

RESEARCH ARTICLE

Cooperative Control of Multi-Agent Systems Under Communication Delays and Packet Loss Scenarios

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ABSTRACT This paper addresses the challenging problem of achieving consensus in multi-agent systems (MASs) under the simultaneous influence of communication delays and packet loss. We introduce a novel switched system model that differentiates between delays caused by network-induced temporal factors and those resulting from packet loss, offering a more precise and less conservative analysis compared to traditional unified delay models. A key contribution of this paper is the design of a low-gain controller, developed using a parametric algebraic Riccati equation, which enhances system robustness without requiring recalibration when the number of agents varies. We also employ Lyapunov stability theory to establish constraints that ensure consensus is maintained despite the intermittent and transient nature of packet loss. Theoretical analysis confirms that our enhanced time-delay model effectively reduces system conservatism. The efficacy of the proposed approach is validated through numerical simulations, demonstrating its capability to achieve consensus in MASs even in the presence of dynamic delays and packet dropouts.

INDEX TERMS Multi-agent systems, cooperative control, communication time-delay, packet loss.

I. INTRODUCTION

In the dynamic field of interconnected autonomous entities, the cooperative control of multi-agent systems (MASs) has gained significant momentum, demonstrating substantial versatility across various sectors including transport networks [1], [2], power grid management [3], [4], wireless sensor clusters [5], [6], [7], and drone swarm coordination [8], [9]. However, in the implementation of cooperative control, these systems are often constrained by communication environments. Particularly in complex and dynamic environments, communication delays and packet loss become inevitable phenomena. These adverse factors directly affect the interaction of agents, thus posing threats to the cooperative performance and stability of the system. Therefore, investigating cooperative control of MASs under

communication delays and packet loss scenarios holds both profound theoretical importance and practical value.

In recent years, works both domestically and internationally have conducted a series of studies on cooperative control of multi-agent systems under communication delays and packet loss scenarios. These studies encompass optimization of control algorithms, improvement of communication protocols, enhancement of system robustness, and application of the exact system [10], [11], [12]. However, despite some achievements, numerous unresolved issues persist. For instance, how to effectively mitigate the impact of communication delays and packet loss on cooperative control while ensuring system stability; how to design more efficient and robust cooperative control strategies to adapt to complex and dynamic communication environments, etc.

Addressing the aforementioned issues, a favored approach in this context is distributed coordinated control based on sampled-data. This strategy concentrates on utilizing local data from immediate neighbors as opposed to

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relying on a centralized model that requires collective information. The benefits of this periodic control scheme include resource preservation and sustained research interest due to its effectiveness [13], [14], [15], [16]. A crucial challenge within MASs is achieving consensus, where agents are aligned to reach a universal agreement within a defined or indefinite time frame. This topic has been extensively explored, highlighting significant contributions such as the influence of interconnectivity in undirected networks on achieving consensus, synchronization of fractional reaction-diffusion complex networks and learning control, and similar findings for directed networks with a spanning tree structure [17], amidst switching topologies and delays.

Research has also delved into scenarios involving consensus amidst random network agreements [18], [19], the impacts of measurement noise [20], [21], and systems with time delays [22], [23], [24], [25], [26], including non-linear MASs [27], [28], [29], and finite-time consensus [30], [31]. The utilization of stochastic matrix theory and system theory has been crucial in enhancing the understanding and solutions for consensus, with Lyapunov's stability functions [32] being a notable method used by researchers.

While previous studies have primarily addressed general aspects of consensus in MASs with or without communication delays, issues such as insufficient bandwidth, sensor-related noise, packet loss, or communication latency can critically impair the functionality of MASs, leading them towards potentially unstable states. Pioneering works have looked into these issues; for instance, Wang et al. [33] and Zhang et al. [34] focused on the consensus challenges posed by time delays and both deterministic and random packet losses in linear MASs. Further explorations by Kikuchi et al. [35] addressed average consensus in discrete MASs with the consideration of time-varying delays and data losses assumed to follow a Bernoulli sequence. References [36], [37], [38], [39] propose novel control methods and theoretical analysis for addressing system uncertainties, specifically in the context of random packet losses and time-varying delays to effectively respond to consensus challenges. However, the consensus problems in continuous MASs under the combined constraints of communication failures and time delays remain underexplored, a gap this study aims to bridge.

This paper aims to address the dual challenge of time delays and packet loss in MASs. The main contributions are outlined thus:

1) Unlike previous studies that focus solely on either packet losses [34], [40], [41], [42] or time delays [27], [43], [44], [45], this study embodies the broader scenario of sampled-data MASs impacted simultaneously by packet losses and time delays.

2) This paper proposes a novel switched system framework to distinguish between time delays arising from computation and communication, and those resulting from packet loss. Unlike conventional modeling approaches that treat both

types of delays as a single issue [23], [46], this method leads to less conservative results.

3) This paper introduces a low-gain controller, which offers a significant advancement over the high-dimensional matrix inequality solutions proposed in previous works [47], [48]. This approach enhances system robustness and obviates the need for recalibration of the controller when the number of agents changes. By employing Lyapunov functions, the paper effectively addresses the complexities associated with constructing a Lyapunov functional for the consensus problem in the presence of both time delays and packet losses, thereby making a substantial contribution to the field.

Here is the proposed paper structure: Section II outlines the problem statement and introduces key terms and preliminary information. Section III details the methods behind the suggested sampled-data controller and corroborates its stability. Section IV illustrates the efficacy of the proposed controller via numerical simulations, and Section V encapsulates the study's pivotal revelations and its scholarly contribution.

II. PROBLEM STATEMENT

We explore the dynamics of MASs consisting of N agents. Each individual agent within the system is characterized by its state $x_i \in \mathbb{R}^n$ and control input $u_i \in \mathbb{R}^m$. The evolution of the state variables over time is given by the following equation:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, 2, \dots, N \quad (1)$$

where the system matrices $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are constant, describing the underlying dynamics of the agents.

Furthermore, in the context of MASs, the communication between agents is characterized by an undirected graph. We denote a graph as $\mathcal{G} = (V, E, A)$, where $V = \{1, 2, \dots, N\}$ represents the index set of N agents, $E \in V \times V$ is the set of edges, and $A = [a_{ij}]$ is the adjacency matrix. The presence of a positive element a_{ij} in the adjacency matrix indicates an information exchange between agents i and j , while $a_{ij} = 0$ implies no direct communication between them. Note that there is no communication flow from an agent to itself, resulting in $a_{ii} = 0$. A graph $\mathcal{G}$ is considered connected if there exists a path between any two agents i and j within V . For any agent i , its set of neighbors, denoted as N_i , consists of all agents $j \in V$ for which there exists an edge (j, i) . The degree of a node i is defined as $\deg(a_{ij})$. Additionally, the Laplacian matrix of an undirected graph V is defined as $\mathcal{L} = D - A$, where $D = \text{diag}\{\deg(1), \dots, \deg(N)\}$. Notably, in the case of undirected graphs, the Laplacian matrix L is symmetric and positive semi-definite. It is worth mentioning that the sum of each row of $\mathcal{L}$ is zero, expressed as $L\mathbf{1}_N = 0$, indicating that the vector $\mathbf{1}_N$ corresponds to the eigenvalue 0. When the undirected graph $\mathcal{G}$ is connected, all its eigenvalues, except for one eigenvalue of 0, are positive. These eigenvalues can be arranged in ascending order as $0 = \lambda_1(\mathcal{L}) < \lambda_2(\mathcal{L}) \leq \lambda_3(\mathcal{L}) \leq \dots \leq \lambda_N(\mathcal{L})$.

The control architecture of the MASs is depicted in Fig. 1. This figure provides a clear illustration of the connections between the agents, sensors, actuators, and the controller.

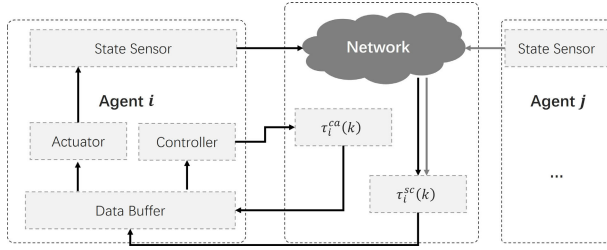


FIGURE 1. The control architecture of the MASs.

The data acquisition process is facilitated by samplers that collect data and transmit it through the communication network to the controller and actuators. In this setup, sensors operate in a time-driven manner with sampling intervals determined by predefined time periods, specifically employing periodic sampling. In contrast, the controllers, buffers, and actuators function in an event-driven manner, responding promptly to incoming signals. Upon receipt of these signals, the controllers, buffers, and actuators initiate the necessary actions or adjustments, thereby ensuring the effective operation of the system.

This assumption holds for the majority of control systems, as evidenced by previous studies [46], [49]. It is assumed that all agents in the system have synchronized sampling periods, denoted as $h > 0$. This synchronization ensures that each agent samples data at the same instants. However, in a networked environment, the sampled data from agent i and its neighboring agent j may arrive at the controller of agent i at different times, despite having been sampled simultaneously. To tackle this problem, a buffer is utilized on the controller's side to temporarily store data received from agent i 's neighboring agents. This buffer refrains from forwarding the sampled data to the controller until it has collected data from all of the neighbors of agent i . Time stamps on the data samples play a crucial role in distinguishing between samples taken at different sampling instances.

We define $\tau_{ij}^{sc}(k)$ as the communication delay experienced by agent i when receiving data from its neighboring agent j at the k -th sampling instant. Additionally, considering that the controllers of agents usually have data buffers, we define $\tau_i^{sc}(k)$ as the delay for controller i to receive the k -th sampling data. To maintain synchronous operations among all agents, a buffer is used at the actuator end to store the received state data from neighbors until all agents have completed the k -th round of information exchange between controllers and actuators. The communication delay between controller i and actuator i is represented by $\tau_i^{ca}(k)$. Therefore, the total delay experienced by the k -th sampled data across all agents in the multi-agent system can be expressed as $\tau_k = \max\{\tau_i^{sc}(k) + \tau_i^{ca}(k) \mid i = 1, 2, \dots, N\}$. This approach ensures that all agents operate on the same timescale, thus facilitating coordinated actions within the MAS.

Therefore, in a multi-agent system, the total delay experienced by the k -th sampling data can be expressed as $\tau_k = \max\{\tau_i^{sc}(k) + \tau_i^{ca}(k) \mid i = 1, 2, \dots, N\}$.

This approach ensures that all agents operate on the same time scale, thereby facilitating coordinated actions within the multi-agent system.

We formulate the control protocol as follows:

$$u_i(t) = K \sum_{j \in \mathcal{N}_i} a_{ij} (x_j(kh) - x_i(kh)) \quad (2)$$

for $t \in [\tau_k, \tau_{k+1})$, where K denotes the controller gain, which will be specified later. h is the time interval of state sampling of agents.

To maintain generality, we assume the existence of a positive constant d_3 such that $h + \tau_{k+1} \leq d_3$. This assumption ensures that there is ample time for successful packet transmission to the actuators, thus preventing any scenario where data might not be successfully transmitted.

Remark 1: In this paper, we propose that if the condition $kh + \tau_k > (k+1)h + \tau_{k+1}$ is met, the k -th packet is considered lost. As a result, only the most recent signal is regarded as valid for transmission to the actuator. This criterion ensures that $kh + \tau_k < (k+1)h + \tau_{k+1}$, thereby addressing packet disorder effectively. Thus, the controller design encapsulated by Eq. (2) accounts for both time delays and packet losses.

Key assumptions regarding the system and communication topology are outlined as follows.

Assumption 1: The controllability of (A, B) holds, where A and B are system matrices.

Assumption 2: The undirected graph $\mathcal{G}$ representing the communication topology is connected.

The Assumption 1 is a fundamental requirement for any controlled system, ensuring the possibility of steering the system's dynamics through appropriate control actions.

The Assumption 2 pertains to the connectivity of the communication network among agents. It is essential for the successful implementation of consensus algorithms, even in the absence of time delays and packet losses. Previous studies have emphasized the significance of these assumptions in the context of consensus problems, as discussed in relevant literature [33], [47].

Definition 1: In MAS (1), consensus is defined as the condition where the states of all agents converge to a common value over time, despite possible differences in their initial conditions. Formally, for a system of N agents with state vectors $x_i(t)$ for $(i = 1, 2, \dots, N)$, consensus is achieved if:

$$\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall i, j \in \{1, 2, \dots, N\} \quad (3)$$

Denote

$$\begin{aligned} x(t) &= [x_1^T(t), x_2^T(t), \dots, x_N^T(t)] \\ u(t) &= [u_1^T(t), u_2^T(t), \dots, u_N^T(t)] \end{aligned} \quad (4)$$

The system dynamics described by Eq. (1), when governed by the controller specified in (2), can be formulated as follows:

$$\begin{cases} \dot{x}(t) = (I_N \otimes A)x(t) + (I_N \otimes B)u(t) \\ u(t) = -(\mathcal{L} \otimes K)x(t - d(t)) \\ x(\theta) = \phi(\theta), \theta \in [-2d_3, 0] \end{cases} \quad (5)$$

where $x(\theta)$ represents a continuous vector-valued function defined over $\theta \in [-2d_3, 0]$ to avoid confusion with $x(t)$, $\phi(\theta)$ is used, $d(t) = t - kh$ and $t \in [kh + \tau_k, (k+1)h + \tau_k + 1)$. Denote $d_1 = \min \tau_k$, it holds that $0 \leq d(t) \leq d_3$.

Remark 2: The parameter $d(t)$ in the system captures the combined effect of time delays arising from both network delays and packet loss. It is important to note that different types of time delay may have distinct upper bounds. For instance, in some scenarios, the upper bound of the time-delay function $d(t)$ may be denoted as d_2 , while in other situations, it may be represented as d_3 .

Additionally, for a given set of systems, consensus can be ensured if the time delay remains within a certain range. However, if this delay exceeds a specified threshold, achieving consensus is not guaranteed. In this paper, we assume that when the time delay $d(t)$ caused by network transmission meets the condition $d_1 \leq d(t) < d_2$, consensus among the agents can be achieved within this time delay. On the other hand, when the time delay $d(t)$ caused by packet loss satisfies $d_1 \leq d(t) \leq d_3$, achieving consensus among the agents may not be guaranteed.

To ensure the stability of the system, specific constraints must be imposed on both the frequency and duration of transmission packet loss. These constraints are vital in assessing the viability of the packet loss scenario. To accurately describe the frequency and period of deterministic packet loss, we introduce the following notations:

We define T_s as the total duration when $d(t)$ lies within the interval $[d_1, d_2]$. Similarly, define T_u as the total duration when $d(t)$ falls within the range $[d_2, d_3]$, i.e., $T_u = \bigcup_{k=1}^{\infty} [\min\{kh + d_2, (k+1)h + \tau_{k+1}\}]$. We also introduce $T_s(t_0, t)$, which represents the total duration during which $d(t)$ is within $[d_1, d_2]$ over the time interval $[t_0, t)$. Similarly, $T_u(t_0, t)$ represents the total time during which $d(t)$ falls within $[d_2, d_3]$ over the period $[t_0, t)$. For convenience, we assume that $[a, a) = a$.

III. MAIN RESULTS

To ensure clarity in the subsequent discussions, we introduce the following notations.

Define σ as $\min_{2 \leq i \leq N} \lambda_i(\mathcal{L}) = \lambda_2(\mathcal{L})$, and let $\sigma_i = \lambda_i(\mathcal{L})\sigma^{-1}$. Clearly, it holds $\sigma_i \leq 1$.

We select a consensus gain matrix $K = \sigma^{-1}B^T\Phi(\vartheta)$, where $\Phi(\vartheta)$ satisfies the parametric algebraic Riccati equation (ARE):

$$A^T\Phi(\vartheta) + \Phi(\vartheta)A - \Phi(\vartheta)BB^T\Phi(\vartheta) = -\vartheta\Phi(\vartheta)$$

In this context, $\vartheta > -2\min\{\text{Re}(\lambda(A))\}$, where $\text{Re}(\lambda(A))$ represents the set of real parts of the eigenvalues of matrix A . For brevity, we denote $\Phi = \Phi(\vartheta)$ in the subsequent statements. Additionally, we introduce useful lemmas that are instrumental in deriving our results.

A. LEMMAS

Lemma 1 ([50]): Consider a system characterized by the pair (A, B) which is controllable. Let ϑ be a scalar parameter

satisfying the condition $\vartheta > -2\min\{\text{Re}(\lambda(A))\}$. Under this condition, the corresponding algebraic Riccati equation has a unique positive definite solution $\Phi(\vartheta)$. This matrix $\Phi(\vartheta)$ is known as a low gain matrix and exhibits the property that $\lim_{\vartheta \rightarrow 0^+} \Phi(\vartheta) = 0$ if all the eigenvalues of the matrix A have zero real parts. Further,

$$\text{Tr}(B^T\Phi(\vartheta)B) = n\vartheta + 2\text{Tr}(A) \quad (6)$$

$$\Phi(\vartheta)BB^T\Phi(\vartheta) \leq (n\vartheta + 2\text{Tr}(A))\Phi(\vartheta) \quad (7)$$

and

$$\begin{aligned} \text{Tr}(\Phi^T(\vartheta)A\Phi^{-1}(\vartheta)A^T) &\leq \frac{1}{2}(n\vartheta + 2\text{Tr}(A))^2 + \vartheta\text{Tr}(A) \\ &\quad - \frac{1}{2}\vartheta(n\vartheta + 2\text{Tr}(A)) - \text{Tr}((A^T)^2) \end{aligned} \quad (8)$$

Lemma 2 ([51]): For a positive definite matrix $M > 0$, two scalars α_1 and α_2 such that $\alpha_1 > \alpha_2$, and a vector function $w : [\alpha_1, \alpha_2] \rightarrow R^n$, then it holds

$$\begin{aligned} &(\int_{\alpha_1}^{\alpha_2} w^T(s)ds)M(\int_{\alpha_1}^{\alpha_2} w^T(s)ds) \\ &\leq (\alpha_2 - \alpha_1) \int_{\alpha_1}^{\alpha_2} w^T(s)Mw(s)ds \end{aligned} \quad (9)$$

Lemma 3 ([33]): Let (A, B) be controllable with $A_i = A - \sigma_i BB^T\Phi(\vartheta)$, then for $i = 1, 2, \dots, N$, it holds

$$A_i^T\Phi(\vartheta)A_i \leq \left[(\sigma_i^2 n^2 + \frac{1}{2}\sigma_i n - \frac{1}{2}\sigma_i n^2)\vartheta^2 + \sigma_i \text{Tr}(A^T)^2 \right] \Phi(\vartheta) \quad (10)$$

Lemma 4 ([52]): Consider a continuous function $y(t) : [t_0 - \tau, \infty) \rightarrow [0, \infty)$, where $\tau > 0$ is a constant. The function $y(t)$ satisfies the differential inequality

$$\dot{y}(t) \leq -ay(t) + b \sup_{t-\tau \leq s \leq t} y(s), \quad \forall t \geq t_0 \quad (11)$$

where $a > b$ are two positive constants. Assume $\eta > 0$ is the unique non-negative solution to the equation $\eta - a + be^{\eta\tau} = 0$. In this situation, $y(t)$ satisfies the inequality

$$y(t) \leq y(t_0)e^{-\eta(t-t_0)}, \quad \forall t \geq t_0, \quad (12)$$

implying an exponential decay of $y(t)$ over time.

Additionally, if the constants a and b satisfy $a \leq b$, referencing the result from [53], it follows that

$$y(t) \leq e^{v(t-t_0)}y(t_0), \quad \forall t \geq t_0 \quad (13)$$

where $v = b - a \geq 0$. This result indicates that, in the scenario where $a \leq b$, $y(t)$ grows at most exponentially with a rate determined by the difference v .

B. STABILITY ANALYSIS

To derive our results, we leverage the properties of the communication graph to decouple the coupled system (5) into multiple independent subsystems. We transform the overall system dynamics into a set of uncoupled systems, simplifying the analysis and enabling a more straightforward investigation of each uncoupled subsystem.

Consider a left eigenvector $p = [p_1, p_2, \dots, p_N]^T$ of L corresponding to $\lambda_1(L) = 0$. We define the non-singular matrix by

$$S = \begin{bmatrix} p_1 & p_{[2]}^T \\ -1_{N-1} & I_{N-1} \end{bmatrix} \quad (14)$$

where $p_{[2]} = [p_2, \dots, p_N]^T$. By (14), we have

$$\epsilon(t) = \begin{bmatrix} \xi(t) \\ \epsilon_{[2]}(t) \end{bmatrix} = (S \otimes I_n)x(t) \quad (15)$$

where $\epsilon_{[2]}(t) = [\epsilon_2^T(t), \dots, \epsilon_N^T(t)]$, $\epsilon_i = x_i - x_1$, ($i = 2, 3, \dots, N$).

Note that ϵ is a variable related to the state error between agents, given system (5), we define the error dynamics as follows:

$$\dot{\epsilon}(t) = (I_N \otimes A)\epsilon(t) - (SLS^{-1} \otimes BK)\epsilon(t - d(t)) \quad (16)$$

Thus, we have

$$SLS^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & L_2 \end{bmatrix}, \quad (17)$$

where L_2 is a matrix with $N-1$ real eigenvalues such that $0 < \lambda_2(L) \leq \dots \leq \lambda_N(L)$. Based on this observation, we derive

$$\begin{cases} \dot{\xi}(t) = A\xi(t) \\ \dot{\epsilon}_{[2]}(t) = (I_{N-1} \otimes A)\epsilon_{[2]}(t) - (L_2 \otimes BK)\epsilon_{[2]}(t - d(t)) \end{cases} \quad (18)$$

Define $\epsilon_{[2]}(t) = (M \otimes I_n)\epsilon_{[2]}(t)$, where the matrix M satisfies $ML_2M^{-1} = \text{diag}(\lambda_2(L), \lambda_3(L), \dots, \lambda_N(L))$, thus it holds that

$$\dot{\tilde{\epsilon}}_i(t) = A\tilde{\epsilon}_i(t) - \lambda_i(L)BK\tilde{\epsilon}_i(t - d(t)), i = 2, \dots, N \quad (19)$$

which expresses the decoupled dynamics in terms of the eigenvalues $\lambda_i(L)$ of the matrix L for the range $i = 2$ to N . This suggests that if $\lim_{t \rightarrow \infty} \tilde{\epsilon}(t) = 0$, we have $\lim_{t \rightarrow \infty} \epsilon(t) = 0$, which signifies the achievement of consensus. Therefore, to attain the control objective, our focus lies in demonstrating that $\lim_{t \rightarrow \infty} \tilde{\epsilon}(t) = 0$.

Remark 3: Based on the undirected communication topology of Assumption 2, we can obtain the decoupled system (19). If the communication topology $\mathcal{G}$ is directed, we can also decouple the system (1) into the following form by further assuming that $\mathcal{G}$ contains a directed spanning tree:

$$\begin{aligned} \dot{\tilde{\epsilon}}_i(t) &= A\tilde{\epsilon}_i(t) - \lambda_i(L)BK\tilde{\epsilon}_i(t - d(t)) - \delta_iBK\tilde{\epsilon}_{i+1}(t - d(t)), \\ &\text{for } i = 2, \dots, N \end{aligned} \quad (20)$$

where formulation (20) expresses the mutual influence between connected nodes based on the communication links.

$\text{Re}\{\lambda_i(L)\} > 0$, $\lambda_i(L) \in \mathbb{C}$, $\tilde{\epsilon}_{N+1}(t) = 0$, and δ_i takes the value of either 1 or 0, with $\delta_N = 0$, and

$$ML_2M^{-1} = \begin{bmatrix} \lambda_2(L) \delta_2 & & & \\ & \ddots & \ddots & \\ & & \lambda_{N-1}(L) \delta_{N-1} & \\ & & & \lambda_N(L) \end{bmatrix} \quad (21)$$

The eigenvalue of the Laplacian matrix L satisfies the stability condition, then the coupling relationship does not lead to instability. Therefore, the stability of system (20) is equivalent to the stability of

$$\dot{\tilde{\epsilon}}_i(t) = A\tilde{\epsilon}_i(t) - \lambda_i(L)BK\tilde{\epsilon}_i(t - d(t)), i = 2, \dots, N \quad (22)$$

Based on the analysis in the problem statement, it can be deduced that the system (1), can be represented as a switched system encounters packet loss, where the switching behavior is regulated by the time-delay functions. With this framework, we can proceed to explore the conditions necessary to ensure consensus in MASs with deterministic packet losses and time delays.

Theorem 1: Given Assumptions 1 and 2, consensus in the system described by (1) can be attained despite the presence of packet losses and time delays. This is accomplished using the consensus protocol described by (2), where the controller gain is given by $K = \sigma^{-1} - B^T\Phi(\vartheta)$. In this context, σ is defined as $\lambda_2(L)$, with $\lambda_2(L)$ satisfying condition (5). Additionally, the existence of $\xi_1 > 0$, $0 < d_2 < d_3$ is required for the following inequalities to hold:

$$-\vartheta + d_2^2\Lambda^2 < 0 \quad (23)$$

$$\sup_{t \geq t_0} \frac{-\eta_1 T_s(t, t_0) - \eta_2 T_u(t, t_0)}{t - t_0} = -\xi_1 \quad (24)$$

where $\eta_1 - \vartheta + d_2^2\Lambda^2 e^{\eta_1(d_2+d_3)} \leq 0$, $\eta_2 = \vartheta - d_3^2\Lambda^2 \leq 0$, $\Lambda = \sqrt{\sigma_N(2\text{Tr}(A) + n\vartheta)(\sqrt{\alpha_N}2(2\text{Tr}(A) + n\vartheta))}$, and $\alpha_N = (\sigma_N^2 n^2 + \frac{1}{2}\sigma_N n - \frac{1}{2}\sigma_N n^2)\vartheta^2 + \sigma_N \text{Tr}(A^T)^2$.

Proof: Considering the switched system framework and to reduce the computational complexity, by selecting the Lyapunov function $V(t) = \tilde{\epsilon}_i^T(t)\Phi\tilde{\epsilon}_i(t)$, the derivative of V along the solution of (19) can be computed. This calculation yields

$$\begin{aligned} \dot{V}(t) &= 2\tilde{\epsilon}_i^T(t)\Phi(A\tilde{\epsilon}_i(t) - \lambda_i(L)BK\tilde{\epsilon}_i(t - d(t))) \\ &= \tilde{\epsilon}_i^T(\Phi A + A^T\Phi - \lambda_i(L)\Phi BK - \lambda_i(L)(BK)^T\Phi)\tilde{\epsilon}_i(t) \\ &\quad + 2\lambda_i(L)\tilde{\epsilon}_i^T(t)\Phi BK(\tilde{\epsilon}_i(t) - \tilde{\epsilon}_i(t - d(t))) \\ &\leq -\tilde{\epsilon}_i^T(t)(\vartheta\Phi + \Phi BB^T\Phi - 2\sigma_i\Phi BB^T\Phi)\tilde{\epsilon}_i(t) \\ &\quad + \sigma_i\tilde{\epsilon}_i^T(t)\Phi BB^T\Phi\tilde{\epsilon}_i(t) + \sigma_i\epsilon_{1i}^T(t)\Phi BB^T\Phi\epsilon_{1i}(t) \\ &\leq -\vartheta\tilde{\epsilon}_i^T(t)\Phi\tilde{\epsilon}_i(t) + \sigma_i\epsilon_{1i}^T(t)\Phi BB^T\Phi\epsilon_{1i}(t) \end{aligned} \quad (25)$$

where $\epsilon_{1i}(t) = \tilde{\epsilon}_i(t) - \tilde{\epsilon}_i(t - d(t))$.

By Lemma 1, it yields $\Phi BB^T\Phi \leq (2\text{Tr}(A)_n\vartheta)\Phi$, then

$$\dot{V}(t) = -\vartheta V(t) + (2\text{Tr}(A) + n\vartheta)\sigma_i\epsilon_{1i}^T(t)\Phi\epsilon_{1i}(t) \quad (26)$$

Using the Jensen inequality, we have

$$\begin{aligned}
 \varepsilon_{1i}^T(t) \Phi \varepsilon_{1i}(t) &= \left(\int_{t-d(t)}^t \dot{\tilde{\varepsilon}}_i(s) ds \right)^T \Phi \left(\int_{t-d(t)}^t \dot{\tilde{\varepsilon}}_i(s) ds \right) \\
 &\leq d(t) \int_{t-d(t)}^t \left((A - \lambda_i(\mathcal{L})BK) \tilde{\varepsilon}_i(s) - \lambda_i(\mathcal{L})BK \varepsilon_{1i}(s) \right)^T \\
 &\quad \times \Phi \left(A - \lambda_i(\mathcal{L})BK \tilde{\varepsilon}_i(s) - \lambda_i(\mathcal{L})BK \varepsilon_{1i}(s) \right) \\
 &\leq (1+r_1) \bar{d} \int_{t-d(t)}^t \tilde{\varepsilon}_i^T(s) A_i^T \Phi A_i \tilde{\varepsilon}_i(s) ds \\
 &\quad + (1 + \frac{1}{r_1}) \bar{d} \sigma_i^2 (2\text{Tr}(A) + n\vartheta)^2 \int_{t-d(t)}^t \varepsilon_{1i}^T(s) \Phi \varepsilon_{1i}(s) ds
 \end{aligned} \quad (27)$$

Here, $\bar{d}$ represents the upper limit of the time-delay function $d(t)$, such that $d(t) \leq \bar{d}$.

Base on Lemma 3, we obtain

$$\begin{aligned}
 A_i^T \Phi A_i &\leq \left((\sigma_i^2 n^2 + \frac{1}{2} \sigma_i n - \frac{1}{2} \sigma_i n^2) \vartheta^2 + \sigma_i (\text{Tr}(A^2)) \right) \Phi \\
 &= \left((\sigma_i n - \frac{(1-n)^2}{2})^2 - \frac{(1-n)^2}{4} \right) \vartheta^2 + \sigma_i (\text{Tr}(A^2)) \Phi \\
 &\leq \left((\sigma_N n - \frac{(1-n)^2}{2})^2 - \frac{(1-n)^2}{4} \right) \vartheta^2 + \sigma_N (\text{Tr}(A^2)) \Phi \\
 &\triangleq \alpha_N \Phi
 \end{aligned} \quad (28)$$

Then inequality (27) can be formulated by

$$\begin{aligned}
 \varepsilon_{1i}^T(t) \Phi \varepsilon_{1i}(t) &\leq (1+r_1) \bar{d}^2 \alpha_N \sup_{t-\bar{d} \leq s \leq t} V(s) + (1 + \frac{1}{r_1}) \sigma_N^2 \\
 &\quad \times 4(2\text{Tr}(A) + n\vartheta)^2 \bar{d}^2 \sup_{t-(\bar{d}+d_3) \leq s \leq t} V(s)
 \end{aligned} \quad (29)$$

Let $r_1 = ([2(2\text{Tr}(A) + n\vartheta)] / \sqrt{\alpha_N})$, we have

$$\dot{V}(t) \leq -\vartheta \tilde{\varepsilon}_i^T(t) \Phi \tilde{\varepsilon}_i(t) + \bar{d}^2 \Lambda^2 \sup_{t-(\bar{d}+d_3) \leq s \leq t} \tilde{\varepsilon}_i^T(s) \Phi \tilde{\varepsilon}_i(s) \quad (30)$$

where $\Lambda = \sqrt{\sigma_N (2\text{Tr}(A) + n\vartheta)} (\sqrt{\alpha_N} + 2(2\text{Tr}(A) + n\vartheta))$

Two cases of (30) are considered as following.

1) *Case 1* ($\bar{d} < (\sqrt{\vartheta}/\Lambda)$): Let $\phi(\tilde{\eta}_1) = \tilde{\eta}_1 - \vartheta + \bar{d}^2 \Lambda^2 e^{\tilde{\eta}_1(\bar{d}+d_3)}$, clearly, $\phi(0) < 0$, $\lim_{\tilde{\eta}_1 \rightarrow +\infty} \phi(\tilde{\eta}_1) = +\infty$ and $\dot{\phi}(\tilde{\eta}_1) = 1 + \bar{d}^2 \Lambda^2 (d + d_3) e^{\tilde{\eta}_1(\bar{d}+d_3)} > 0$. This implies that there is precisely one solution to the equation $\phi(\tilde{\eta}_1) = 0$.

Drawing from Lemma 4, it becomes evident that

$$V(t) \leq V(t_0) e^{-\tilde{\eta}_1(t-t_0)}, \quad t \geq t_0 \quad (31)$$

where $\tilde{\eta}_1 > 0$ is determined $\tilde{\eta}_1 - \vartheta + \bar{d}^2 \Lambda^2 e^{\tilde{\eta}_1(\bar{d}+d_3)} = 0$.

Case 2 ($\bar{d} \geq \sqrt{\vartheta}/\Lambda$): Applying the results from Lemma 5, we find that

$$V(t) \leq V(t_0) e^{-\tilde{\eta}_2(t-t_0)} \quad (32)$$

where $\tilde{\eta}_2 \leq 0$ is defined by $\tilde{\eta}_2 = \vartheta - \bar{d}^2 \Lambda^2$.

Next, we turn to the initial time interval $[h + \tau_1, \min\{h + d_2, 2h + \tau_2\}]$. Let $t_0 = h + \tau_1$ be denoted. Referring to inequality (31), we can infer that

$$V(t) \leq V(t_0) e^{-\eta_1(t-(h+\tau_1))} \quad (33)$$

If $[\min\{h + d_2, 2h + \tau_2\}, 2h + \tau_2] \neq \emptyset$, according to inequality (32), for $t \in [\min\{h + d_2, 2h + \tau_2\}, 2h + \tau_2]$, we have

$$\begin{aligned}
 V(t) &\leq V(h + d_2) e^{-\eta_2(t-(h+d_2))} \\
 &\leq V(t_0) e^{-\eta_1(h+d_2-t_0)} e^{-\eta_2(t-(h+d_2))}
 \end{aligned} \quad (34)$$

where $\eta_2 = \vartheta - d_2^2 \Lambda^2 \leq 0$.

By repeating the aforementioned procedure, we obtain

$$\begin{aligned}
 V(t) &\leq \|V_{t_0}\| e^{-\eta_1 T_s(t, t_0)} e^{-\eta_2 T_u(t, t_0)} \\
 &\leq \|V_{t_0}\| e^{(-\eta_1 \frac{T_s(t, t_0)}{t-t_0} - \eta_2 \frac{T_u(t, t_0)}{t-t_0})(t-t_0)}
 \end{aligned} \quad (35)$$

According to condition (23), we have

$$-\eta_1 T_s(t, t_0) - \eta_2 T_u(t, t_0) \leq -\xi(t - t_0) \quad (36)$$

It is obvious that

$$\lambda_{\min}(\Phi) \|\tilde{\varepsilon}_i(t)\|^2 \leq V(t) \leq \|V_{t_0}\| e^{\xi_1(t-t_0)} \quad (37)$$

i.e.,

$$\|\tilde{\varepsilon}_i(t)\|^2 \leq \frac{\lambda_{\max}(\Phi) \sup_{t_0-2d_3 \leq s \leq t_0} \|\tilde{\varepsilon}_i(s)\|^2}{\lambda_{\min}(\Phi)} \quad (38)$$

This implies that as t approaches infinity, $\lim_{t \rightarrow \infty} \tilde{\varepsilon}_i(t) = 0$. In other words, consensus can be achieved in the MASs described by (1), despite the presence of packet losses and time delays.

Remark 4: As observed in Theorem 1, it can be deduced that d_2 serves as the permissible upper limit for time delay in the system. That is to say, if the time delay exceeds d_2 , it may jeopardize the stability of the system. It is evident that d_2 increases with an increase in the decay rate α of the original system, which aligns with conventional wisdom.

Remark 5: The parameter d_3 denotes the maximum allowable time delay due to consecutive packet losses. The relationship between the maximum allowable consecutive packet losses and switching frequency is described in condition (24). Therefore, to maintain system stability, the system cannot remain in an unstable state for extended periods, which also means that packet loss cannot be too frequent. In the control gain $K = \left(\frac{1}{\min_{i=2, \dots, N} \text{Re}(\lambda_i(\mathcal{L}))} \right) B^T \Phi(\vartheta)$, the control law relies on the smallest real part of the eigenvalues of the Laplacian matrix (i.e., the real part of $\mathcal{L}$). The use of the real part helps to handle the complex eigenvalue distribution in directed graphs, by leveraging control gain K , allowing the original control theory for undirected graphs to be extended to

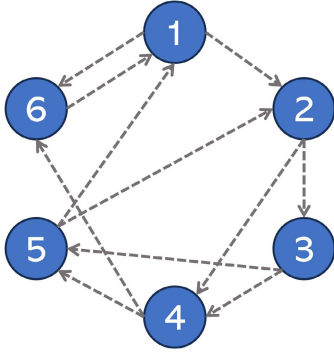


FIGURE 2. The communication topology of the MAS in our simulation.

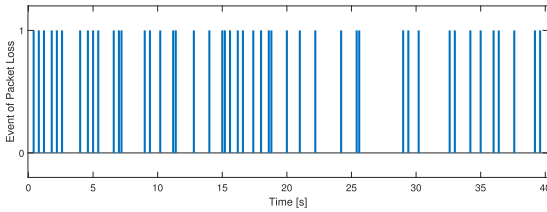


FIGURE 3. The event of transmission packet loss from agent 1 and agent 2.

directed graphs, while still maintaining the connectivity and control performance of the system.

In the case where different types of time delays are not distinguished, the time delay function $d(t)$ in the system can be described by the inequality $0 \leq d(t) \leq \tilde{d}_3$. Based on this, we can derive the following results.

Corollary 1: Given Assumptions 1 and 2, we can deduce that the system described by Eq. (1) can achieve consensus through the implementation of the protocol specified by Eq. (2). This is contingent upon the controller gain being set to $K = \sigma^{-1} B^T \Phi(\vartheta)$, where σ represents the second smallest eigenvalue of the Laplacian matrix, denoted as $\lambda_2(\mathcal{L})$, and $\Phi(\vartheta)$ is a solution to the algebraic Riccati equation. The time delay $d(t)$ must adhere to the condition $0 \leq d(t) \leq \tilde{d}_3$, with $\tilde{d}_3$ bounded by $\sqrt{\vartheta}/\Lambda$. The parameter Λ is calculated using the expression $\Lambda = \sqrt{\sigma_N(2\text{Tr}(A) + n\vartheta)}(\sqrt{\alpha_N} + 2(2\text{Tr}(A) + n\vartheta))$, where α_N is defined as $\alpha_N = (\sigma_N^2 n^2 + \frac{1}{2}\sigma_N n - \frac{1}{2}\sigma_N n^2)\vartheta^2$. Moreover, in the special case where all eigenvalues of the system matrix A are zero, it is crucial for the time delay $d(t)$ to satisfy $0 \leq d(t) \leq \tilde{d}_3$, with the stricter bound of $\tilde{d}_3 < 1/\rho$. Here, ρ is defined as $\rho = \sqrt{\sigma_N n} \left(\sqrt{\sigma_N^2 n^2 + \frac{1}{2}\sigma_N n - \frac{1}{2}\sigma_N n^2} + 2n\vartheta \right)$.

Remark 6: Delving into Corollary 1 reveals a significant insight: $\tilde{d}_3 \leq d_3$. This suggests that ignoring the distinctions between different types of time delays might result in a smaller upper-bound estimate for the time delay. Consequently, when more detailed information about the nature of the time delays is taken into account, the resulting analysis can be less conservative. To substantiate this observation, we provide theoretical evidence demonstrating

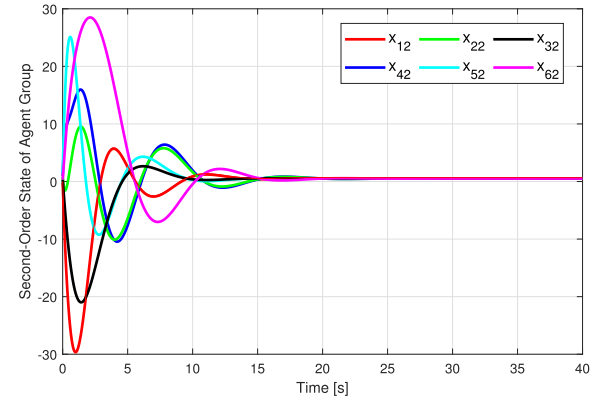
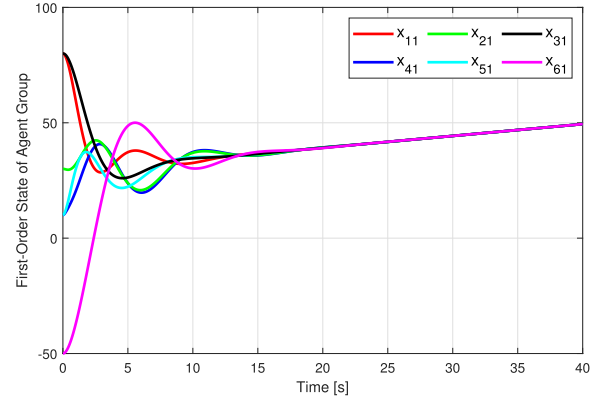


FIGURE 4. The consensus of the first and second-order state of agents in MAS.

that incorporating nuanced time delay characteristics leads to more precise and potentially less conservative estimates, improving the accuracy of the system's assessment.

Remark 7: It is worthwhile to draw a comparison with the findings presented in [34], which focused exclusively on scenarios involving packet losses. Our paper, however, offers a broader analysis that includes not only packet losses but also additional complexities such as time delays. Therefore, the system described in [34] can be viewed as a special case within the more comprehensive framework we explore in this paper. This expanded analysis provides a more thorough understanding of the system's behavior under a wider range of conditions.

IV. NUMERIC SIMULATION

In this section, we demonstrate the aforementioned theoretical results through numeric simulation. We examine a group of six agents, each operating under the general linear dynamics outlined in system (1), where the parameters for these agents are defined as

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The communication topology of the agent network is illustrated in Fig. 2. Based on this topology, the corresponding

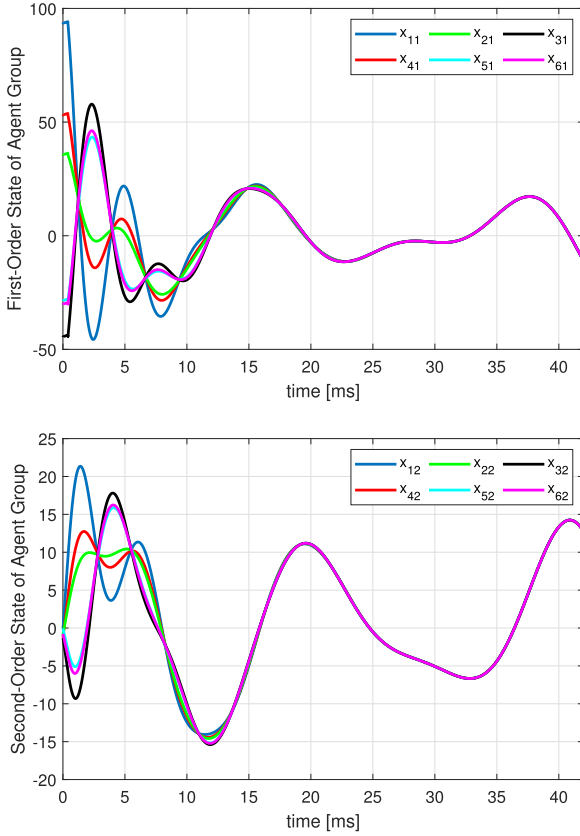


FIGURE 5. The consensus of the first and second-order state of agents in the second simulation.

Laplacian matrix is defined as follows:

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & -1 \\ 0 & 2 & -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & -1 & -1 & 0 \\ 0 & 0 & 0 & 2 & -1 & -1 \\ -1 & -1 & 0 & 0 & 2 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Given that $\vartheta = 0.1$, we can derive the matrix Φ by solving matrix (24), resulting

$$\Phi = \begin{bmatrix} 0.2 & -0.05 \\ -0.05 & 0.1(0.1^2 + 2) \end{bmatrix}.$$

The smallest real part of the eigenvalue of the Laplacian matrix $\mathcal{L}$ can handle the complex eigenvalue distribution in directed graphs. By designing the control gain K , the control theory originally developed for undirected graphs can be extended to directed graphs. The control gain matrix K is determined as $K = (\lambda_2(\mathcal{L}))^{-1} B^T \Phi = [0.4387, -0.0081]$. According to condition (19), we have $d_2 < 0.2039$ s. In this paper, we choose $d_2 = 0.1$ s, based on condition (20), if $(T_s(t_0, t)/t - t_0) = 0.9$ and $(T_u(t_0, t)/t - t_0) = 0.1$, it follows that $d_3 < 0.26$, $\eta_1 = 0.012$ and $\eta_2 = -0.168$. Fig. 3 presents a schematic of the packet loss transmission from agent 1 to agent 2, where a value of “1” represents a packet loss at that specific time and a value of “0” denotes

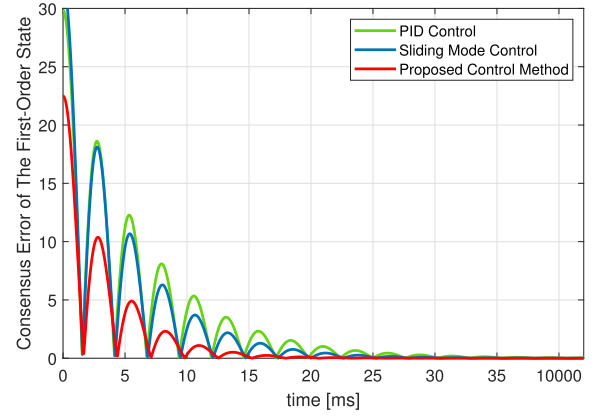


FIGURE 6. The consensus error of the first-order state of agents in the second simulation.

no packet loss. The system is capable of achieving consensus despite packet losses and time delays, as demonstrated in Fig. 4. The simulation results are derived from the initial condition

$$[x_1(t), \dots, x_6(t)]^T = \cos(t) \begin{bmatrix} 2 & 1 & 1.5 & 2.5 & 3 & 3.5 \\ 3 & 4 & 0.5 & 2 & 1.5 & 2.5 \end{bmatrix}.$$

With a sampling period of $h = 0.05$, the condition $d_3 \leq 0.28$ implies that the system can tolerate a maximum of two consecutive packet losses. Specifically, this condition ensures that within the sampling interval of 0.05, the system remains stable and performs within acceptable bounds even when up to two consecutive packets are lost. This result highlights the robustness of the system to packet loss, as it indicates that the occurrence of such events does not lead to significant degradation in performance or stability, provided the loss is within this specified threshold.

In the second experiment, we increased the communication delay and packet loss rate of the multi-agent system (as shown in Fig. 2). We set d_2 and d_3 to 0.2s and 0.25s, respectively, and the random packet loss rate was set to 0.15. The first-order and second-order state changes of the multi-agent system are shown in Fig. 5. It can be seen that our proposed control algorithm still enables the system to achieve consensus. In Fig. 6, we compared the cooperative error of our proposed control algorithm with PID and sliding mode control. It is evident that our control algorithm achieves smaller cooperative error and faster consensus stabilization.

V. CONCLUSION

This paper explored the consensus problem of MASs subjected to dynamic communication delays and packet loss scenarios. To differentiate and effectively manage network-induced temporal delays and packet loss, we have adopted a methodology based on switched systems. This approach allowed us to craft a controller with a reduced gain formulated through the application of a parametric algebraic Riccati equation, thereby addressing the inherent complexities of such network conditions. By leveraging

Lyapunov stability theory, we established constraints that ensure the maintenance of consensus despite the intermittent and transient nature of packet losses in MASs. Our theoretical analysis has shown that utilizing an enhanced time-delay model reduces the conservative assumptions typically associated with such systems, thereby improving the robustness of our control strategy. Future work can expand on this foundation by exploring additional enhancements to the controller and addressing other types of network imperfections to further enhance the robustness and efficacy of MASs.

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