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SURVEY

Gaussian Filtering With False Data Injection and Randomly Delayed Measurements

SUMANTA KUMAR NANDA[1], GUDDU KUMAR[1], AMIT KUMAR NAIK[1], MOHAMMED ABDEL-HAFEZ[2], VIMAL BHATIA[1,3,4], (Senior Member, IEEE), ONDREJ KREJCAR[3,5], AND ABHINOY KUMAR SINGH[6], (Member, IEEE)

[1]Department of Electrical Engineering, Indian Institute of Technology Indore, Indore 453552, India
[2]Electrical and Communication Engineering Department, College of Engineering, United Arab Emirates University, Al Ain, United Arab Emirates
[3]Faculty of Informatics and Management, Univerzita Hradec Králové, 500 03 Hradec Králové, Czech Republic
[4]School of Electronic and Information Engineering, Soochow University, Suzhou, Jiangsu 215006, China
[5]Malaysia-Japan International Institute of Technology, Universiti Teknologi Malaysia Kuala Lumpur, Kuala Lumpur 54100, Malaysia
[6]Department of Electrical Engineering, Indian Institute of Technology Patna, Patna 801106, India

Corresponding author: Mohammed Abdel-Hafez (mhafez@uaeu.ac.ae)

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ABSTRACT State estimation in cyber-physical systems is a challenging task involving integrating physical models and measurements to estimate dynamic states accurately in practical machine-to-machine and IoT deployments. However, integrating advanced wireless communication and intelligent measurements has increased vulnerability of external intrusion through a centralized server. This study addresses the challenge of Gaussian filtering for a specific type of stochastic nonlinear system vulnerable to cyber attacks and delayed measurements. These attacks occur randomly when data is transmitted from sensor nodes to remote filter nodes. To address this issue, a new cyber attack model is proposed that combines false data injection attacks and delayed measurement into a unified framework. The study also analyzes the stochastic stability of the proposed filter and establishes sufficient conditions to ensure that the filtering error remains bounded even in the presence of randomly occurring cyber attacks and delayed measurements. The proposed methodology is demonstrated and compared with other widely used approaches using simulated data to highlight its effectiveness and usefulness.

INDEX TERMS Delay measurement, FDI, Gaussian filtering, nonlinear Bayesian filtering.

I. INTRODUCTION

Filtering is a recursive process for state estimation of dynamical systems from noisy measurements [1]. A popular nonlinear filtering method [2], Gaussian filter comprises of prediction and update steps, and is based on Bayesian approximation method. It approximates the unknown prior and posterior probability density functions (PDFs) as Gaussian, and characterizes them with mean and covariance [3]. The computation of mean and covariance involves intractable integrals, which are numerically approximated during the filtering process. Some popular Gaussian filters are extended Kalman filter (EKF) [4], unscented Kalman filter (UKF) [5], Cubature Kalman filter (CKF) [6], Gauss-Hermite filter (GHF) [7], and others. An alternate choice of Gaussian filtering is particle filtering, which is beyond the scope of this paper due to its huge computational complexity [8].

In this context, Gaussian filtering, a commonly used state estimation technique, often underperforms or fails to produce accurate results in the presence of irregularities [9]. In this paper, we consider two complex irregularities namely unknown delay which is caused by data propagation time, and ‘‘cyber attacks’’, in which intruders inject false data into the true measurement data.

The cyber attacks occur in cyber-physical systems (CPS) [10]. To connect geographically dispersed sensors, the CPS

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utilizes wireless communication networks. The extensive controller components and complex CPS networks makes them susceptible to security threats in both physical and cyber domains through cyber attacks [11], [13], [14], [15]. In a related study, [12] and [13] presented improved Lyapunov-Krasovskii functions (LKFs) with fuzzy membership functions to estimate the performance degradation caused by denial-of-service (DoS) attacks in T-S fuzzy networked control systems, whereas [14] and [15] proposed robust estimators to estimate the states accurately in presence of false data injection (FDI) attacks.

The term "FDI attack" is used in this paper to refer to cyber attacks. The FDI attacks can occur due to various reasons [16], including: i) exploitation of vulnerabilities in the communication protocols or software components that can allow unauthorized access, ii) sensor-level attacks, iii) altering data packets by tampering with the communication channels, and iv) manipulating the control algorithms. As an example scenario, let us consider a sensor-level attacks, where the attack is to mislead and deceive the target system via secretly compromising measurements (generally, by a small margin). Such manipulation of measurement data adversely harms the performance of the proposed filtering algorithms [17].

In the literature, there are some contributions which independently handle both the irregularities: delay and FDI attack. For example, [18], [19], and [20] address the delays to different extents. They reformulate the measurement model to incorporate the delay possibilities and re-derive the traditional Gaussian filtering method for the reformulated measurement models. While [18] and [19] handle only small delays (up to two sampling intervals), [20] addresses larger delays. As compared to these methods, a different approach is adopted in [21] and [22], where the delay (or delay probability) is identified using likelihood, and the traditional Gaussian filtering method is readjusted (in time) accordingly. Furthermore, for handling the FDI attacks, [23] extends the traditional EKF to address this problem. However, it does not apply to other Gaussian filters, such as UKF, CKF, and GHF, which provide higher accuracy. In later developments, [14] and [15] introduced a generalized Gaussian filtering method for handling the FDI attacks. They reformulate the measurement model stochastically to incorporate the possibility of FDI attacks and re-derive the traditional Gaussian filtering accordingly. As [14] and [15] are generalized extensions of Gaussian filtering, it applies to any existing Gaussian filters, such as the EKF, CKF, and GHF.

Although [14], [15], [16], [18], [19], [20], [21], [22], [23], and [24] address the delay and cyber-attacks independently, they fail to handle their simultaneous occurrences while the simultaneous occurrences cannot be ignored in practice. In addition to the above literature, recent studies in [25], [26], [27], and [28] handle both delay and cyber-attack simultaneously. Reference [25] presented security-oriented filtering that considers two types of disturbances: randomly occurring sensor saturations and FDI attacks. The occurrence of an FDI attack is modeled using the Bernoulli process, which accurately captures the characteristics of their presence. Another recent contribution [26] addresses the secure particle filtering problem for a specific category of discrete-time nonlinear CPSs. The adversaries launch a variety of attacks, including DoS attacks, FDI attacks, and flipping attacks, which manifest randomly. Furthermore, [27] investigates the feasibility of a coordinated attack known as a time-delay and FDI attack on CPS. This coordinated attack combines the detrimental effects of time-delay attacks and FDI attacks in a synchronized manner, resulting in increased potency compared to each individual attack alone. However, assessing the stealthiness and effectiveness of such a coordinated attack presents significant challenges, and the current formulation is restricted to linear systems, requiring further advancements for nonlinear dynamical systems. Additionally, [28] faces difficulty in setting an upperbound of the delay which may affect the filtering performance.

In this paper, for the first time, we redesign the traditional Gaussian filtering method to simultaneously handle delay and FDI attacks in a nonlinear dynamical system. We named the proposed method Gaussian filtering with delay and FDI attack (GFDF). The proposed GFDF reformulates the traditional measurement model using Bernoulli, Geometric, and Gaussian random variables to incorporate the possibility of delay and FDI attack. The proposed GFDF subsequently re-derives the traditional Gaussian filtering method for the reformulated measurement model. The re-derivation of the traditional Gaussian filtering method (for the modified measurement model) mainly requires re-deriving the expressions of measurement estimate, measurement covariance, and state-measurement cross-covariance. Interestingly, the proposed GFDF is a general extension of Gaussian filtering, which applies to any existing Gaussian filters, such as the EKF, CKF, and GHF. We study the stability of the proposed GFDF for its EKF-based formulation. Moreover, we validate the improved accuracy of the proposed GFDF for its CKF-based formulation.

This article presents several noteworthy contributions and innovations outlined as follows:

- A stochastic model is developed to capture the simultaneous occurrences of FDI attacks and delayed measurements.
- An advanced filtering approach is proposed to improve estimation accuracy where FDI attacks and measurement delays simultaneously occur.
- The need for ambiguous selection of an upper bound of delay could be avoided [28], further contributing to improved accuracy.
- A stochastic stability analysis is derived for the EKF-based formulation of the proposed filtering method.

II. PROBLEM DESCRIPTION

At first, let us consider process and measurement models, expressed as

$$\mathbf{x}_k = \Phi_{k-1}(\mathbf{x}_{k-1}) + \boldsymbol{\omega}_{k-1}, \tag{1}$$

$$\mathbf{z}_k = \Psi_k(\mathbf{x}_k) + \boldsymbol{\zeta}_k, \tag{2}$$

where $\mathbf{x}_k \in \mathbb{R}^n$ is state variable, $\boldsymbol{\omega}_k \in \mathbb{R}^n$ is process error, and $\Phi_k : \mathbf{x}_{k-1} \to \mathbf{x}_k$ is general propagation function representing the state dynamics at the k^{th} instant. While $\mathbf{z}_k \in \mathbb{R}^r$ is measurement equation, $\boldsymbol{\zeta}_k \in \mathbb{R}^r$ is measurement error, and $\Psi_k : \mathbf{x}_k \to \mathbf{z}_k$ is general measurement propagation function at the k^{th} instant with $k \in \{1, 2, \ldots.\}$. Moreover, without loss of generality, the errors $\boldsymbol{\omega}_k$ and $\boldsymbol{\zeta}_k$ are assumed independent and uncorrelated with approximation zero mean Gaussian with covariances $\mathbf{Q}_k$ and $\mathbf{R}_k$, respectively. Please note that the scope of this article is limited to Gaussian approximated distribution.

As mentioned earlier, the true measurement $\mathbf{z}_k$ may be subject to various irregularities, such as delayed measurement or FDI attacks. These irregularities can result in the observed measurement $\mathbf{y}_k$. Consequently, the fundamental aim of filtering is to deduce the current state $\mathbf{x}_k$ based on the received measurement data $\mathbf{y}_k$.

Attackers frequently make intermittent changes to measurement data to conceal their intrusion. We hypothesize that data is manipulated by FDI and delayed measurement at a particular time instant. To accommodate these factors, we modify the measurement model equation (2) for $\mathbf{y}_k$ by utilizing the following modeling techniques:

- To capture the occurrence of data alterations, we introduce two Bernoulli random variables, β_k and α_k. Our approach comprises of the following steps: i) employing a likelihood test to identify instances with no data alteration and to estimate the value of β_k. ii) performing a correlation analysis to detect delayed measurements and estimate the value of α_k. iii) infer the presence of FDI attacks and estimate the true dynamical states.
- After detecting an FDI attack, it is possible to model the uncertain false data using a Gaussian distribution. The true measurement is randomly altered through an amplification/attenuation multiplicative process. To do this, appropriate Gaussian distribution is identified using data preprocessing techniques, as outlined in [14].
- Incorporating the Geometric distribution is an effective method for accounting for uncertain measurement delays caused by limited resources. This approach enables delayed measurements to be incorporated into the analysis without requiring prior knowledge of the maximum delay. Moreover, using the Geometric distribution to model instantaneous delays is an effective way to handle larger delays.

It is worth noting that the data pre-analysis and preprocessing are performed separately and not included in the proposed methodology.

Throughout rest of this manuscript, we will use the notations Θ' and p' to denote $(1-\Theta)$ and $(1-p)$, respectively, for any random variable Θ and any probability p. This notation applies to all random variables and probabilities.

The Bernoulli random variables β_k and α_k are subject to the following notation

$$\begin{cases} P(\beta_k = 1) = \mathbb{E}[\beta_k] = \mathbb{E}\left[(\beta_k)^m\right] = p_a \\ P(\alpha_k = 1) = \mathbb{E}[\alpha_k] = \mathbb{E}\left[(\alpha_k)^m\right] = p_d \end{cases} \tag{3}$$

where $\mathbb{E}[\cdot]$ denotes the statistical expectation operator and $m \in \mathbb{R}$ is a constant. Moreover, p_a and p_d denote the probabilities of no-attack and FDI attack, respectively. Similar to equation (3), corresponding to $P(\beta_k = 0) = p'_a$, we get $\mathbb{E}\left[\beta'_k\right] = \mathbb{E}\left[(\beta'_k)^m\right] = p'_a$ and $\mathbb{E}\left[(\beta_k - p_a)^2\right] = \mathbb{E}\left[(\beta_k)^2\right] - \mathbb{E}[\beta_k]^2 = p_a p'_a$. Similar conclusions for α_k, corresponding to $P(\alpha_k = 0) = p'_d$, we get $\mathbb{E}\left[\alpha'_k\right] = \mathbb{E}\left[(\alpha'_k)^m\right] = p'_d$ and $\mathbb{E}\left[(\alpha_k - p_d)^2\right] = \mathbb{E}\left[(\alpha_k)^2\right] - \mathbb{E}[\alpha_k]^2 = p_d p'_d$. In addition, we can represent d-delay at time t_k using a Geometric random variable denoted by $G_{d,k}$. This random variable accounts for delays of up to d sampling intervals. The probability of obtaining a value of one for each entry of $G_{d,k}$ can be denoted by p_g. This random variable follows:

$$\begin{cases} P\left(G_{d,k}(i) = 1\right) = \mathbb{E}\left[(G_{d,k}(i))^m\right] = \Gamma_i \\ P\left(G_{d,k}(i) = 0\right) = \mathbb{E}\left[(G_{d,k}(i)')^m\right] = \Gamma'_i \\ \mathbb{E}\left[\left(G_{d,k}(i) - \Gamma_i\right)^2\right] = \Gamma_i \Gamma'_i, \end{cases} \tag{4}$$

where $\Gamma_i = (p'_g)^{i-1} p_g$ is the probability of i-delay $\forall i \in \{1, 2, \cdots, d\}$ at t_k.

Identifying and countering false data is crucial to avoid its effects on filtering. Various techniques can be employed to achieve this. Stochastic quantitative methods are instrumental in mitigating the impact of false data, as stochastic rules can be more effective in the presence of unknown intruders and arbitrary data injection. Pre-analysis rules, such as heuristic rules and normalization methods, can also aid in identifying and normalizing false data. In addition, stochastic heuristics can determine the probability of receiving a measurement and adjust amplification, and attenuation factors accordingly. Using appropriate heuristic rules and normalization methods, false data can be closely approximated as Gaussian data. Specifically, if false data is injected at time t_k, it can be represented as Δ_k, which can be approximated as $\mathbb{N}(\hat{\delta}, \Sigma_\delta)$ in the event of an FDI attack at t_k. It is important to note that $\mathbb{E}[\Delta_k^2] = \Sigma_\delta + \hat{\delta}^2$ is used in the next section of the discussion. As discussed in [14], $\hat{\delta}$ and Σ_δ is computed using predefined heuristic rules and normalization methods.

To model the received measurement $\mathbf{y}_k$ and address the issue at hand, we adopt the modeling strategy depicted in equation (5). Additionally, the random variables are assumed to be independent and uncorrelated. Specifically, we express $\mathbf{y}_k$ as $\mathbf{y}_k = \beta_k \mathbf{z}_k + \beta'_k[\alpha_k \Delta_k \mathbf{z}_k + \alpha'_k \sum_{i=1}^{d} G_{d,k}(i)\mathbf{z}_{k-i}]$. This equation can be further simplified to yield

$$\mathbf{y}_k = (\beta_k + \beta'_k \alpha_k \Delta_k)\mathbf{z}_k + \beta'_k \alpha'_k \sum_{i=1}^{d} G_{d,k}(i)\mathbf{z}_{k-i}. \tag{5}$$

TABLE 1. α_k **and** β_k **values for different attacks.**

Stochastic parameters	Form of attack
$\beta_k = 1,\ \alpha_k \in \{0,1\}$	No-attack
$\beta_k = 0,\ \alpha_k = 1$	FDI-attack
$\beta_k = 0,\ \alpha_k = 0$	Delayed data

In Table 1, we present the values of α_k and β_k corresponding to different types of attacks. This study aims to re-derive the traditional Gaussian filtering method from a modified measurement model.

Remark 1: In cyber-attacks, this modified measurement model is useful to counter both data replay attacks and FDI attacks jointly. Moreover, this approach ensures data security and integrity in various contexts and comprehensively controls cyber-attacks.

III. DESIGN METHODOLOGY OF GFDF

In this section, a proposed Gaussian filtering strategy is introduced to handle FDI and delayed measurements that occur concurrently. Prior research has shown that irregular measurements can affect the filtering accuracy in a way unrelated to the system's state dynamics. Therefore, the proposed filtering strategy only requires the re-derivation of measurement-related parameters because the traditional Gaussian filters prediction is independent of measurement, and only the update step is influenced by measurement. Specifically, the measurement estimate, covariance, and cross-covariance for a true measurement are denoted as $\hat{\mathbf{z}}_{k|k-1}$, $\mathbf{P}^{\mathbf{zz}}_{k|k-1}$, and $\mathbf{P}^{\mathbf{xz}}_{k|k-1}$, respectively. Similarly, the corresponding parameters for an actually received measurement are denoted as $\hat{\mathbf{y}}_{k|k-1}$, $\mathbf{P}^{\mathbf{yy}}_{k|k-1}$, and $\mathbf{P}^{\mathbf{xy}}_{k|k-1}$, respectively. The proposed Gaussian filter replaces the parameters for a true measurement with those for an actually received measurement. In the subsequent discussion, the authors derive the parameters for the modified measurement model that accounts for FDI and delayed measurements that occur concurrently.

We derive $\hat{\mathbf{y}}_{k|k-1}$, $\mathbf{P}^{\mathbf{yy}}_{k|k-1}$, and $\mathbf{P}^{\mathbf{xy}}_{k|k-1}$ with respect to $\mathbf{y}_k$ (modeled in equation (5)) through three subsequent lemmas.

Lemma 1: The estimation of the measurement $\mathbf{y}_k$ in the presence of both delays and cyber-attacks can be described as follows:

$$\hat{\mathbf{y}}_{k|k-1} = (p_a + p'_a p_d \hat{\delta})\hat{\mathbf{z}}_{k|k-1} + p'_a p'_d \sum_{i=1}^{d} \Gamma_i \hat{\mathbf{z}}_{k-i|k-1}. \quad (6)$$

Proof: Let us denote $\hat{\mathbf{y}}_{k|k-1} = \mathbb{E}\big[\mathbf{y}_k\big]$, for $\mathbf{y}_k$ given in equation (5), computed as

$$\hat{\mathbf{y}}_{k|k-1} = \mathbb{E}\Big[(\beta_k + \beta'_k \alpha_k \Delta_k)\mathbf{z}_k + \beta'_k \alpha'_k \sum_{i=1}^{d} G_{d,k}(i)\mathbf{z}_{k-i}\Big].$$

Please note that β_k, α_k, Δ_k, and $G_{d,k}(i)$ characterize measurement irregularities and they are independent of $\mathbf{z}_k$, which defines measurement values at the current instant. Since, $\mathbb{E}\big[\mathbf{z}_k\big] = \hat{\mathbf{z}}_{k|k-1}$, after simplification of β_k, α_k and $G_{d,k}$, we get

$$\hat{\mathbf{y}}_{k|k-1} = \mathbb{E}\left[\beta_k + \beta'_k \alpha_k \Delta_k\right]\hat{\mathbf{z}}_{k|k-1} + \mathbb{E}\left[\alpha'_k \beta'_k\right] \sum_{i=1}^{d} \mathbb{E}\left[G_{d,k}(i)\right]\hat{\mathbf{z}}_{k-i|k-1}.$$

Substituting $\mathbb{E}[\alpha_k]$, $\mathbb{E}[\beta_k]$, and $\mathbb{E}\big[G_{d,k}(i)\big]$ from equations (3) and (4), and their subsequent discussions, the above equation reduces to equation (6).

Lemma 2: The covariance matrix $\mathbf{P}^{\mathbf{yy}}_{k|k-1}$ for $\mathbf{y}_k$ can be given in the form of equation (7), as shown at the bottom of the page.

Proof: The covariance matrix $\mathbf{P}^{\mathbf{yy}}_{k|k-1}$ is given as

$$\mathbf{P}^{\mathbf{yy}}_{k|k-1} = \mathbb{E}\left[(\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1})(\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1})^T\right]. \quad (8)$$

We can express the difference between $\mathbf{y}_k$ and $\hat{\mathbf{y}}_{k|k-1}$, as given in equations (5) and (6), respectively, using the equation (9), as shown at the bottom of the next page. By substituting $\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1}$ from equation (9) into equation (8), we obtain $\mathbf{P}^{\mathbf{yy}}_{k|k-1} = \sum_{i=1}^{4}\sum_{j=1}^{4} \mathbb{E}\left[J_i J_j^T\right]$. In stochastic filtering theory, it is common to simplify calculations by assuming stochastic independence properties for independent random variables. Applying this assumption, we can easily conclude that $\mathbb{E}[J_i J_j^T] = 0\ \forall i \neq j$. As an example, we can write $\mathbb{E}[J_1 J_2^T] = \mathbb{E}[(\beta_k + \beta'_k \alpha_k \Delta_k)(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1})(\beta_k + \beta'_k \alpha_k \Delta_k - (p_a + p'_a p_d \hat{\delta})]\hat{\mathbf{z}}_k^T$, which can be rewritten as $\mathbb{E}[J_1 J_2^T] = \mathbb{E}[(\beta_k + \beta'_k \alpha_k \Delta_k)]\mathbb{E}[(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1})]\mathbb{E}[(\beta_k + \beta'_k \alpha_k \Delta_k - (p_a + p'_a p_d \hat{\delta})]\mathbb{E}[\hat{\mathbf{z}}_k^T]$. After further simplification and substituting the values from equation (3), we get $\mathbb{E}[J_1 J_2^T] = 0$. Similarly, we can trivially conclude for other expressions $\mathbb{E}[J_i J_j^T]\ \forall\ i \neq j$. Thus,

$$\mathbf{P}^{\mathbf{yy}}_{k|k-1} = \sum_{i=1}^{4} \mathbb{E}\left[J_i J_i^T\right]. \quad (10)$$

$$\mathbf{P}^{\mathbf{yy}}_{k|k-1} = (p_a + p'_a p_d(\Sigma_\delta + \hat{\delta}^2) + 2p_a p'_d \hat{\delta})\mathbf{P}^{\mathbf{zz}}_{k|k-1} + (p_a p'_a + p'_a p_d \hat{\delta}^2(1 - p'_a p_d) + p'_a p_d \Sigma_\delta)\hat{\mathbf{z}}_{k|k-1}\hat{\mathbf{z}}^T_{k|k-1} + p'_a p'_d \sum_{i=1}^{d} \Gamma_i \mathbf{P}^{\mathbf{zz}}_{k-i|k-1} + \sum_{i=1}^{d}\left(p'_a p'_d \Gamma_i (1 - p'_a p'_d \Gamma_i)\right)\hat{\mathbf{z}}_{k-i|k-1}\hat{\mathbf{z}}^T_{k-i|k-1} + \sum_{i \neq j=1}^{d}\left(p'_a p'_d (p'_g)^{i+j-2} p_g^2 (1 - p'_a p'_d (p'_g)^{i+j-2} p_g^2)\right)\hat{\mathbf{z}}_{k-i|k-1}\hat{\mathbf{z}}^T_{k-j|k-1}. \quad (7)$$

We now derive $\mathbb{E}[J_iJ_i^T]\ \forall i \in \{1, 2, \cdots, 4\}$, which we add later to obtain $\mathbf{P}^{\mathbf{yy}}_{k|k-1}$.

For J_1 given in equation (9), we can write

$$\mathbb{E}\left[J_1J_1^T\right] = \mathbb{E}\left[(\beta_k+\beta'_k\alpha_k\Delta_k)^2(\mathbf{z}_k-\hat{\mathbf{z}}_{k|k-1})(\mathbf{z}_k-\hat{\mathbf{z}}_{k|k-1})^T\right].$$

Please note that α_k and β_k are independent of $\mathbf{z}_k$ and $\hat{\mathbf{z}}_{k|k-1}$. Moreover, as α_k and β_k are independent Bernoulli random variables, we obtain $\mathbb{E}\left[(\beta_k+\beta'_k\alpha_k\Delta_k)^2\right] = \mathbb{E}\left[(\beta_k^2+\beta_k'^2\alpha_k^2\Delta_k^2+2\beta_k\beta'_k\alpha_k\Delta_k)\right]$. Substituting the values from equation (3) and $\Delta_k^2 = \Sigma_\delta + \hat{\delta}$, we get $\mathbb{E}\left[(\beta_k+\beta'_k\alpha_k\Delta_k)^2\right] = p_a + p'_a p_d(\Sigma_\delta + \hat{\delta}^2) + 2p_a p'_d\hat{\delta}$. Subsequently, the above equation is simplified as

$$\mathbb{E}\left[J_1J_1^T\right] = (p_a + p'_a p_d(\Sigma_\delta+\hat{\delta}^2) + 2p_a p'_d\hat{\delta})P^{\mathbf{zz}}_{k|k-1}. \quad (11)$$

Similarly, for J_2 given in equation (9), we obtain

$$\mathbb{E}\left[J_2J_2^T\right] = \mathbb{E}\left[(\beta_k+\beta'_k\alpha_k\Delta_k-(p_a+p'_a p_d\hat{\delta}))^2\hat{\mathbf{z}}_{k|k-1}\hat{\mathbf{z}}^T_{k|k-1}\right],$$

which is further simplified as

$$\mathbb{E}\left[J_2J_2^T\right] = \left(p_a p'_a + p'_a p_d\hat{\delta}^2(1-p'_a p_d) + p'_a p_d\Sigma_\delta\right)\hat{\mathbf{z}}_{k|k-1} \times \hat{\mathbf{z}}^T_{k|k-1}. \quad (12)$$

Now substituting J_3 from equation (9) into $\mathbb{E}\left[J_3J_3^T\right]$ to obtain

$$\mathbb{E}\left[J_3J_3^T\right] = \mathbb{E}\Bigg[\sum_{i=1}^{d}\beta'_k\alpha'_k G_{d,k}(i)(\mathbf{z}_{k-i}-\hat{\mathbf{z}}_{k-i|k-1}) \times \sum_{j=1}^{d}\beta'_k\alpha'_k G_{d,k}(j)(\mathbf{z}_{k-j}-\hat{\mathbf{z}}_{k-j|k-1})^T\Bigg].$$

Here are some notes we ought to consider: 1) $\mathbf{z}_{k-i}$ and $\mathbf{z}_{k-j}$ are independent of $\forall i \neq j$, 2) $G_{d,k}(i)$ and $G_{d,k}(j)$ are independent of $\forall i \neq j$, and 3) α_k and β_k are independent of each other, and also independent of $\mathbf{z}_{k-i}$ and $G_{d,k}(i)\ \forall i \in \{1, 2, \cdots, d\}$. Based on their independently derived properties, we can simplify the above equation further as

$$\mathbb{E}\left[J_3J_3^T\right] = p'_a p'_d\sum_{i=1}^{d}\Gamma_i\mathbf{P}^{\mathbf{zz}}_{k-i|k-1}. \quad (13)$$

To this end, for J_4 given in equation (9), we get

$$\mathbb{E}\left[J_4J_4^T\right] = \mathbb{E}\left[\sum_{i=1}^{d}\left(\beta'_k\alpha'_k G_{d,k}(i) - p'_a p'_d\Gamma_i\right)^2\hat{\mathbf{z}}_{k-i|k-1}\hat{\mathbf{z}}^T_{k-i|k-1}\right]. \quad (14)$$

According to various independence properties for independent random variables, the above equation can be written as follows:

$$\begin{aligned}\mathbb{E}\left[J_4J_4^T\right] &= \sum_{i=j=1}^{d} p'_a p'_d\Gamma_i(1-p_a p'_d\Gamma_i)\hat{\mathbf{z}}_{k-i|k-1}\hat{\mathbf{z}}^T_{k-i|k-1}\\ &+ \sum_{i\neq j=1}^{d}\left(p'_a p'_d(p'_g)^{i+j-2}p_g^2\right)\left(1-\left(p'_a p'_d(p'_g)^{i+j-2}p_g^2\right)\right)\\ &\times \hat{\mathbf{z}}_{k-i|k-1}\hat{\mathbf{z}}^T_{k-j|k-1}.\end{aligned} \quad (15)$$

Substituting $\mathbb{E}[J_1J_1^T]$, $\mathbb{E}[J_2J_2^T]$, $\mathbb{E}[J_3J_3^T]$, and $\mathbb{E}[J_4J_4^T]$, from equations (11), (12), (13), and (15), respectively, into equation (10), $\mathbf{P}^{\mathbf{yy}}_{k|k-1}$ can be expressed in the form of equation (7). □

Lemma 3: The cross-covariance matrix between $\mathbf{x}_k$ and $\mathbf{y}_k$ can be obtained as

$$\mathbf{P}^{\mathbf{xy}}_{k|k-1} = (p_a + p'_a p_d\hat{\delta})\mathbf{P}^{\mathbf{xz}}_{k|k-1} + \sum_{i=1}^{d}p'_a p'_d\Gamma_i\mathbf{P}^{\mathbf{xz}}_{k-i|k-1}. \quad (16)$$

Proof: For $\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1}$ given in equation (9), we get $\mathbf{P}^{\mathbf{xy}}_{k|k-1} = \sum_{i=1}^{4}\mathbb{E}\left[(\mathbf{x}_k-\hat{\mathbf{x}}_{k|k-1})J_i^T\right]$. As $\hat{\mathbf{z}}_{k|k-1}$, $\hat{\mathbf{z}}_{k-i|k-1}$, and $\hat{\delta}$ are constants and $\mathbf{x}_k$ is independent of Δ_k, we can conclude that $\sum_i\mathbb{E}\left[(\mathbf{x}_k-\hat{\mathbf{x}}_{k|k-1})J_i^T\right] = 0,\ \forall i \in \{2, 4\}$, giving

$$\mathbf{P}^{\mathbf{xy}}_{k|k-1} = \mathbb{E}\left[(\mathbf{x}_k-\hat{\mathbf{x}}_{k|k-1})J_1^T\right] + \mathbb{E}\left[(\mathbf{x}_k-\hat{\mathbf{x}}_{k|k-1})J_3^T\right]. \quad (17)$$

To this end, for J_1 given in equation (9), we obtain

$$\mathbb{E}\left[(\mathbf{x}_k-\hat{\mathbf{x}}_{k|k-1})J_1^T\right] = \mathbb{E}\big[(\mathbf{x}_k-\hat{\mathbf{x}}_{k|k-1})(\beta_k+\beta'_k\alpha_k\Delta_k) \times (\mathbf{z}_k-\hat{\mathbf{z}}_{k|k-1})^T\big].$$

$$\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1} = \underbrace{(\beta_k+\beta'_k\alpha_k\Delta_k)(\mathbf{z}_k-\hat{\mathbf{z}}_{k|k-1})}_{J_1} + \underbrace{(\beta_k+\beta'_k\alpha_k\Delta_k-p_a-p'_a p_d\hat{\delta})\hat{\mathbf{z}}_{k|k-1}}_{J_2} + \underbrace{\sum_{i=1}^{d}\beta'_k\alpha'_k G_{d,k}(i)(\mathbf{z}_{k-i}-\hat{\mathbf{z}}_{k-i|k-1})}_{J_3} + \underbrace{\sum_{i=1}^{d}\left(\beta'_k\alpha'_k G_{d,k}(i)-p'_a p'_d\Gamma_i\right)\hat{\mathbf{z}}_{k-i|k-1}}_{J_4}. \quad (9)$$

which is simplified as

$$\mathbb{E}\left[(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1})J_1^T\right] = (p_a + p'_a p_d \hat{\delta})P^{\mathbf{xz}}_{k|k-1}. \tag{18}$$

Moreover, for J_3 given in equation (9), we get

$$\begin{aligned}\mathbb{E}&\left[\left(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}\right) J_3^T\right] \\ &= \mathbb{E}\Big[\left(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}\right) \\ &\quad \times \Big(\sum_{i=1}^{d} \beta'_k \alpha'_k G_{d,k}(i)(\mathbf{z}_{k-i} - \hat{\mathbf{z}}_{k-i|k-1})^T\Big)\Big].\end{aligned}$$

Applying the independent property, we get the following after a few simplifications and rearrangements.

$$\begin{aligned}\mathbb{E}&\left[(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1})J_3^T\right] \\ &= \mathbb{E}\left[\beta'_k \alpha'_k\right] \sum_{i=1}^{d} \Big(\mathbb{E}\left[G_{d,k}(i)\right] \\ &\quad \times \mathbb{E}\left[(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1})(\mathbf{z}_{k-i} - \hat{\mathbf{z}}_{k-i|k-1})^T\right]\Big).\end{aligned}$$

As $\mathbb{E}\left[\beta'_k \alpha'_k\right] = p'_a p'_d$ and $\mathbb{E}\left[G_{d,k}(i)\right] = \Gamma_i$, we obtain

$$\mathbb{E}\left[(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1})J_3^T\right] = \sum_{i=1}^{d} p'_a p'_d \Gamma_i \mathbf{P}^{\mathbf{xz}}_{k-i|k-1}. \tag{19}$$

Substituting equations (18) and (19) into equation (17), we get $\mathbf{P}^{\mathbf{xy}}_{k|k-1}$ in the form of equation (16). □

In light of the above discussion, we have derived a new method known as GFDF to counter cyber-attacks on measurements and delay measurements concurrently. This method replaces the traditional Gaussian filters estimated measurement vector $\hat{\mathbf{z}}_{k|k-1}$, the covariance matrix of the measurement error $\mathbf{P}^{\mathbf{zz}}_{k|k-1}$, and the cross-covariance matrix between the state and measurement $\mathbf{P}^{\mathbf{xz}}_{k|k-1}$ with their counterparts for the altered measurements, namely the estimated measurement vector $\hat{\mathbf{y}}_{k|k-1}$, the covariance matrix of the measurement error $\mathbf{P}^{\mathbf{yy}}_{k|k-1}$, and the cross-covariance matrix between the state and measurement $\mathbf{P}^{\mathbf{xy}}_{k|k-1}$. These values are computed by utilizing the three lemmas. Incorporating measurement irregularities into GFDF allows more precise estimation of the system state even under attack and delayed measurement. The estimated measurement vector $\hat{\mathbf{y}}_{k|k-1}$ considers the impact of the attack on the measurement as well as delayed measurement, while the covariance matrix of the measurement error $\mathbf{P}^{\mathbf{yy}}_{k|k-1}$ captures the measurement uncertainty caused by these irregularities. The cross-covariance matrix between the state and measurement $\mathbf{P}^{\mathbf{xy}}_{k|k-1}$ reflects the relationship between the system state and the altered measurement. For estimating the status of systems affected by cyber-attacks, GFDF is more resilient and accurate. Through the inclusion of the effects of these irregularities in the estimation process, the system becomes more resilient and remains uninterrupted in the scenario of malicious attacks.

Remark 2: The proposed GFDF utilizes some estimate and covariance expressions from past instants, which increases its storage requirement.

IV. STABILITY OF THE GFDF

This section undertakes a stochastic stability analysis of the proposed filter, utilizing the concept of "exponential boundedness in mean square." To accomplish this, we opt for the EKF-based formulation of the proposed filtering algorithm, abbreviated as EKFDF. To begin, we construct a dynamic model for the estimation error of the EKFDF. Subsequently, we demonstrate that the estimation error of the EKFDF remains exponentially bounded in the mean square. To ensure stability, the associated parameters must be bounded, for which a detailed explanation is given in the later part of the manuscript. Prior to continuing, we review the conventional notion used to evaluate the aforementioned stability concept [29].

Statement 1: A stochastic process is said to be exponential bounded in mean square sense if there are real numbers $\theta_1 > 0$, $\theta_2 > 0$, $\phi' > 0$, and $0 < \kappa \leq 1$, and there exists a positive definite function $\mathbb{V}(\boldsymbol{\xi}_k)$ for a stochastic process $\boldsymbol{\xi}_k$, satisfying the following conditions

$$\begin{cases}\theta_1 \left\|\boldsymbol{\zeta}_k\right\|^2 \leq \mathbb{V}(\boldsymbol{\zeta}_k) \leq \theta_2 \left\|\boldsymbol{\zeta}_k\right\|^2 \\ \mathbb{E}\left[\mathbb{V}(\boldsymbol{\zeta}_k)|\boldsymbol{\zeta}_{k-1}\right] - \mathbb{V}(\boldsymbol{\zeta}_{k-1}) \leq \phi' - \kappa\mathbb{V}(\boldsymbol{\zeta}_{k-1}) \leq 0\end{cases} \tag{20}$$

that jointly conclude

$$\mathbb{E}\left[\left\|\boldsymbol{\zeta}_k\right\|^2\right] \leq \frac{\theta_2}{\theta_1}\mathbb{E}\left[\left\|\boldsymbol{\zeta}_0\right\|^2\right](1-\kappa)^k + \frac{\phi'}{\theta_1}\sum_{i=0}^{k-1}(1-\kappa)^i, \tag{21}$$

where $\|\cdot\|$ denotes the L2-norm. For further elaboration, please refer to [29].

Remark 3: equation (21) is the mathematical definition of "exponential boundedness in a mean square" [29]. Therefore, if the stochastic process $\boldsymbol{\zeta}_k$ satisfies this equation, it is stable in the sense of exponential boundedness. Moreover, equation (21) is inferred from equation (20); thus, $\boldsymbol{\zeta}_k$ must satisfy the conditions in equation (20) to be exponentially stable in mean square.

To proceed with the dynamic model for the estimation error of the proposed filter, we recall the traditional EKF parameters [4].

$$\begin{cases}\hat{\mathbf{x}}_{k|k-1} = \Phi(\hat{\mathbf{x}}_{k-1|k-1}) \\ \mathbf{P}_{k|k-1} = \mathbf{F}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1},\end{cases} \tag{22}$$

where $\hat{\mathbf{x}}_{k|k-1}$ and $\mathbf{P}_{k|k-1}$ represent the predicted state and its error covariance, respectively at t_k; $\mathbf{F}_{k-1}$ represents the Jacobian matrix of $\Phi(\mathbf{x}_{k-1})$. Now consider the measurement update parameters [4]

$$\begin{cases}\hat{\mathbf{z}}_{k|k-1} = \Psi(\hat{\mathbf{x}}_{k|k-1}) \\ \mathbf{P}^{\mathbf{zz}}_{k|k-1} = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k \\ \mathbf{P}^{\mathbf{xz}}_{k|k-1} = \mathbf{P}_{k|k-1}\mathbf{H}_k^T \\ \hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right),\end{cases} \tag{23}$$

with $\mathbf{H}_k$ denoting the Jacobian of $\Psi(\mathbf{x}_k)$.

Moreover, the Taylor series approximations for $\Phi(\mathbf{x}_k)$ and $\Psi(\mathbf{x}_k)$ can be given as

$$\begin{cases} \Phi(\mathbf{x}_k) = \Phi(\hat{\mathbf{x}}_{k|k}) + \mathbf{F}_k \mathbf{e}_{k|k} + \mathscr{F}_t(\mathbf{x}_k, \hat{\mathbf{x}}_{k|k}) \\ \Psi(\mathbf{x}_k) = \Psi(\hat{\mathbf{x}}_{k|k-1}) + \mathbf{H}_k \mathbf{e}_{k|k-1} + \mathscr{H}_t(\mathbf{x}_k, \hat{\mathbf{x}}_{k|k-1}), \end{cases} \tag{24}$$

where $\mathbf{e}_{k|k} = \mathbf{x}_k - \hat{\mathbf{x}}_{k|k}$ and $\mathbf{e}_{k|k-1} = \mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}$ are the estimation and prediction errors, respectively; $\mathscr{F}_t(\mathbf{x}_k, \hat{\mathbf{x}}_{k|k})$ and $\mathscr{H}_t(\mathbf{x}_k, \hat{\mathbf{x}}_{k|k-1})$ denote the respective remainder terms.

The posterior estimate for $\mathbf{y}_k$ is $\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}\left(\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1}\right)$, giving $\mathbf{e}_{k|k} = \mathbf{e}_{k|k-1} - \mathbf{K}(\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1})$. Subsequently, from equations (1), (5), (6), (23), and (24), the dynamical model of $\mathbf{e}_{k|k}$ can be obtained as

$$\mathbf{e}_{k|k} = \bar{\mathscr{A}}_k \mathbf{e}_{k-1|k-1} + \bar{\mathscr{B}}_k + \bar{\mathscr{C}}_k + \bar{\mathscr{D}}_k, \tag{25}$$

where $\bar{\mathscr{A}}_k$, $\bar{\mathscr{B}}_k$, $\bar{\mathscr{C}}_k$, and $\bar{\mathscr{D}}_k$, are given by equation (26), as shown at the bottom of the page.

Following Remark 3, the error (25) should satisfy equation (20) for the EKFDF to be exponentially bounded in the mean square. Let us first introduce the following bounds and conditions required to prove the stochastic stability of EKFDF [29].

- $\mathbf{F}_k$ is non-singular $\forall k$.
- Matrices and vectors are bounded via equation (27), as shown at the bottom of the page, where η_1, η_2, τ_1, τ_2, χ_1, χ_2, ξ, $\mathscr{H}$, $\mathscr{P}_1$, $\mathscr{P}_2$, $\mathscr{Q}_1$, $\mathscr{Q}_2$, $\mathscr{R}_1$, and $\mathscr{R}_2$ are real numbers.

Theorem 1: For the bounds presented in equation (27), the stochastic dynamic model $\mathbf{e}_{k|k}$ (equation (25)) remains exponentially bounded in mean square. Alternatively, it satisfies

$$\mathbb{E}\left[\left\|\mathbf{e}_{k|k}\right\|^2\right] \le \frac{\theta_2}{\theta_1}\mathbb{E}\left[\left\|\mathbf{e}_{0|0}\right\|^2\right](1-\kappa)^k + \frac{\phi'}{\theta_1}\sum_{i=0}^{k-1}(1-\kappa)^i, \tag{28}$$

Proof: We now consider the positive definite function as $\mathbb{V}(\mathbf{e}_{k|k}) = \mathbf{e}_{k|k}^T \mathbf{P}_{k|k} \mathbf{e}_{k|k}$, and substitute $\mathbf{e}_{k|k}$ from equation (25). Thus, we can express $\mathbb{V}(\mathbf{e}_{k|k})$ as given in equation (29), as shown at the bottom of the next page. We now adopt the following steps for proving that $\mathbb{V}(\mathbf{e}_{k|k})$ satisfies the conditions given in equation (20).

- Similar to [30], we obtain $\bar{\mathscr{A}}_k^T \mathbf{P}_{k|k}^{-1} \bar{\mathscr{A}}_k \le (1-\kappa)\mathbf{P}_{k-1|k-1}^{-1}$, which further gives $\mathbf{e}_{k-1|k-1}^T \bar{\mathscr{A}}_k^T \mathbf{P}_{k|k}^{-1} \bar{\mathscr{A}}_k \mathbf{e}_{k-1|k-1} \le (1-\kappa)\mathbb{V}(\mathbf{e}_{k-1|k-1})$.
- Note that $\mathbb{V}(\mathbf{e}_{k|k})$ is scalar. Thus, following [30], we calculate: i) $\bar{\mathscr{C}}_k^T \mathbf{P}_{k|k}^{-1}(2\bar{\mathscr{A}}_k \mathbf{e}_{k-1|k-1} + \bar{\mathscr{C}}_k) \le \lambda_1 \xi^2$, ii) $\bar{\mathscr{B}}_k^T \mathbf{P}_{k|k}^{-1} \bar{\mathscr{B}}_k \le \lambda_2$, iii) $2\bar{\mathscr{D}}_k^T \mathbf{P}_{k|k}^{-1}(\bar{\mathscr{A}}_k \mathbf{e}_{k-1|k-1} + \bar{\mathscr{C}}_k) \le \lambda_3\xi^3 + \lambda_4\xi^2 + \lambda_5\xi + \lambda_6$, and iv) $\bar{\mathscr{D}}_k^T \mathbf{P}_{k|k}^{-1} \bar{\mathscr{D}}_k \le \lambda_7\xi^2 + \lambda_8\xi + \lambda_9$, with $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8$, and λ_9 being expressions in terms of η_1, η_2, τ_1, τ_2, χ_1, χ_2, ξ, $\mathscr{H}$, $\mathscr{P}_1$, $\mathscr{P}_2$, $\mathscr{Q}_1$, $\mathscr{Q}_2$, $\mathscr{R}_1$, and $\mathscr{R}_2$. For more details, please refer to [30].
- The expectation operator afterward gives $\mathbb{E}[2\bar{\mathscr{B}}_k^T \mathbf{P}_{k|k}^{-1}(\bar{\mathscr{A}}_k \mathbf{e}_{k-1|k-1} + \bar{\mathscr{C}}_k + \bar{\mathscr{D}}_k)] = 0$, as $\bar{\mathscr{B}}_k$ comprises the noises $\boldsymbol{\omega}_{k-1}$ and $\boldsymbol{\zeta}_k$.

Following the discussion, we obtain (30), as shown at the bottom of the next page.

Let us now define $\phi' = \lambda_3\xi^3 + (\lambda_1 + \lambda_4 + \lambda_7)\xi^2 + (\lambda_5 + \lambda_8)\xi + \lambda_2 + \lambda_6 + \lambda_9$. Subsequently, the above equation satisfies the second condition of equation (20).

Let us now apply the inverse operator and multiply $\mathbf{e}_{k|k}^T$ and $\mathbf{e}_{k|k}$ to the inequality of $\mathbf{P}_{k|k}$ given in equation (27). Thus, we get

$$\frac{1}{\mathscr{P}_2}\|\mathbf{e}_{k|k}\|^2 \le \mathbb{V}(\mathbf{e}_{k|k}) \le \frac{1}{\mathscr{P}_1}\|\mathbf{e}_{k|k}\|^2. \tag{31}$$

Substituting $\theta_1 = 1/\mathscr{P}_u$ and $\theta_2 = 1/\mathscr{P}_L$, the above inequality satisfies the first condition of equation (20).

$$\begin{cases} \bar{\mathscr{A}}_k = (\mathbf{I} - (p_a + p_a' p_d \hat{\delta})\mathbf{K}\mathbf{H}_k)\mathbf{F}_{k-1} \\ \bar{\mathscr{B}}_k = \boldsymbol{\omega}_{k-1} - \mathbf{K}\left((\beta_k + \beta_k'\alpha_k\Delta_k)\boldsymbol{\zeta}_k + \beta_k'\alpha_k'\sum_{i=1}^{d} G_{d,k}(i)\boldsymbol{\zeta}_{k-i}\right) \\ \bar{\mathscr{C}}_k = \mathscr{F}_t(\mathbf{x}_k, \hat{\mathbf{x}}_{k|k}) - \mathbf{K}\left((\beta_k + \beta_k'\alpha_k\Delta_k)\mathscr{H}_t(\mathbf{x}_k, \hat{\mathbf{x}}_{k|k-1}) + \beta_k'\alpha_k'\sum_{i=1}^{d} G_{d,k}(i)\mathscr{H}_t(\mathbf{x}_{k-i}, \hat{\mathbf{x}}_{k-i|k-1})\right) \\ \bar{\mathscr{D}}_k = -\mathbf{K}\Big[(\beta_k + \beta_k'\alpha_k\Delta_k)\mathbf{H}_k\mathbf{e}_{k|k-1} + \beta_k'\alpha_k'\sum_{i=1}^{d} G_{d,k}(i)\mathbf{H}_{k-i}\mathbf{e}_{k-i|k-1} + ((\beta_k + \beta_k'\alpha_k\Delta_k) - (p_a + p_a'p_d\hat{\delta})) \\ \qquad \times\Psi(\hat{\mathbf{x}}_{k|k-1}) + \sum_{i=1}^{d}(\beta_k'\alpha_k' G_{d,k}(i) - p_a'p_d'\Gamma_i)\Psi(\hat{\mathbf{x}}_{k-i|k-1}) + (p_a + p_a'p_d\hat{\delta})\mathbf{H}_k\mathbf{F}_{k-1}\mathbf{e}_{k|k-1}\Big]. \end{cases} \tag{26}$$

$$\begin{cases} \|\boldsymbol{\omega}_k\| \le \eta_1, \quad \|\boldsymbol{\zeta}_k\| \le \eta_2, \quad \|\mathscr{F}_t(\mathbf{x}_{k-1}, \hat{\mathbf{x}}_{k-1|k-1})\| \le \tau_1\|\mathbf{x}_{k-1} - \hat{\mathbf{x}}_{k-1|k-1}\|^2, \quad \|\mathscr{H}_t(\mathbf{x}_k, \hat{\mathbf{x}}_{k|k-1})\| \le \tau_2\|\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}\|^2 \\ \|\mathbf{F}_k\| \le \chi_1, \quad \|\mathbf{H}_k\| \le \chi_2, \quad \left\|\mathbf{x}_{k-1} - \hat{\mathbf{x}}_{k-1|k-1}\right\| = \left\|\mathbf{e}_{k-1|k-1}\right\| \le \xi, \quad \left\|\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}\right\| = \left\|\mathbf{e}_{k|k-1}\right\| \le \xi \\ \left\|\Psi(\hat{\mathbf{x}}_{k|k-1})\right\| \le \mathcal{H}, \quad \mathcal{P}_1\mathbf{I} \le \mathbf{P}_{k|k} \le \mathbf{P}_{k|k-1} \le \mathcal{P}_2\mathbf{I}, \quad \mathcal{Q}_1\mathbf{I} \le \mathbf{Q}_k \le \mathcal{Q}_2\mathbf{I}, \quad \text{and } \mathcal{R}_1\mathbf{I} \le \mathbf{R}_k \le \mathcal{R}_2\mathbf{I}, \end{cases} \tag{27}$$

We now emphasize that equations (30) and (31) altogether satisfy equation (20). Thus, for chosen $\mathbb{V}(\mathbf{e}_{k|k}) = \mathbf{e}_{k|k}^T \mathbf{P}_{k|k} \mathbf{e}_{k|k}$, the estimation error $\mathbf{e}_{k|k}$ (equation (25)) satisfies equation (21), which concludes the exponential boundedness of $\mathbf{e}_{k|k}$. Therefore, the EKFDF remains exponentially bounded in mean square if the inequalities presented in equation (27) holds true.

V. SIMULATION AND RESULTS

In this section, we use the CKF-based formulation of the proposed GFDF to solve nonlinear filtering problem. To compare its performance, benchmark filters including: i) ordinary CKF [6], ii) CKF with FDI attack handling [14], and iii) CKF with delayed measurement handling [20], [21] were considered. We will refer to the CKF-based formulations of as CKF_FA [14], MLCKF [21], and CKF_GD [20].

To validate the improved accuracy of the proposed method as compared to its existing techniques, 200 Monte Carlo (M_c) simulations were conducted. The average of root mean square error (RMSE) and average of mean absolute error (MAE) were selected as performance metrics along with the corresponding execution time. The RMSE and MAE are obtained as

$$\text{RMSE}(\mathbf{x}_k) = \sqrt{\frac{1}{\text{M}_c} \sum_{i=1}^{\text{M}_c} \left\| \hat{\mathbf{x}}_{k|k}^i - \mathbf{x}_k^i \right\|_2^2} \tag{32}$$

$$\text{MAE}(\mathbf{x}_k) = \frac{1}{\text{M}_c} \sum_{i=1}^{\text{M}_c} \left| \hat{\mathbf{x}}_{k|k}^i - \mathbf{x}_k^i \right| \tag{33}$$

We present the results for two simulation examples, such as multiple sinusoid estimation and power system state estimation problem.

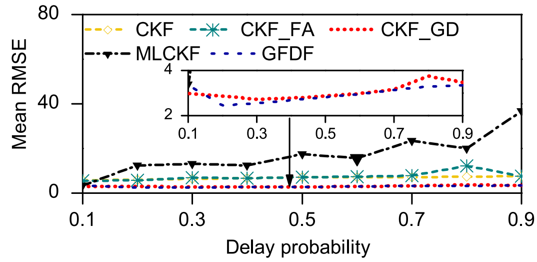
A. MULTIPLE SINUSOIDS ESTIMATION

Consider a problem of estimating multiple sinusoids [14], [20]. The state space model is given as

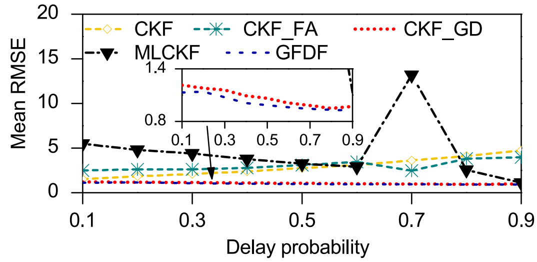
$$\mathbf{x}_k = \mathbf{I}\mathbf{x}_{k-1} + \boldsymbol{\omega}_{k-1} \tag{34}$$

$$\mathbf{z}_k = \left[\sum_{j=1}^{3} a_{j,k} \cos(2\pi f_{j,k} k\tau), \sum_{j=1}^{3} a_{j,k} \sin(2\pi f_{j,k} k\tau) \right]^T + \boldsymbol{\zeta}_k, \tag{35}$$

where $\mathbf{x} = [f_1, f_2, f_3, a_1, a_2, a_3]^T$ contains the frequencies f_i and amplitudes a_i of three sinusoids; $\tau = 0.25$ *ms* is the sampling time. The covariance matrices are assigned as $\mathbf{Q} = diag([\sigma_f^2\ \sigma_f^2\ \sigma_f^2\ \sigma_a^2\ \sigma_a^2\ \sigma_a^2])$ and $\mathbf{R} = diag([\sigma_r^2\ \sigma_r^2])$.



(a) Frequency



(b) Amplitude

FIGURE 1. Case 1: ARMSE comparison of CKF, MLCKF, CKF_FA, CKF_GD, and GFDF against varying delay probabilities.

We perform the simulation over 400 time-steps for two cases i) $p_a = 0.2$, $\sigma_f = \sqrt{25}\ mHz$, $\sigma_a = \sqrt{0.8}\ mV$, and $\sigma_r = \sqrt{0.9}\ V$ and ii) Case 2: $p_a = 0.5$, $\sigma_f = \sqrt{0.9}\ Hz$, $\sigma_a = \sqrt{0.1}\ mV$, and $\sigma_r = \sqrt{0.1}\ V$. The true initial state is chosen as $\mathbf{x}_0 = [200, 500, 1000, 3, 4, 3]^T$, the estimate $\hat{\mathbf{x}}_{0|0}$ are considered to be normally distributed with mean $\mathbf{x}_0$ and covariance $\mathbf{P}_{0|0} = diag\big([20, 20, 20, 0.5, 0.5, 0.5]\big)$. During the simulation, we set Δ_k to follow a normal distribution with a mean of 0.5 and a variance of 0.4. In the presence of an attack, we assume $p_d = 0.5$. The mean RMSEs for amplitudes and frequencies are compared.

The mean RMSEs for all filters for two cases are presented in Figures 1 and 2. The figures indicate that the proposed GFDF achieves improved accuracy compared to all existing filters. The computation times relative to CKF, MLCKF, CKF_FA, CKF_GD, and GFDF are 1, 1.4937, 1.3254, 1.3253, and 1.3289, respectively. This implies that the computational time of the proposed method is slightly increased compared to the traditional CKF, however remains comparable to existing CKF extensions for handling these irregularities.

We presented the results obtained in this study and derived some noteworthy conclusions. Figures 1 and 2 demonstrate

$$\begin{aligned}\mathbb{V}(\mathbf{e}_{k|k}) = {} & \mathbf{e}_{k-1|k-1}^T \bar{\mathscr{A}}_k^T \mathbf{P}_{k|k}^{-1} \bar{\mathscr{A}}_k \mathbf{e}_{k-1|k-1} + \bar{\mathscr{C}}_k^T \mathbf{P}_{k|k}^{-1} \big(2\bar{\mathscr{A}}_k \mathbf{e}_{k-1|k-1} + \bar{\mathscr{C}}_k\big) + 2\bar{\mathscr{B}}_k^T \mathbf{P}_{k|k}^{-1} \big(\bar{\mathscr{A}}_k \mathbf{e}_{k-1|k-1} + \bar{\mathscr{C}}_k + \bar{\mathscr{D}}_k\big) \\ & + \bar{\mathscr{B}}_k^T \mathbf{P}_{k|k}^{-1} \bar{\mathscr{B}}_k + 2\bar{\mathscr{D}}_k^T \mathbf{P}_{k|k}^{-1} \big(\bar{\mathscr{A}}_k \mathbf{e}_{k-1|k-1} + \bar{\mathscr{C}}_k\big) + \bar{\mathscr{D}}_k^T \mathbf{P}_{k|k}^{-1} \bar{\mathscr{D}}_k.\end{aligned} \tag{29}$$

$$\mathbb{E}[\mathbb{V}(\mathbf{e}_{k|k})|\mathbf{e}_{k-1|k-1}] - \mathbb{V}(\mathbf{e}_{k-1|k-1}) \leq \lambda_3 \xi^3 + (\lambda_1 + \lambda_4 + \lambda_7)\,\xi^2 + (\lambda_5 + \lambda_8)\,\xi + \lambda_2 + \lambda_6 + \lambda_9 - \kappa \mathbb{V}(\mathbf{e}_{k-1|k-1}). \tag{30}$$

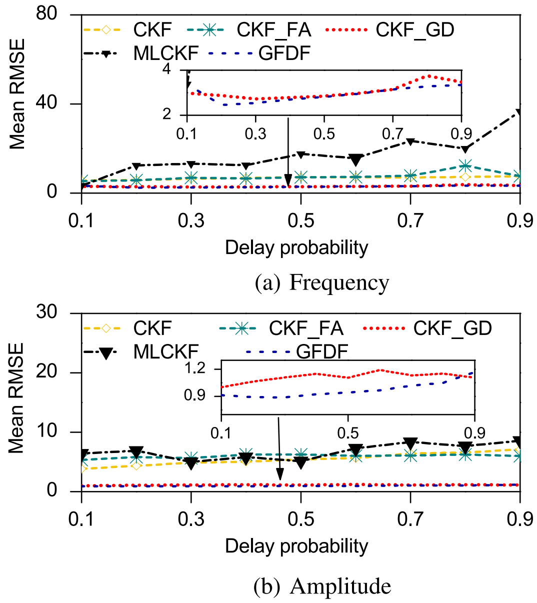


FIGURE 2. Case 2: ARMSE comparison of CKF, MLCKF, CKF_FA, CKF_GD, and GFDF against varying delay probabilities.

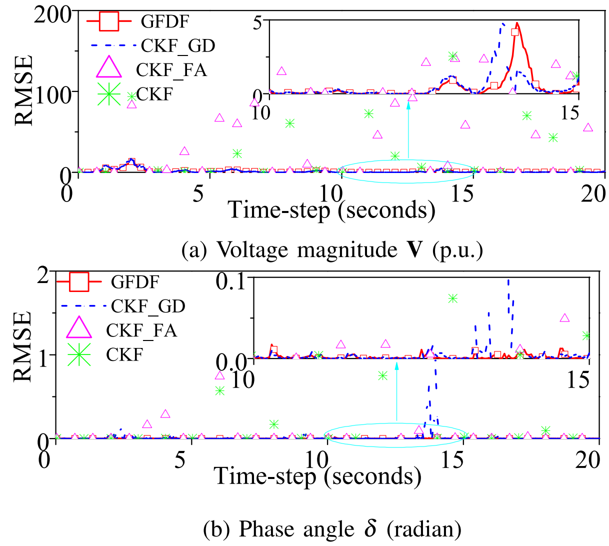


FIGURE 3. RMSE comparison of CKF, CKF_FA, CKF_GD, and GFDF at Bus-9 for 0.3 delay probabilities.

that the accuracy, as measured by ARMSE, deteriorates with an increase in delay probability (p_g), as expected. This implies that the filtering performance is not significantly impacted if only a few measurements experience delay while others are time-synchronized. Similar observations have been made in [20] and [21]. It is important to note that this trend may vary depending on the system dynamics and environmental factors. To this end, it can be inferred that the current FDI attack methodology fails to address delayed measurements adequately. On the other hand the proposed GFDF method demonstrates its viability in simultaneously addressing the challenges posed by FDI attacks and delayed measurements.

B. POWER SYSTEM STATE ESTIMATION

In this section, we estimate the power system states, namely the voltage magnitude and phase angle at each bus or node, using a limited number of noisy measurements. The analysis is performed on the IEEE 14-bus benchmark power system network. Please refer to [31] and [32] for more details.

Table 2 presents the measurement locations of phasor measurement units (PMUs) and remote terminal units (RTUs). The PMUs are placed at specific locations to measure voltage and current phasors ($\mathbf{V}_r$, $\mathbf{V}_i$, $\mathbf{I}_r$, and $\mathbf{I}_i$), while the RTUs provide power injections ($\mathbf{P}^i$ and $\mathbf{Q}^i$) at the installed buses and power flows ($\mathbf{P}^f$ and $\mathbf{Q}^f$) through the specified branches. The detailed expressions for these measurements can be found in [31]. This simulation considers that RTU data packets are updated every two seconds, and 30 PMU data packets were received between two adjacent RTU data packets. The proposed GFDF-based PSSE was implemented over 40 seconds at the PMU scan rate by incorporating the most recent PMU sensor data with the last available RTU sensor data.

The power system state dynamics are simplified as a random walk model due to the low likelihood of significant changes occurring between successive PMU scans. This simulation considers a 3% randomly changing load condition throughout the simulation period. For validating the proposed approach for large and random voltage fluctuations, a significantly larger process noise covariance, $\mathbf{Q}_k = 9 \times 10^{-6}\mathbf{I}_{n\times n}$, is considered rather than a value mentioned in [32]. Let us consider δ_v^r, δ_{pi}^r, and δ_{pf}^r represent the standard deviations of sensor noises for RTU measured voltage, power injection, and power flow, respectively, and δ_v^p and δ_i^p are the corresponding values for PMU measured voltage and current, respectively. To characterize sensor noises, the following values were considered [31]: $\delta_v^r = 0.001$, $\delta_{pi}^r = 0.02$, $\delta_{pf}^r = 0.02$, $\delta_v^p = 0.001$, and $\delta_i^p = 0.001$. The simulations were carried out with true initial bus voltages of $\mathbf{x}_0 = 1\angle 0°$ and the PSSE was performed with $\hat{\mathbf{x}}_{0|0} = \mathbf{x}_0$ and $\mathbf{P}_{0|0} = 10^{-6}\mathbf{I}_{n\times n}$. During the simulation, we set Δ_k to follow a normal distribution with a mean of 0.5 and a variance of 0.4. In the presence of an attack, we assume $p_a = p_d = 0.5$.

We conducted a comparative study of the proposed GFDF-based PSSE and other existing filters, as shown in Figures 3 and 4, and Table 3. The RMSE plots for voltage magnitudes and voltage phase angles at bus 9 ($\mathbf{V}_{k,9}$ and $\delta_{k,9}$) are displayed in Figure 3 and 4 for delay probabilities of 0.3 and 0.5, respectively. These plots reveal that the CKF_FA and CKF filter has significantly higher RMSEs as it did not consider falsely injected data, resulting in poor performance compared to other filters. Furthermore, Figures 3 and 4 demonstrate that CKF_GD exhibit inferior estimation performance compared to the proposed GFDF-based PSSE.

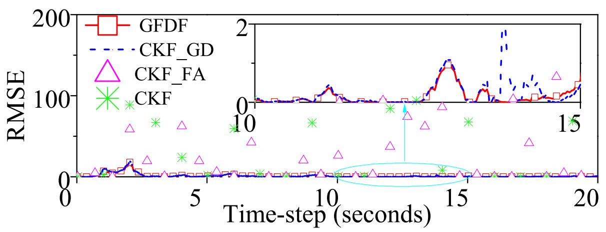
Table 3 presents ARMSE and AMAE of all considered filters for varying probabilities of delay. The results show that

TABLE 2. PMU and RTU measurements locations for the IEEE 14-bus benchmark power system networks.

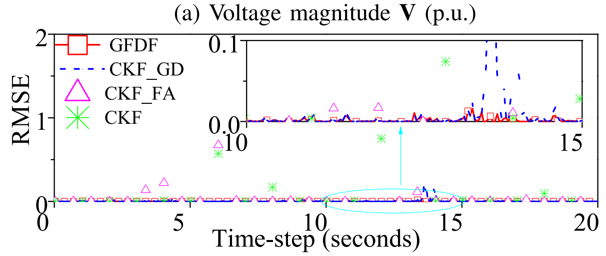
PMU		RTU
$\mathbf{V}_r$, $\mathbf{V}_i$, $\mathbf{I}_r$, and $\mathbf{I}_i$	$\mathbf{P}^i$ and $\mathbf{Q}^i$	$\mathbf{P}^f$ and $\mathbf{Q}^f$
2, 7, 9, 13	3, 5, 13, 14	1-5, 2-1, 2-5, 3-4, 4-5, 4-7, 6-11, 6-12, 6-13, 8-7, 9-4, 9-7, 9-10, 9-14, 10-11, 12-13, 13-14

TABLE 3. Performance indices (in 10^{-3}) of the proposed GFDF-based PSSE with the existing benchmark filters obtained by averaging the voltage magnitude (V) and phase angle δ across all buses.

Delay probability (p_g)	Error	V				δ			
		GFDF	CKF_GD	CKF_FA	CKF	GFDF	CKF_GD	CKF_FA	CKF
0.1	ARMSE	22	24	114	508	1.7	2.7	34.1	80.6
0.2		22.1	24.9	113.7	508	1.4	2.6	34.1	80.8
0.3		22.9	24.8	113.6	508.5	1.1	2.2	34.1	80.9
0.4		21.8	23.6	113.4	508	0.9	1.9	34.4	80.8
0.5		22.7	24	113.4	508.6	0.9	2	34.3	81.1
0.1	AMAE	3.55	3.61	18.4	361	0.238	0.366	1.88	22.33
0.2		3.575	3.905	18.28	360.7	0.18	0.27	1.84	21.92
0.3		3.68	3.97	18.27	361.2	0.16	0.23	1.82	21.75
0.4		3.49	3.70	18.21	361.1	0.148	0.212	1.825	21.643
0.5		3.491	3.70	18.21	361.27	0.143	0.2	1.815	21.608



(a) Voltage magnitude **V** (p.u.)



(b) Phase angle δ (radian)

FIGURE 4. RMSE comparison of CKF, CKF_FA, CKF_GD, and GFDF at Bus-9 for 0.5 delay probabilities.

the proposed GFDF-based PSSE achieves the lowest ARMSE and AMAE. Moreover, the table indicates that the filtering performance deteriorates with an increase in the probability of delay.

The computation times relative to CKF, CKF_FA, CKF_GD, and GFDF are 1, 1.051, 1.063, and 1.171, respectively. This implies that the computational time of the proposed GFDF is marginally higher than traditional CKF but remains comparable to other existing CKF extensions for handling these irregularities.

VI. DISCUSSION AND CONCLUSION

The practical applications of nonlinear estimation and filtering are vast, spanning fields such as defence, power, and network systems. However, the widely accepted Gaussian filtering method falls short in accounting for irregular measurements caused by delay and cyber-attacks. Given such irregularities commonly observed in practical measurements, the applying an advanced Gaussian filtering method becomes crucial.

We introduce a method that employs stochastic modeling of delayed and cyber-attack measurements to meet this need. The proposed stochastic model employs a Bernoulli random variable to indicate whether a measurement has been altered, either through an FDI attack or delay. We then redesigned the traditional Gaussian filtering method to account for these modified measurements.

The detailed analysis demonstrates that the proposed method outperforms traditional Gaussian filtering, resulting in improved estimation accuracy even in the presence of both delay and cyber-attacks. However, it is important to note that the computational budget and storage requirement of the proposed method are higher compared to the traditional Gaussian filtering method.

REFERENCES

[1] A. H. Jazwinski, *Stochastic Processes and Filtering Theory*. North Chelmsford, MA, USA: Courier Corp., 2007.

[2] B. Jia and M. Xin, *Grid-Based Nonlinear Estimation and Its Applications*. Boca Raton, FL, USA: CRC Press, 2019.

[3] S. Bhaumik and P. Date, *Nonlinear Estimation: Methods and Applications With Deterministic Sample Points*. Boca Raton, FL, USA: CRC Press, 2019.

[4] B. D. Anderson and J. B. Moore, *Optimal Filtering*. North Chelmsford, MA, USA: Courier Corp., 2012.

[5] S. J. Julier and J. K. Uhlmann, ''Unscented filtering and nonlinear estimation,'' *Proc. IEEE*, vol. 92, no. 3, pp. 401–422, Mar. 2004.

[6] I. Arasaratnam and S. Haykin, ''Cubature Kalman filters,'' *IEEE Trans. Autom. Control*, vol. 54, no. 6, pp. 1254–1269, Jun. 2009.

[7] I. Arasaratnam, S. Haykin, and R. J. Elliott, ''Discrete-time nonlinear filtering algorithms using Gauss–Hermite quadrature,'' *Proc. IEEE*, vol. 95, no. 5, pp. 953–977, May 2007.

[8] G. Kumar, P. Date, R. B. Pachori, R. Swaminathan, and A. K. Singh, ''Wrapped particle filtering for angular data,'' *IEEE Access*, vol. 10, pp. 90287–90298, 2022.

[9] A. Teixeira, I. Shames, H. Sandberg, and K. H. Johansson, ''A secure control framework for resource-limited adversaries,'' *Automatica*, vol. 51, pp. 135–148, Jan. 2015.

[10] Z. Zhao, P. X. Liu, and J. Gao, ''Model-based fault diagnosis methods for systems with stochastic process—A survey,'' *Neurocomputing*, vol. 513, pp. 137–152, Nov. 2022.

[11] H. Fawzi, P. Tabuada, and S. Diggavi, ''Secure estimation and control for cyber-physical systems under adversarial attacks,'' *IEEE Trans. Autom. Control*, vol. 59, no. 6, pp. 1454–1467, Jun. 2014.

[12] X. Cai, K. Shi, K. She, S. Zhong, Y. C. Soh, and Y. Yu, ''Performance error estimation and elastic integral event triggering mechanism design for T–S fuzzy networked control system under DoS attacks,'' *IEEE Trans. Fuzzy Syst.*, vol. 31, no. 4, pp. 1327–1339, Apr. 2023.

[13] X. Cai, K. Shi, K. She, S. Zhong, and Y. Tang, ''Quantized sampled-data control tactic for T-S fuzzy NCS under stochastic cyber-attacks and its application to truck-trailer system,'' *IEEE Trans. Veh. Technol.*, vol. 71, no. 7, pp. 7023–7032, Jul. 2022.

[14] A. K. Singh, S. Kumar, N. Kumar, and R. Radhakrishnan, ''Bayesian approximation filtering with false data attack on network,'' *IEEE Trans. Aerosp. Electron. Syst.*, vol. 58, no. 2, pp. 976–988, Apr. 2022.

[15] S. Chen, Q. Zhang, D. Lin, and S. Wang, ''A class of nonlinear Kalman filters under a generalized measurement model with false data injection attacks,'' *IEEE Signal Process. Lett.*, vol. 29, pp. 1187–1191, 2022.

[16] S. Padhan and A. K. Turuk, ''Design of false data injection attacks in cyber-physical systems,'' *Inf. Sci.*, vol. 608, pp. 825–843, Aug. 2022.

[17] A. Tajer, ''False data injection attacks in electricity markets by limited adversaries: Stochastic robustness,'' *IEEE Trans. Smart Grid*, vol. 10, no. 1, pp. 128–138, Jan. 2019.

[18] A. Hermoso-Carazo and J. Linares-Pérez, ''Extended and unscented filtering algorithms using one-step randomly delayed observations,'' *Appl. Math. Comput.*, vol. 190, no. 2, pp. 1375–1393, Jul. 2007.

[19] A. Hermoso-Carazo and J. Linares-Pérez, ''Unscented filtering algorithm using two-step randomly delayed observations in nonlinear systems,'' *Appl. Math. Model.*, vol. 33, no. 9, pp. 3705–3717, Sep. 2009.

[20] G. Kumar, S. K. Nanda, A. K. Verma, V. Bhatia, and A. K. Singh, ''Nonlinear Gaussian filtering with network-induced delay in measurements,'' *IEEE Trans. Aerosp. Electron. Syst.*, vol. 58, no. 6, pp. 5974–5981, Dec. 2022.

[21] R. Esmzad and R. Mahboobi Esfanjani, ''Bayesian filter for nonlinear systems with randomly delayed and lost measurements,'' *Automatica*, vol. 107, pp. 36–42, Sep. 2019.

[22] G. Kumar, V. G. Yamalakonda, and A. K. Singh, ''Fractionally delayed Bayesian approximation filtering under non-Gaussian noisy environment,'' *IEEE Trans. Aerosp. Electron. Syst.*, early access, Apr. 11, 2023, doi: 10.1109/TAES.2023.3266176.

[23] S. Liu, G. Wei, Y. Song, and Y. Liu, ''Extended Kalman filtering for stochastic nonlinear systems with randomly occurring cyber attacks,'' *Neurocomputing*, vol. 207, pp. 708–716, Sep. 2016.

[24] S. K. Nanda, G. Kumar, V. Bhatia, and A. K. Singh, ''Kalman filtering with delayed measurements in non-Gaussian environments,'' *IEEE Access*, vol. 9, pp. 123231–123244, 2021.

[25] D. Wang, Z. Wang, B. Shen, and F. E. Alsaadi, ''Security-guaranteed filtering for discrete-time stochastic delayed systems with randomly occurring sensor saturations and deception attacks,'' *Int. J. Robust Nonlinear Control*, vol. 27, no. 7, pp. 1194–1208, May 2017.

[26] W. Song, Z. Wang, J. Wang, F. E. Alsaadi, and J. Shan, ''Secure particle filtering for cyber-physical systems with binary sensors under multiple attacks,'' *IEEE Syst. J.*, vol. 16, no. 1, pp. 603–613, Mar. 2022.

[27] Z. Zhang, R. Deng, P. Cheng, and Q. Wei, ''On feasibility of coordinated time-delay and false data injection attacks on cyber–physical systems,'' *IEEE Internet Things J.*, vol. 9, no. 11, pp. 8720–8736, Jun. 2022.

[28] A. Sargolzaei, B. C. Allen, C. D. Crane, and W. E. Dixon, ''Lyapunov-based control of a nonlinear multiagent system with a time-varying input delay under false-data-injection attacks,'' *IEEE Trans. Ind. Informat.*, vol. 18, no. 4, pp. 2693–2703, Apr. 2022.

[29] K. Reif, S. Gunther, E. Yaz, and R. Unbehauen, ''Stochastic stability of the discrete-time extended Kalman filter,'' *IEEE Trans. Autom. Control*, vol. 44, no. 4, pp. 714–728, Apr. 1999.

[30] A. K. Naik, G. Kumar, P. K. Upadhyay, P. Date, and A. K. Singh, ''Gaussian filtering for simultaneously occurring delayed and missing measurements,'' *IEEE Access*, vol. 10, pp. 100746–100762, 2022.

[31] A. Sharma, S. C. Srivastava, and S. Chakrabarti, ''A cubature Kalman filter based power system dynamic state estimator,'' *IEEE Trans. Instrum. Meas.*, vol. 66, no. 8, pp. 2036–2045, Aug. 2017.

[32] A. M. L. da Silva, M. B. D. C. Filho, and J. F. de Queiroz, ''State forecasting in electric power systems,'' *IIEE Proc. C Gener., Transmiss. Distrib.*, vol. 130, no. 5, pp. 237–244, Sep. 1983.

SUMANTA KUMAR NANDA received the B.Tech. degree in electrical engineering from the Biju Pattanaik University of Technology, India, in 2010, and the M.Tech. degree from the Department of Electrical Engineering, Indian Institute of Technology ISM Dhanbad, India, in 2015. He is currently pursuing the Ph.D. degree with the Department of Electrical Engineering, Indian Institute of Technology Indore, India. His research interest includes the Kalman filter and its application for power system state estimation and epidemic state estimation.

GUDDU KUMAR received the B.Tech. degree in electronics and communication engineering from the West Bengal University of Technology, West Bengal, India, in 2014, the M.Tech. degree from the Department of Electronics and Communication Engineering, NIT Patna, India, in 2019, and the Ph.D. degree in electrical engineering from IIT Indore, MP, India. He is currently a Post-Doctoral Research with the University of Oulu, Finland. His research interests include estimation and filtering, communication and sensing.

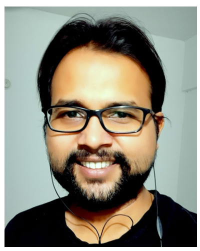
AMIT KUMAR NAIK received the B.E. degree in electrical and electronics engineering from the Bhilai Institute of Technology Raipur, Raipur, India, in 2016, and the M.Tech. degree in power systems from the Maulana Azad National Institute of Technology Bhopal, India, in 2019. He is currently pursuing the Ph.D. degree with the Department of Electrical Engineering, Indian Institute of Technology Indore, Indore, India. His research works focus on nonlinear filtering and stochastic stability analysis.

MOHAMMED ABDEL-HAFEZ received the B.Sc., M.Sc., and Ph.D. degrees in electrical and electronic engineering from Eastern Mediterranean University (EMU), Northern Cyprus, Turkey, in June 1992, August 1994, and November 1997, respectively. From 1992 to 1997, he was a Research Engineer with the Department of Electrical and Electronic Engineering, EMU. In 1995, he was an Instructor with the Department of Electrical Engineering, Jerusalem University. From 1997 to 1999, he was a Data Communication Section Manager with Palestine Telecommunication Company (PALTEL). In August 1999, he joined the Centre for Wireless Communications, University of Oulu, as a Senior Research Scientist and a Project Manager. He is currently an Associate Professor of electrical engineering with United Arab Emirates University, United Arab Emirates. He is also an Associate Professor with the Department of Electrical Engineering (Communications), United Arab Emirates University. He is a frequent Visiting Professor with the Centre for Wireless Communications, University of Oulu, Oulu, Finland. His research interests include performance analysis of wireless communication systems, future broadband wireless systems, wireless sensor networks, the IoT, smart grid communication, 5G radio access, cooperative communication systems, cognitive radio networks, advanced receivers' algorithms, digital signals processing, and ultra-wide band (UWB) systems.

VIMAL BHATIA (Senior Member, IEEE) received the Ph.D. degree from the Institute for Digital Communications, The University of Edinburgh, Edinburgh, U.K., in 2005.

During the Ph.D. degree, he received the IEE Fellowship for collaborative research with the Department of Systems and Computer Engineering, Carleton University, Canada. He is currently a Professor with the Indian Institute of Technology (IIT) Indore, India. He is also an Adjunct Faculty with IIT Delhi and IIIT Delhi, India. He is also a DRISHTI CPS Chair Professor with IIT Indore, and also chair Professor at the the School of Electronic and Information Engineering, Soochow University, China. He has worked with various IT companies for over 11 years both in India and the U.K. He has delivered many talks, and tutorials, conducted faculty development programs for the World Bank's NPIU TEQIP-III, and invited talk at WWRF46-Paris. He is a PI/co-PI/coordinator for external projects with funding of over U.S. $20 million from MeitY, DST, UKIERI, MoE, AKA, IUSSTF, and KPMG. He has more than 350 peer-reviewed publications and has filed 13 patents (with eight granted). He has supervised 18 awarded Ph.D. theses. His research interests include communications, non-Gaussian non-parametric signal processing, machine/deep learning with applications to communications, and photonics. He is a fellow of IETE and OSI. He is a Young Faculty Research Fellow from MeitY, Government of India. He was a recipient of the Prof. SVC Aiya Memorial Award, in 2019. He is a certified SCRUM Master. He is a reviewer of IEEE, Elsevier, Wiley, Springer, and IET. He is also an Associate Editor of *IETE Technical Review*, *Frontiers in Communications and Networks*, *Frontiers in Signal Processing*, and IEEE Wireless Communications Letters.

ONDREJ KREJCAR received the Ph.D. degree in technical cybernetics from the Technical University of Ostrava, Czech Republic, in 2008. From 2016 to 2020, he was the Vice-Dean of Science and Research with the Faculty of Informatics and Management, University of Hradec Kralove (UHK), Czech Republic. He has been the Vice-Rector for science and creative activities of UHK, since June 2020. He is currently a Full Professor of systems engineering and informatics with the Center for Basic and Applied Research, Faculty of Informatics and Management, UHK, and a Research Fellow with the Malaysia-Japan International Institute of Technology, University Technology Malaysia, Kuala Lumpur, Malaysia. He is also the Director of the Center for Basic and Applied Research, UHK. His research interests include control systems, smart sensors, ubiquitous computing, manufacturing, wireless technology, portable devices, biomedicine, image segmentation and recognition, biometrics, technical cybernetics, ubiquitous computing, biomedicine (image analysis), as well as biotelemetric system architecture (portable device architecture, wireless biosensors), and development of applications for mobile devices with the use of remote or embedded biomedical sensors. He has been a Management Committee Member Substitute at Project COST CA16226, since 2017. He has been the Vice-Leader and a Management Committee Member of WG4 at Project COST CA17136, since 2018. Since 2019, he has been the Chairperson of the Program Committee of the KAPPA Program and a Technological Agency of the Czech Republic as a Regulator of the EEA/Norwegian Financial Mechanism in the Czech Republic, from 2019 to 2024. From 2014 to 2019, he was the Deputy Chairperson of Panel 7 (Processing Industry, Robotics, and Electrical Engineering) of the Epsilon Program, Technological Agency of the Czech Republic. Since 2020, he has been the Chairperson of Panel 1 (Computer, Physical and Chemical Sciences) of the ZETA Program, Technological Agency of the Czech Republic. He is also on the editorial board of the *Sensors* (MDPI) IF journal (Q1/Q2 at JCR) and several other ESCI-indexed journals. In 2018, he was the 14th Top Peer-Reviewer in Multidisciplinary in the World according to Publons, and a Top Reviewer in the Global Peer Review Awards 2019 by Publons.



ABHINOY KUMAR SINGH (Member, IEEE) received the B.Tech. degree in electrical and electronics engineering from the Cochin University of Science and Technology (CUSAT), Kochi, India, in 2012, and the Ph.D. degree in electrical engineering from the Indian Institute of Technology Patna, Patna, India, in August 2016. He worked as a Post-Doctoral Researcher at McGill University, Canada, between 2017 to 2018. Before joining IIT Patna, he also worked at Shiv Nadar University, India and IIT Patna, India, in different Teaching Positions. He has supervised several Ph.D. students, one has been awarded the degree and one has submitted the thesis. He is PI/Co-PI for external projects (ongoing/completed) with funding over U.S. $ 2.5 million. He is currently working at IIT Patna as an Assistant Professor. He has published more than 40 peer-reviewed papers, including 25 journals, two book chapters, and 16 conferences. His research interests include stochastic control theory, estimation/filtering, and continuous glucose monitoring. He was a recipient of Inspire Faculty Award in 2018, which is awarded by the Department of Science and Technology, Government of India. He is a reviewer of several journals belonging to the publishers like IEEE, Elsevier, IET, and Springer.