{B}_j(s), \tag{9}$$

where $L_i(s) = I - \frac{2Re(z_i)}{s + \bar{z}_i} \gamma_i \gamma_i^H$, $\tilde{B}_j(s) = I - \frac{2Re(p_j)}{s + \bar{p}_j} \varsigma_j \varsigma_j^H$, γ_i is the unitary vector which represents the direction of the non-minimum phase zeros, ς_j is the unitary vector which represents the direction of the unstable poles. For any transfer function matrix $\mathbb{B}(s)$, the scaled transfer function matrix is defined as: $\mathbb{B}_\lambda(s) \triangleq \mathbb{B}(s + \lambda)$.

According to (6), combining the relevant conclusions of matrix multiplication, one has:

$$\tilde{X} M - e^{-\tau s} \tilde{Y} N = I, \ -\tilde{N} M + e^{-\tau s} \tilde{M} N = 0. \tag{10}$$

By combining equations (5), (7), and (10), it can be calculated that:

$$\begin{aligned} A_1 &= I - G(I - K_2 d_r FG)^{-1} K_1 \\ &= I - (1-\sigma)^{-1} e^{-\tau s} F^{-1} N \Theta. \end{aligned} \tag{11}$$

At the same time, A_2 and A_3 can be calculated using the same calculation method as A_1:

$$\begin{aligned} A_2 &= (1-\sigma)^{-1} e^{-\tau s} F^{-1} N \left(\tilde{X} - e^{-\tau s} R \tilde{N}\right) D^{-1}, \\ A_3 &= (1-\sigma)^{-1} e^{-\tau s} F^{-1} N \left(\tilde{Y} - R\tilde{M}\right). \end{aligned} \tag{12}$$

It can be obtained from (3), (4), (11), and (12):

$$\begin{aligned} J_\lambda &= \mathbb{E}\left\{\left[e^{-\lambda t} e^T(t)\right]\left[e^{-\lambda t} e(t)\right]\right\} \\ &= \|e\|_2^2 = \|r - y\|_2^2 \\ &= \|A_1 r - A_2 n_1 - A_3 Q\|_2^2 \\ &= \left\|A_1 \frac{\Sigma}{s+\lambda}\right\|_2^2 + \left\|A_2 \frac{\Xi}{s+\lambda}\right\|_2^2 + \left\|A_3 \frac{\Lambda}{s+\lambda}\right\|_2^2. \end{aligned} \tag{13}$$

III. MAIN RESULT

The optimal modified performance J_λ^* can be expressed by all possible stabilizing controllers (denoted by $\mathcal{K}$). According to [23], the relationship between optimal modified performance J_λ^* and modified performance J_λ can be defined as:

$$J_\lambda^* = \inf_{K \in \mathcal{K}} J_\lambda, \tag{14}$$

where $\inf(\cdot)$ stands for the lower bound, which is a fundamental concept in mathematical analysis and is defined on the basis of the lower bound. If there is a maximum lower bound among all those lower bounds, it is called the infimum of$\cdot$.

Combining equations (13) and (14), J_λ^* can be obtained:

$$J_\lambda^* = \inf_{K\in\mathcal{K}} \left\|A_1 \frac{\Sigma}{s+\lambda}\right\|_2^2 + \left\|A_2 \frac{\Xi}{s+\lambda}\right\|_2^2 + \left\|A_3 \frac{\Lambda}{s+\lambda}\right\|_2^2. \tag{15}$$

At the same time, J_λ^* can be rephrased as: $J_\lambda^* = J_{1\lambda}^* + J_{2\lambda}^* + J_{3\lambda}^*$, where $J_{1\lambda}^* = \inf_{\Theta\in\mathbb{R}\mathcal{H}_\infty} \left\|A_1 \frac{\Sigma}{s+\lambda}\right\|_2^2$, $J_{2\lambda}^* = \inf_{R\in\mathbb{R}\mathcal{H}_\infty} \| A_2 \frac{\Xi}{s+\lambda} \|_2^2$, $J_{3\lambda}^* = \inf_{R\in\mathbb{R}\mathcal{H}_\infty} \left\|A_3 \frac{\Lambda}{s+\lambda}\right\|_2^2$.

Theorem 1: The simulation frame diagram of the given NCSs is shown in Figure 1, it is assumed that the unstable poles and non-minimum phase zeros satisfy the following conditions: $p_j \in \mathbb{C}_+$, $j = 1, \cdots, m$, and $z_i \in \mathbb{C}_+$, $i = 1, \cdots, n$. r is independent of n_1, then the optimal modified performance of the NCSs for which the transfer function of the given plant satisfies (5) is:

$$\begin{aligned} J_\lambda^* = &\sum_{i,j=1}^{m} \varepsilon_i^2 \sum_{i=1}^{N_z} \prod_{k=1}^{i} |g(s_k)|^2 \frac{e^{2\tau(z_i-\lambda)} 4Re(z_i)\, Re(z_j)}{\bar{z}_i z_j (\bar{z}_i + z_j - 2\lambda)} \\ &+ \sum_{i=1}^{m} \frac{\xi_i^2}{12} \sum_{i,j=1}^{N_p} \frac{e^{2\tau(p_i-\lambda)} 4Re(p_i)\, Re(p_j)}{\bar{p}_i p_j (\bar{p}_i + p_j - 2\lambda)} tr\left(\phi_j \phi_i^H\right) \\ &+ \sum_{i=1}^{m} \epsilon_i^2 \sum_{i,j=1}^{N_z} \frac{e^{2\tau(z_i-\lambda)} 4Re(z_i)\, Re(z_j)}{\bar{z}_i z_j (\bar{z}_i + z_j - 2\lambda)} tr\left(\varphi_j \varphi_i^H\right) \end{aligned} \tag{16}$$

where

$$g(s_i) = I - \frac{2Re(z_i)}{z_i} \gamma_i \gamma_i^H, \quad \Delta = \prod_{i=1}^{N_z} g(s_i),$$

$$\mathcal{C}_j = \prod_{k=1}^{j-1} \left(\tilde{B}_{mk}(p_j - \lambda)\right)^{-1}, \quad \mathcal{M}_j = \prod_{k=j}^{N_p} \left(\tilde{B}_{mk}(p_j - \lambda)\right)^{-1},$$

$$\mathcal{A}_i = \prod_{k=1}^{i} L_i^{-1}(z_i - \lambda), \quad \mathcal{B}_i = \prod_{k=i+1}^{N_z} L_i^{-1}(z_i - \lambda),$$

$$\phi_j = (1-\sigma)^{-1} L_z^{-1}(p_j - \lambda) F^{-1}(p_j - \lambda)\, \mathcal{C}_j \zeta_j \zeta_j^H \mathcal{M}_j,$$

$$\varphi_j = N_n(z_i - \lambda) M^{-1}(z_i - \lambda) D^{-1}(z_i - \lambda)\, \mathcal{A}_i \gamma_i \gamma_i^H \mathcal{B}_i,$$

λ stands for modified factor, τ means time-delay.

Proof: First of all, calculate $J_{1\lambda}^*$. Combining $J_{1\lambda}^*$ and (8), it can be concluded that:

$$J_{1\lambda}^* = \inf_{\Theta\in\mathbb{R}\mathcal{H}_\infty} \left\|\left[I - e^{-\tau s} L_z N_n \Theta\right] \frac{\Sigma}{s+\lambda}\right\|_2^2. \tag{17}$$

Due to L_z is an all pass factor, $J_{1\lambda}^*$ can be rewritten as: $J_{1\lambda}^* = \inf_{\Theta\in\mathbb{R}\mathcal{H}_\infty} \left\|\left[L_z^{-1} e^{\tau s} - N_n \Theta\right] \frac{\Sigma}{s+\lambda}\right\|_2^2$. Define $g(s_i) = I - \frac{2Re(z_i)}{z_i} \gamma_i \gamma_i^H$, $\Delta = \prod_{i=1}^{N_z} g(s_i)$, $J_{1\lambda}^*$ can be obtained:

$$J_{1\lambda}^* = \inf_{\Theta\in\mathbb{R}\mathcal{H}_\infty} \left\|\left[e^{\tau s}\left(L_z^{-1} - \Delta\right) + e^{\tau s}\Delta - N_n \Theta\right] \frac{\Sigma}{s+\lambda}\right\|_2^2. \tag{18}$$

Since $\left[e^{\tau s}\left(L_z^{-1} - \Delta\right)\right] \frac{\Sigma}{s+\lambda} \in H_2^\perp$, $\left[e^{\tau s}\Delta - N_n \Theta\right] \frac{\Sigma}{s+\lambda} \in H_2$, one has [23]:

$$\begin{aligned} J_{1\lambda}^* = &\left\|e^{\tau s}\left(L_z^{-1} - \Delta\right) \frac{\Sigma}{s+\lambda}\right\|_2^2 \\ &+ \inf_{\Theta\in\mathbb{R}\mathcal{H}_\infty} \left\|\left[e^{\tau s}\Delta - N_n \Theta\right] \frac{\Sigma}{s+\lambda}\right\|_2^2. \end{aligned} \tag{19}$$

Because of $\Theta \in \mathbb{R}\mathcal{H}_\infty$, it is possible to choose the appropriate Θ to make $\inf_{\Theta\in\mathbb{R}\mathcal{H}_\infty} \left\|\left[e^{\tau s}\Delta - N_n \Theta\right] \frac{\Sigma}{s+\lambda}\right\|_2^2 = 0$.

According to [25], it can be obtained that:

$$J_{1\lambda}^{*}=\sum_{i=1}^{m}\varepsilon_i^2\sum_{i,j=1}^{N_z}\prod_{k=1}^{i}\left|g\left(s_k\right)\right|^2\frac{e^{2\tau(z_i-\lambda)}4Re\left(z_i\right)Re\left(z_j\right)}{\bar{z}_iz_j\left(\bar{z}_i+z_j-2\lambda\right)}. \tag{20}$$

Then, calculate $J_{2\lambda}^{*}$. Using the same method as solving $J_{1\lambda}^{*}$, $J_{2\lambda}^{*}$ can be obtained:

$$J_{2\lambda}^{*}=\inf_{R\in\mathbb{R}\mathcal{H}_\infty}\left\|\left(N_n\tilde{Y}\Xi-N_nR\tilde{M}\Xi\right)\frac{1}{s+\lambda}\right\|_2^2 \tag{21}$$

Define $\tilde{M}\Xi=\tilde{M}_m\tilde{B}_m$, $\tilde{B}_m(s)=\prod_{j=1}^{N_p}\tilde{B}_{mj}(s)$, where $\tilde{B}_{mj}(s)=I-\frac{2Re(p_j)}{s+p_j}\zeta_j\zeta_j^H$, $\zeta_j=\frac{\Xi^{-1}F\varsigma_j}{\|\Xi^{-1}F\varsigma_j\|}$, then, $J_{2\lambda}^{*}$ can be rewritten as:

$$J_{2\lambda}^{*}=\inf_{R\in\mathbb{R}\mathcal{H}_\infty}\left\|\left(N_n\tilde{Y}\Xi B_m^{-1}-N_nR\tilde{M}_m\right)\frac{1}{s+\lambda}\right\|_2^2. \tag{22}$$

Using partial factorization, one has:

$$\begin{aligned}N_n\tilde{Y}\Xi B_m^{-1}&=\sum_{j=1}^{N_p}N_n\tilde{Y}\Xi\mathcal{C}_j\left(\tilde{B}_{mj}^{-1}-I\right)\mathcal{M}_j+R_1\\&=\sum_{j=1}^{N_p}N_n\tilde{Y}\Xi\mathcal{C}_j\nabla\mathcal{M}_j+R_1\\&\quad-\sum_{j=1}^{N_p}N_n\tilde{Y}\Xi\mathcal{C}_j\frac{2Re\left(p_j\right)}{p_j}\zeta_j\zeta_j^H\mathcal{M}_j,\end{aligned}$$

where $\mathcal{C}_j=\prod_{k=1}^{j-1}\left(\tilde{B}_{mk}\left(p_j-\lambda\right)\right)^{-1}$, $\mathcal{M}_j=\prod_{k=j}^{N_p}\left(\tilde{B}_{mk}\left(p_j-\lambda\right)\right)^{-1}$, $R_1\in\mathbb{R}\mathcal{H}_\infty$, $\nabla=\left(I+\frac{2Re(p_j)}{s-p_j+\lambda}\zeta_j\zeta_j^H-I+\frac{2Re(p_j)}{p_j}\zeta_j\zeta_j^H\right)$.

Based on the above calculation, $J_{2\lambda}^{*}$ can be obtained:

$$\begin{aligned}J_{2\lambda}^{*}=\inf_{R\in\mathbb{R}\mathcal{H}_\infty}&\left\|\sum_{j=1}^{N_p}N_n\tilde{Y}\Xi\mathcal{C}_j\nabla\mathcal{M}_j\frac{1}{s+\lambda}\right.\\&\left.+\left[-\sum_{j=1}^{N_p}N_n\tilde{Y}\Xi\mathcal{C}_j\frac{2Re\left(p_j\right)}{p_j}\zeta_j\zeta_j^H\mathcal{M}_j+R_1-N_nR\tilde{M}_m\right]\frac{1}{s+\lambda}\right\|_2^2.\end{aligned} \tag{23}$$

Since $\left[\sum_{j=1}^{N_p}N_n\tilde{Y}\Xi\mathcal{C}_j\nabla\mathcal{M}_j\right]\frac{1}{s+\lambda}\in H_2^{\perp}$, $\left[R_1-\sum_{j=1}^{N_p}N_n\tilde{Y}\Xi\mathcal{C}_j\frac{2Re(p_j)}{p_j}\zeta_j\zeta_j^H\mathcal{M}_j-N_nR\tilde{M}_m\right]\frac{1}{s+\lambda}\in H_2$, one has:

$$\begin{aligned}J_{2\lambda}^{*}&=\left\|\sum_{j=1}^{N_p}N_n\tilde{Y}\Xi\mathcal{C}_j\nabla\mathcal{M}_j\frac{1}{s+\lambda}\right\|_2^2+\inf_{R\in\mathbb{R}\mathcal{H}_\infty}\\&\times\left\|\left[R_1-\sum_{j=1}^{N_p}N_n\tilde{Y}\Xi\mathcal{C}_j\frac{2Re\left(p_j\right)}{p_j}\zeta_j\zeta_j^H\mathcal{M}_j-N_nR\tilde{M}_m\right]\frac{1}{s+\lambda}\right\|_2^2.\end{aligned} \tag{24}$$

Because of $R_1, R\in\mathbb{R}\mathcal{H}_\infty$, it is possible to choose the appropriate R_1, R to make

$$\inf_{R\in\mathbb{R}\mathcal{H}_\infty}\left\|\left[R_1-\sum_{j=1}^{N_p}N_n\tilde{Y}\Xi\mathcal{C}_j\frac{2Re\left(p_j\right)}{p_j}\zeta_j\zeta_j^HM_j-N_nR\tilde{M}_m\right]\frac{1}{s+\lambda}\right\|_2^2=0.$$

According to (10), it can be obtained that:

$$\begin{aligned}\tilde{Y}&=-N^{-1}\\&=\frac{-e^{-\tau s}}{(1-\sigma)}N_n^{-1}\left(p_j-\lambda\right)L_z^{-1}\left(p_j-\lambda\right)F^{-1}\left(p_j-\lambda\right).\end{aligned}$$

At the same time, $J_{2\lambda}^{*}$ can be calculated:

$$J_{2\lambda}^{*}=\sum_{i=1}^{m}\frac{\xi_i^2}{12}\sum_{i,j=1}^{N_p}\frac{e^{2\tau(p_i-\lambda)}4Re\left(p_i\right)Re\left(p_j\right)}{\bar{p}_ip_j\left(\bar{p}_i+p_j-2\lambda\right)}tr\left(\phi_j\phi_i^H\right), \tag{25}$$

where $\phi_j=(1-\sigma)^{-1}L_z^{-1}\left(p_j-\lambda\right)F^{-1}\left(p_j-\lambda\right)\mathcal{C}_j\zeta_j\zeta_j^H\mathcal{M}_j$.

At last, calculate $J_{3\lambda}^{*}$. Using the same method as solving $J_{1\lambda}^{*}$ and $J_{2\lambda}^{*}$, $J_{3\lambda}^{*}$ can be obtained:

$$J_{3\lambda}^{*}=\inf_{R\in\mathbb{R}\mathcal{H}_\infty}\left\|\left[N_n\left(\tilde{X}-e^{-\tau s}R\tilde{N}\right)D^{-1}\right]\frac{\Lambda}{s+\lambda}\right\|_2^2. \tag{26}$$

Define $\tilde{N}D^{-1}\Lambda=N_nL_z$, where $L_z(s)=\prod_{i=1}^{N_z}L_i(s)$, $L_i(s)=I-\frac{2Re(z_i)}{s+\bar{z}_i}\gamma_i\gamma_i^H$, then, $J_{3\lambda}^{*}$ can be rewritten as:

$$J_{3\lambda}^{*}=\inf_{R\in\mathbb{R}\mathcal{H}_\infty}\left\|\left[e^{\tau s}N_n\tilde{X}D^{-1}\Lambda L_z^{-1}-N_nRN_n\right]\frac{1}{s+\lambda}\right\|_2^2. \tag{27}$$

Using partial factorization, one has:

$$\begin{aligned}&e^{\tau s}N_n\tilde{X}D^{-1}\Lambda L_z^{-1}\\&\quad=\sum_{i=1}^{N_z}e^{\tau(z_i-\lambda)}N_n\left(z_i-\lambda\right)\tilde{X}\left(z_i-\lambda\right)D^{-1}\left(z_i-\lambda\right)\\&\quad\Lambda\mathcal{A}_i\left(L_i^{-1}-I\right)\mathcal{B}_i\\&\quad+R_2,\end{aligned}$$

where $R_2 \in \mathbb{RH}_\infty$, $\mathcal{A}_i = \prod_{k=1}^{i} L_i^{-1}(z_i - \lambda)$, $\mathcal{B}_i = \prod_{k=i+1}^{N_z} L_i^{-1}(z_i - \lambda)$.

Based on the above calculation, $J_{3\lambda}^*$ can be obtained:

$$J_{3\lambda}^* = \inf_{R \in \mathbb{RH}_\infty} \|[\sum_{i=1}^{N_z} e^{\tau(z_i-\lambda)} N_n \tilde{X} D^{-1} \Lambda \mathcal{A}_i \left(L_i^{-1} - I\right) \mathcal{B}_i + R_2 - N_n R N_n] \frac{1}{s+\lambda} \|_2^2. \quad (28)$$

Since $[\sum_{i=1}^{N_z} e^{\tau(z_i-\lambda)} N_n \tilde{X} D^{-1} \Lambda \mathcal{A}_i \left(L_i^{-1} - I\right) \mathcal{B}_i] \frac{1}{s+\lambda} \in H_2^\perp$, $[R_2 - N_n R N_n] \frac{1}{s+\lambda} \in H_2$, one has:

$$J_{3\lambda}^* = \left\| \left[\sum_{i=1}^{N_z} e^{\tau(z_i-\lambda)} N_n \tilde{X} D^{-1} \Lambda \mathcal{A}_i \left(L_i^{-1} - I\right) \mathcal{B}_i \right] \frac{1}{s+\lambda} \right\|_2^2 + \inf_{R \in \mathbb{RH}_\infty} \left\| [R_2 - N_n R N_n] \frac{1}{s+\lambda} \right\|_2^2. \quad (29)$$

Because of $R_2, R \in \mathbb{RH}_\infty$, it is possible to choose the appropriate R_2, R to make $\inf_{R \in \mathbb{RH}_\infty} \left\| [R_2 - N_n R N_n] \frac{1}{s+\lambda} \right\|_2^2 = 0$. According to (10), it can be obtained that:

$$\tilde{X}(z_i - \lambda) = M^{-1}(z_i - \lambda).$$

At the same time, $J_{3\lambda}^*$ can be calculated:

$$J_{3\lambda}^* = \sum_{i=1}^{m} \epsilon_i^2 \sum_{i,j=1}^{N_z} \frac{e^{2\tau(z_i-\lambda)} 4Re(z_i) Re(z_j)}{\bar{z}_i z_j (\bar{z}_i + z_j - 2\lambda)} tr\left(\varphi_j \varphi_i^H\right), \quad (30)$$

where $\varphi_j = N_n(z_i - \lambda) M^{-1}(z_i - \lambda) D^{-1}(z_i - \lambda) \mathcal{A}_i \gamma_i \gamma_i^H \mathcal{B}_i$.

By combining $J_{1\lambda}^*$, $J_{2\lambda}^*$, and $J_{3\lambda}^*$, it can obtain:

$$\begin{aligned} J_\lambda^* &= J_{1\lambda}^* + J_{2\lambda}^* + J_{3\lambda}^* \\ &= \sum_{i=1}^{m} \varepsilon_i^2 \sum_{i,j=1}^{N_z} \prod_{k=1}^{i} |g(s_k)|^2 \frac{e^{2\tau(z_i-\lambda)} 4Re(z_i) Re(z_j)}{\bar{z}_i z_j (\bar{z}_i + z_j - 2\lambda)} \\ &+ \sum_{i=1}^{m} \frac{\xi_i^2}{12} \sum_{i,j=1}^{N_p} \frac{e^{2\tau(p_i-\lambda)} 4Re(p_i) Re(p_j)}{\bar{p}_i p_j (\bar{p}_i + p_j - 2\lambda)} tr\left(\phi_j \phi_i^H\right) \\ &+ \sum_{i=1}^{m} \epsilon_i^2 \sum_{i,j=1}^{N_z} \frac{e^{2\tau(z_i-\lambda)} 4Re(z_i) Re(z_j)}{\bar{z}_i z_j (\bar{z}_i + z_j - 2\lambda)} tr\left(\varphi_j \varphi_i^H\right) \end{aligned} \quad (31)$$

where

$$g(s_i) = I - \frac{2Re(z_i)}{z_i} \gamma_i \gamma_i^H, \quad \Delta = \prod_{i=1}^{N_z} g(s_i),$$

$$\mathcal{C}_j = \prod_{k=1}^{j-1} \left(\tilde{B}_{mk}(p_j - \lambda)\right)^{-1}, \quad \mathcal{M}_j = \prod_{k=j}^{N_p} \left(\tilde{B}_{mk}(p_j - \lambda)\right)^{-1},$$

$$\mathcal{A}_i = \prod_{k=1}^{i} L_i^{-1}(z_i - \lambda), \quad \mathcal{B}_i = \prod_{k=i+1}^{N_z} L_i^{-1}(z_i - \lambda),$$

$$\phi_j = (1-\sigma)^{-1} L_z^{-1}(p_j - \lambda) F^{-1}(p_j - \lambda) \mathcal{C}_j \zeta_j \zeta_j^H \mathcal{M}_j,$$

$$\varphi_j = N_n(z_i - \lambda) M^{-1}(z_i - \lambda) D^{-1}(z_i - \lambda) \mathcal{A}_i \gamma_i \gamma_i^H \mathcal{B}_i,$$

λ stands for modified factor, τ means time-delay.

This completes the proof. ■

The modified factor λ satisfies the following conditions:

$$Re(p_j) - \lambda > 0, \quad Re(z_i) - \lambda > 0,$$

the proof is given in [32].

IV. ILLUSTRATIVE EXAMPLE

Numerical examples in this part verify the validity of the theoretical results.

Example 1: Assuming the given plant is as follows:

$$G(s) = \begin{bmatrix} \frac{s-k}{s+2} & 0 \\ 0 & \frac{1}{(s+2)(s-3)} \end{bmatrix} e^{-\tau s}.$$

The following information is known from the given plant expression: the non-minimum phase zero and its direction are $z = k$, $\gamma_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ respectively, the unstable pole and its direction are $p = 3$, $\varsigma_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$ respectively. Meanwhile, assuming $\lambda = 1$, $\tau = 0.5$, $\varepsilon_i^2 = 5$, $\xi_i^2 = 9$, $\epsilon_i^2 = 3$. The encoding, decoding, and bandwidth are $D(s) = \begin{pmatrix} \frac{s+1}{s-2} & 0 \\ 0 & \frac{s+2}{s-2} \end{pmatrix}$, $D^{-1}(s) = \begin{pmatrix} \frac{s-2}{s+1} & 0 \\ 0 & \frac{s-2}{s+2} \end{pmatrix}$, and $F(s) = \begin{pmatrix} \frac{\Phi}{s+\Phi} & 0 \\ 0 & \frac{\Phi}{s+\Phi} \end{pmatrix}$ respectively, where Φ is the rate of bandwidth. When $\Phi = 10$, $F^{-1}(s) = \begin{pmatrix} \frac{s+10}{10} & 0 \\ 0 & \frac{s+10}{10} \end{pmatrix}$.

It is not difficult to infer from the provided information: $L_z^{-1}(s) = \begin{pmatrix} \frac{p+k}{p-k} & 0 \\ 0 & 1 \end{pmatrix}$, $N_n(s) = \begin{pmatrix} \frac{1}{s+2} & 0 \\ 0 & \frac{1}{s+2} \end{pmatrix}$, $M^{-1}(s) = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{s-3} \end{pmatrix}$, $g(s_k) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, $|g(s_k)|^2 = 1$. Through simple calculations, it can be concluded that:

$$tr\left(\gamma_j \gamma_i^H\right) = (1-\sigma)^{-2} \frac{121\left[(3+k)^2 + (3-k)^2\right]}{100(3-k)^2},$$

$$tr\left(\zeta_j \zeta_i^H\right) = \frac{(k-3)^2}{k^2(k+3)^2} + \frac{1}{(k+3)^4}.$$

Therefore, according to *Theorem 1*, one has:

$$J_\lambda^* = \frac{10}{k-3} e^{k-3} + \frac{363e\left[(3+k)^2 + (3-k)^2\right]}{200(1-\sigma)^2(3-k)^2} + \frac{6e^{k-3}}{k-3}\left[\frac{(k-3)^2}{k^2(k+3)^2} + \frac{1}{(k+3)^4}\right].$$

When the probability of packet dropout σ is taken as $\frac{1}{5}$, $\frac{3}{4}$ and $\frac{1}{3}$ respectively, J_{λ}^{*} can be obtained:

$$J_{1\lambda}^{*} = \frac{10}{k-3}e^{k-3} + \frac{363e\left[(3+k)^2+(3-k)^2\right]}{128(3-k)^2} + \frac{6e^{k-3}}{k-3}\left[\frac{(k-3)^2}{k^2(k+3)^2}+\frac{1}{(k+3)^4}\right],$$

$$J_{2\lambda}^{*} = \frac{10}{k-3}e^{k-3} + \frac{726e\left[(3+k)^2+(3-k)^2\right]}{25(3-k)^2} + \frac{6e^{k-3}}{k-3}\left[\frac{(k-3)^2}{k^2(k+3)^2}+\frac{1}{(k+3)^4}\right],$$

$$J_{3\lambda}^{*} = \frac{10}{k-3}e^{k-3} + \frac{3267e\left[(3+k)^2+(3-k)^2\right]}{800(3-k)^2} + \frac{6e^{k-3}}{k-3}\left[\frac{(k-3)^2}{k^2(k+3)^2}+\frac{1}{(k+3)^4}\right].$$

The optimal modified performance of NCSs affected by different probability of packet dropout, non-minimum phase zeros constraints and unstable poles is shown in Figure 2. According to Figure 2, the optimal modified performance of NCSs varies with the change of probability of packet dropout, non-minimum phase zeros constraints and unstable poles. The greater probability of packet dropout is, the worse the optimal modified performance of the NCSs will be. And the optimal modified performance tends to be infinity when the unstable poles moves closer to the non-minimum phase zeros.

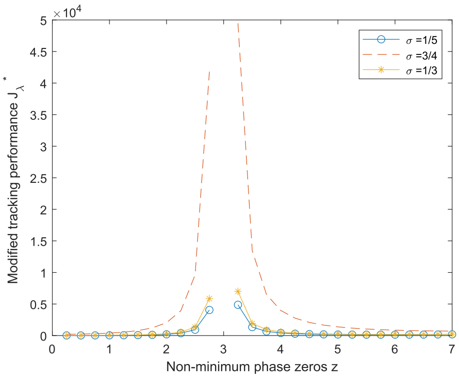

FIGURE 2. The optimal modified performance of NCSs with different probability of packet dropout.

When the probability of packet dropout σ is taken as $\frac{1}{2}$, J_{λ}^{*} can be obtained:

$$J_{4\lambda}^{*} = \frac{10}{k-3}e^{k-3} + \frac{362e\left[(3+k)^2+(3-k)^2\right]}{50(3-k)^2} + \frac{6e^{k-3}}{k-3}\left[\frac{(k-3)^2}{k^2(k+3)^2}+\frac{1}{(k+3)^4}\right].$$

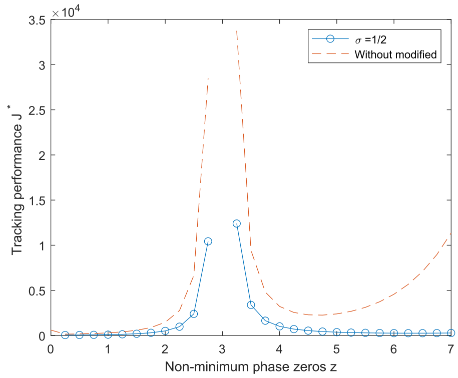

FIGURE 3. The optimal performance of NCSs with and without modified performance.

If the modified performance is not introduced, the optimal tracking performance expression of NCSs is as follows:

$$J^{*} = \sum_{i,j=1}^{m}\varepsilon_i^2\sum_{i=1}^{N_z}\frac{e^{2\tau z_i}4Re(z_i)Re(z_j)}{(\bar{z}_i+z_j)\bar{a}_i a_j} + \sum_{i=1}^{m}\frac{\xi_i^2}{12}\sum_{i,j=1}^{N_p}\frac{e^{2\tau p_i}4Re(p_i)Re(p_j)}{(\bar{p}_i+p_j)\bar{b}_i b_j}tr\left(\phi_j\phi_i^H\right) + \sum_{i=1}^{m}\epsilon_i^2\sum_{i,j=1}^{N_z}\frac{e^{2\tau z_i}4Re(z_i)Re(z_j)}{(\bar{z}_i+z_j)\bar{a}_i a_j}tr\left(\varphi_j\varphi_i^H\right).$$

Under the same conditions, the optimal tracking performance of NCSs is obtained:

$$J_1^{*} = 10e^{k} + \frac{362e^2\left[(3+k)^2+(3-k)^2\right]}{50(3-k)^2} + 6e^{k}\left[\frac{(k-3)^2}{k^2(k+3)^2}+\frac{1}{(k+3)^4}\right].$$

Figure 3 is a comparison of the performance of the NCSs with and without modified performance. It can be seen from Figure 3 that the performance of the NCSs with modified performance is better than that of the NCSs without modified performance.

Example 2: Assuming the given plant is as follows:

$$G(s) = \begin{bmatrix} \frac{s-4}{s+1} & 0 \\ 0 & \frac{s+2}{(s+1)(s-5)} \end{bmatrix} e^{-\tau s}.$$

The following information is known from the given plant expression: the non-minimum phase zero and its direction are $z = 4$, $\gamma_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ respectively, the unstable pole and its direction are $p = 5$, $\varsigma_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$ respectively. Meanwhile, assuming $\lambda = 1$, $\tau = 0.5$, $\varepsilon_i^2 = 1$, $\xi_i^2 = 6$, $\epsilon_i^2 = 2$. The encoding and decoding are $D(s) = \begin{pmatrix} \frac{s+4}{s-1} & 0 \\ 0 & \frac{s+1}{s-1} \end{pmatrix}$, and $D^{-1}(s) = \begin{pmatrix} \frac{s-1}{s+4} & 0 \\ 0 & \frac{s-1}{s+1} \end{pmatrix}$. It is not difficult to infer from the provided information:

$L_z^{-1}(s) = \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix}$, $N_n(s) = \begin{pmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1+2}{s+1} \end{pmatrix}$, $M^{-1}(s) = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{s-5} \end{pmatrix}$, $g(s_k) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, $|g(s_k)|^2 = 1$. Through simple calculations, it can be concluded that: $tr\left(\gamma_j\gamma_i^H\right) = 82(1-\sigma)^{-2}F^{-2}\left(p_j-\lambda\right)$, $tr\left(\zeta_j\zeta_i^H\right) = \frac{1289}{12544}$.

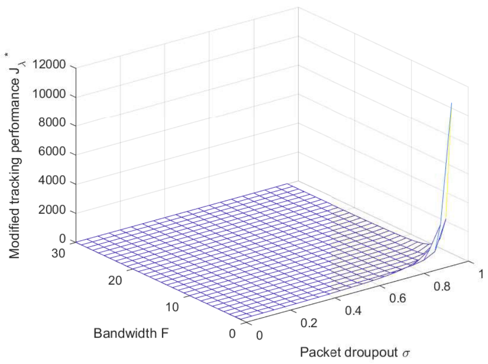

FIGURE 4. The optimal modified performance of NCSs with packet dropout and bandwidth constraints.

Therefore, according to *Theorem 1*, one has:

$$J_{5\lambda}^* = \frac{10139}{9408}e^3 + \frac{1}{2}e^4(1-\sigma)^{-2}F^{-2}\left(p_j-\lambda\right).$$

As shown in Figure 4, the optimal modified performance of the NCSs is limited by packet dropouts and bandwidth. The greater the packet dropouts rate, the worse the optimal modified performance of the system; the smaller the bandwidth, the worse the optimal modified performance of the system.

Under the same conditions, when the probability of packet dropout σ is taken as $\frac{1}{2}$, quantization noise ξ is unknown, $F^{-1}\left(p_j-\lambda\right) = \begin{pmatrix} \frac{9}{5} & 0 \\ 0 & \frac{9}{5} \end{pmatrix}$, $tr\left(\gamma_j\gamma_i^H\right) = \frac{6642}{100}$, $tr\left(\zeta_j\zeta_i^H\right) = \frac{1289}{12544}$. $J_{6\lambda}^*$ can be obtained:

$$J_{6\lambda}^* = \frac{10139}{9408}e^3 + \frac{3321}{1200}\xi_i^2e^4.$$

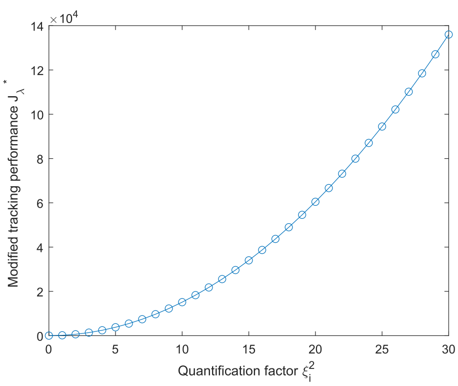

FIGURE 5. The diagram block of NCSs with quantization noise.

As shown in Figure 5, the optimal modified performance of the NCSs is limited by quantization noise. The greater the quantization noise, the worse the optimal modified performance of the system.

Example 3: An example of the inverted pendulum model is shown in Figure 6. The transfer function of the inverted pendulum model is given by [34]:

$$G(s) = \begin{bmatrix} \frac{s-z}{M(s+p)} & 0 \\ 1 & \frac{s+z}{s^2(s-p)} \end{bmatrix} e^{-\tau s}.$$

where $z = \sqrt{\frac{g}{l}}$ represents the nonminimum phase zeros, $p = \sqrt{\frac{g}{l} + \frac{mg}{Ml}}$ is the unstable poles, g stands for the acceleration of gravity. As shown in Figure 6, l, m and M stand for pendulum length, the mass of the ball and the mass of the car respectively.

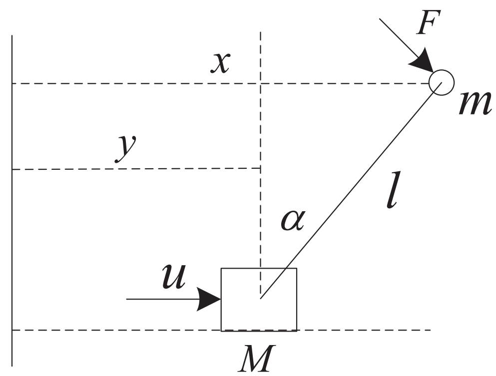

FIGURE 6. Model diagram of inverted pendulum.

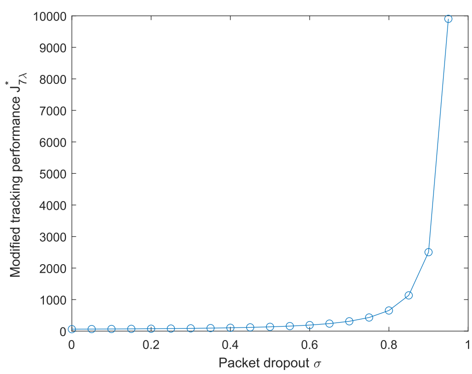

FIGURE 7. The optimal modified performance of NCSs with packet dropout.

When $M = 1, m = 0.21, l = 1.1, g = 9.8$, through a simple calculation, $z = \sqrt{8.91} \approx 3, p = \sqrt{10.89} \approx 3.3$, and their directions are $\gamma_3 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ and $\varsigma_3 = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$. The selection of other unknown parameters is the same as that of *Example 1*, then: $tr\left(\gamma_j\gamma_i^H\right) = (1-\sigma)^{-2}\frac{121}{20}$, $tr\left(\zeta_j\zeta_i^H\right) = \frac{29}{6400}$. According to *Theorem 1*, one has:

$$J_{7\lambda}^* = \frac{32087}{6400}e^2 + \frac{363}{40(1-\sigma)^2}e.$$

According to the example simulation figure 7, it can be seen that the optimal modified performance of the NCSs is limited by packet dropouts, the greater the packet dropouts rate, the worse the optimal modified performance of the system. The results obtained are consistent with the numerical simulation results, so the proposed method is feasible and effective.

V. CONCLUSION

In this paper, the modified performance of 2-DOF controllers MIMO NCSs with channel noise, quantization noise, bandwidth constraints and packet dropout is discussed. Frequency domain analysis, coprime factorization, and various decomposition methods are used to solve the problems encountered in the research process. The results show that the modified performance of NCSs is affected not only by the inherent characteristics of a given plant, but also by channel noise, quantization noise, bandwidth constraints and packet dropout. In the end, two sets of numerical simulations are used to verify the accuracy of the conclusion.

In recent years, network security has become a hot topic in contemporary research. In the future, we will take the optimal modified performance of NCSs with cyber attacks as a new research direction.

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LINGLI CHENG received the M.S. degree in operational research and cybernetics from Hubei Normal University, Huangshi, China, in 2018, and the Ph.D. degree in control science and engineering from the School of Information Science and Engineering, East China University of Science and Technology, Shanghai, China, in 2022. She is currently a Lecturer with the College of Electrical Engineering and Automation, Hubei Normal University. Her research interests include networked control systems, event-triggered mechanisms, and cyber attacks.

HENG ZHAN received the B.S. degree from the College of Information Engineering, Nanchang University. He is currently pursuing the M.S. degree with the College of Electrical Engineering and Automation, Hubei Normal University, Huangshi, China. His research interests include networked systems and signal processing.

YU SHI received the B.S. degree from the College of Electrical Information Engineering, Henan Institute of Engineering. He is currently pursuing the M.S. degree with the College of Electrical Engineering and Automation, Hubei Normal University, Huangshi, China. His research interests include cyber-physical systems and attack detection.

XISHENG ZHAN received the B.S. and M.S. degrees in control theory and control engineering from Liaoning Shihua University, Fushun, China, in 2003 and 2006, respectively, and the Ph.D. degree in control theory and applications from the Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, China, in 2012. He is currently a Professor with the College of Electrical Engineering and Automation, Hubei Normal University. His research interests include networked control systems, robust control, and iterative learning control.