Wei Liu, Martin Haardt, Maria S. Greco,
Christoph F. Mecklenbräuker, and Peter Willett

Twenty-Five Years of Sensor Array and Multichannel Signal Processing

A review of progress to date and potential research directions

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In this article, a general introduction to the area of sensor array and multichannel signal processing is provided, including associated activities of the IEEE Signal Processing Society (SPS) Sensor Array and Multichannel (SAM) Technical Committee (TC). The main technological advances in five SAM subareas made in the past 25 years are then presented in detail, including beamforming, direction-of-arrival (DOA) estimation, sensor location optimization, target/source localization based on sensor arrays, and multiple-input multiple-output (MIMO) arrays. Six recent developments are also provided at the end to indicate possible promising directions for future SAM research, which are graph signal processing (GSP) for sensor networks; tensor-based array signal processing, quaternion-valued array signal processing, 1-bit and noncoherent sensor array signal processing, machine learning and artificial intelligence (AI) for sensor arrays; and array signal processing for next-generation communication systems.

Introduction

Sensor array and multichannel signal processing has a long history, with typical research topics including beamforming and DOA estimation at its early stage and corresponding representative algorithms, including the Capon beamformer/linearly constrained minimum variance (LCMV) beamformer and the MUSIC/ESPRIT algorithms [1], [2], [3], [4], [5]. The past 25 years have seen an explosive growth of research activities in this area, and significant progress has been made in a wide range of theoretical and application areas of sensor array and multichannel signal processing. Although, traditionally, the areas' applications have been mainly limited to the defense sector, such as radar and sonar, today, we can see their impact in everyday life, including beamforming for ultrasound imaging, synthetic aperture radar for remote sensing, vehicular radar (ultrasound and electromagnetic) for autonomous driving, microphone arrays for human-machine interfaces (a good example is the Amazon Echo), and MIMO antenna arrays for Wi-Fi and mobile communications standards (IEEE 802.11n, IEEE 802.11ac, 3G, WiMax, and LTE).

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As a result, the sensor array and multichannel signal processing research area has expanded significantly in the past years, as reflected by the scope of the SPS SAM TC. The SAM TC, formed in 2000, aims to promote activities within the technical fields of sensor array processing and multichannel statistical signal processing [6], including beamforming and space-time adaptive processing; DOA estimation; source separation; target detection; localization and tracking; MIMO signal processing; array processing for radar, sonar, and communications; and many other applications of multisensor and synthetic aperture systems, as indicated by the list of editors' information classification schemes covered by the TC (https://signalprocessingsociety.org/community-involvement/sensor-array-and-multichannel/edics).

The SAM TC organizes two biennial workshops dedicated to the SAM research area: the IEEE International Workshop on Computational Advances in Multisensor Adaptive Processing (CAMSAP), organized in December every odd-numbered year since 2005, and the IEEE Sensor Array and Multichannel Signal Processing Workshop, organized in June/July every even-numbered year since 2002, each accepting 100–200 research papers. Due to the COVID-19 pandemic, CAMSAP 2021, originally scheduled for December 2021, in Costa Rica, was postponed to December 2023. The next SAM workshop (SAM 2024) will be held in the United States, with two possible venues: Oregon State University, Corvallis, OR, and Skamania Lodge, Stevenson, WA. Moreover, at each year's ICASSP conference, the SAM track also receives about 100–200 regular submissions. Currently, there is also the Synthetic Aperture Technical Working Group, which resides under the SAM TC, with the goal of "supporting the maturation of the theoretical framework and the associated empirical techniques that underpin the estimation of parameters of propagating waves through various media using synthetic apertures."

In this article, as it is not possible to give an exhaustive list of all the advances made in the SAM area, we focus on five major topics and introduce the corresponding progress made in tackling their respective technical challenges: beamforming [including robust adaptive beamforming and frequency-invariant beamforming (FIB)], DOA estimation (including sparsity-based and underdetermined DOA estimation), sensor location optimization, target/source localization based on sensor arrays, and MIMO arrays (including MIMO radar and MIMO for wireless communications). The first two are classic SAM topics from the very beginning of SAM research, as mentioned earlier, while the latter three were studied systematically only in the past decades. Then, six new developments in the SAM area are presented to give an indication about possible future research directions, including GSP for sensor networks, tensor-based array signal processing, quaternion-valued array signal processing, 1-bit and noncoherent sensor array signal processing, machine learning and AI for sensor arrays, and array signal processing for next-generation communication systems.

This article is structured as follows. The five main technological advances are introduced in detail in the "Main Technological Advances in the SAM Area" section, followed by the six new developments in the "New Developments in the SAM Area" section and some concluding remarks in the "Concluding Remarks" section.

Main technological advances in the SAM area

In this section, advances made in the five major SAM research topics in the past 25 years are presented, including beamforming, DOA estimation, sensor location optimization, target/source localization based on sensor arrays, and MIMO arrays.

Beamforming

Beamforming is a classic sensor array signal processing problem and a core SAM topic [1], [2], [3], [4], [5], and it has been studied extensively at least for a century. It can be classified into narrowband and wideband beamforming according to the relative bandwidth of the signals, adaptive and fixed beamforming according to its relationship with the received data, and analog and digital beamforming according to its circuits implementation. In the past 25 years, three main developments have been achieved, including robust adaptive beamforming [7], FIB [5], and hybrid beamforming [8], which is a combination of digital and analog beamforming techniques. In this section, we discuss the first two in detail and leave the topic of hybrid beamforming to the section about MIMO arrays.

Robust adaptive beamforming

In general, for the narrowband case, for an M-sensor array with K impinging signals, the received array signals can be formulated into the following form:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{x}(t) = [x_1(t), \ldots, x_M(t)]^T$ is the received signal vector, $\mathbf{A}$ is the steering matrix consisting of K steering vectors $\mathbf{a}(\theta)$ corresponding to the K source signals [θ represents the angle of arrival (AOA) of an arbitrary impinging signal], and $\mathbf{n}(t)$ is the noise vector.

Then, the beamformer output $y(t)$ is given by an instantaneous linear combination of the received spatial samples $x_m(t)$, as follows:

$$y(t) = \sum_{m=1}^{M} x_m(t) w_m^* = \mathbf{w}^H \mathbf{x}(t) \tag{2}$$

where w_m is the weight coefficient for the mth received sensor signal, with the weight vector $\mathbf{w} = [w_1, \ldots, w_M]^T$.

The Capon beamformer, which can be considered a special case of the more general LCMV beamformer [1], [2], [3], [4], [5], can achieve effective adaptive beamforming when the DOA angle θ_0 of the desired signal is exactly known, and the following is the standard formulation:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{a}(\theta_0) = 1 \tag{3}$$

where $\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\}$ is the covariance matrix and $\mathbf{a}(\theta_0)$ is the steering vector of the array at θ_0. In practice, since $\mathbf{R}$ is usually not available, as an approximation, it is replaced by the sample covariance matrix $\hat{\mathbf{R}}$, which is obtained through the finite number of data samples.

However, the Capon beamformer is very sensitive to model mismatch errors, such as DOA error for the desired signal, mutual coupling, general array manifold errors, and finite sample effects in covariance matrix estimation, and therefore, various robust adaptive beamforming techniques have been developed [7]. One well-known technique is diagonal loading, with the weight vector expressed as $\alpha(\hat{\mathbf{R}}+\xi\mathbf{I})^{-1}\mathbf{a}(\theta_0)$, with α being a constant, ξ the diagonal loading factor, and $\mathbf{I}$ the identity matrix.

One prominent development in this area in the past 25 years is the worst-case-based robust adaptive beamformer [9], where, instead of constraining the beamformer response to be unity at the desired signal direction, the response is forced to exceed unity within an uncertainty set of steering vectors, which can be expressed as

$$\min_{\mathbf{w}} \mathbf{w}^H\hat{\mathbf{R}}\mathbf{w} \quad \text{subject to} \quad \left|\mathbf{w}^H\tilde{\mathbf{a}}\right| \geq 1,\ \forall\tilde{\mathbf{a}}\in\mathcal{A}$$
$$\mathcal{A}=\{\tilde{\mathbf{a}}|\tilde{\mathbf{a}}=\mathbf{a}(\theta_0)+\mathbf{e}, \|\mathbf{e}\|\leq\varepsilon\} \qquad (4)$$

where $\tilde{\mathbf{a}}$ is the possible actual steering vector of the desired signal corresponding to the presumed steering vector $\mathbf{a}(\theta_0)$, $\mathcal{A}$ is the full set that $\tilde{\mathbf{a}}$ belongs to, and $\mathbf{e}$ is the steering vector error, with its norm bounded by ε. The problem is then converted to the following form using the worst-case optimization:

$$\min_{\mathbf{w}} \mathbf{w}^H\hat{\mathbf{R}}\mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H\mathbf{a}(\theta_0)\geq\varepsilon\|\mathbf{w}\|+1$$
$$\mathrm{Im}\{\mathbf{w}^H\mathbf{a}(\theta_0)\}=0 \qquad (5)$$

where $\mathrm{Im}\{\cdot\}$ denotes the imaginary part of its argument. Since the signal-to-interference-plus-noise ratio (SINR) of the beamformer output will not change by rotating the weight vector, an alternative formulation can be derived as

$$\min_{\mathbf{w}} \mathbf{w}^H\hat{\mathbf{R}}\mathbf{w} \quad \text{subject to} \quad \mathrm{Re}\{\mathbf{w}^H\mathbf{a}(\theta_0)\}\geq\varepsilon\|\mathbf{w}\|+1 \qquad (6)$$

where $\mathrm{Re}\{\cdot\}$ takes the real part of its argument.

Both the Capon beamformer and the worst-case robust beamformer require estimation of the covariance matrix $\mathbf{R}$, and it is a challenging task when only a small number of snapshots is available; one solution to the problem is the family of iterative adaptive approach-based methods [10], which can still work for the extreme case with only one snapshot.

Another notable contribution for robust adaptive beamforming is based on interference covariance matrix reconstruction and steering vector estimation [11], which has attracted much attention recently, with follow-up works focusing on different ways of reconstructing either or both of the covariance matrices corresponding to the desired signal and interference plus noise, separately.

Frequency-invariant beamforming

For wideband arrays, different from the data model in (1), the received array signals are expressed in the form of convolution (represented by $\star$) [5]:

$$\mathbf{x}(t)=\tilde{\mathbf{A}}\star\mathbf{s}(t)+\mathbf{n}(t) \qquad (7)$$

where the (m, k)th element of the matrix $\tilde{\mathbf{A}}$ is given by $\delta(t-\tau_{m,k})$, with $\tau_{m,k}$ being the time delay of the kth impinging signal at the mth sensor compared to some reference point.

As a result, wideband beamforming is achieved through a series of tapped delay lines (TDLs) or finite-impulse response/infinite-impulse response filters in its discrete form [5]. For wideband beamformers, in general, the beamwidth will increase with the decrease of frequency since the relative aperture of the array becomes smaller for lower frequencies, and therefore, one unique problem for wideband beamforming is how to design a beamformer with a frequency-invariant beam response or beam pattern.

To achieve a frequency-independent beam response, many methods were proposed in the past, and one typical solution is harmonic nesting, where, for a number of frequency bands, different subarrays with appropriate aperture and sensor spacing are operated [4]. In a design proposed in [12], each sensor in the array is followed by its own primary filter, and the outputs of these primary filters share a common secondary filter to form the final output; although the design for a 1D array is relatively simple due to the dilation property of the primary filters, for 2D and 3D arrays, this property is not guaranteed, which makes the general design case very complicated. In [5] and [13], based on a simple Fourier transform relationship, a systematic and consistent approach was developed to design fixed frequency-invariant beamformers for 1D, 2D, and 3D arrays and for both continuous and discrete apertures.

Furthermore, a series of least-squares-based frequency-invariant beamformer design methods were proposed with closed-form solutions and applicable to arbitrary array geometries [5]. In its very basic form, given the desired beam pattern $P_d(\Omega,\theta)$ (Ω is the normalized frequency) and designed response $P(\Omega,\theta)$ (a quadratic function of the beamforming weight vector $\mathbf{w}$) over the frequency range of interest Ω_I and the range of the angle of interest Θ, the design is to minimize the following cost function:

$$\alpha\int_{\Theta}\left|P(\Omega_r,\theta)-P_d(\Omega_r,\theta)\right|^2 d\theta$$
$$+(1-\alpha)\int_{\Omega_I}\int_{\Theta}\left|P(\Omega,\theta)-P(\Omega_r,\theta)\right|^2 d\Omega d\theta \qquad (8)$$

where the first part is the traditional cost function for a least-squares-based design over one reference frequency Ω_r; the second part is the term for measuring the difference between the response of the designed beamformer and its response at the reference frequency Ω_r over the full range of the angle of interest, i.e., the frequency variation of the response; and α trades these off. Note that the first part of the cost function is calculated only at the reference frequency, not the whole Ω_I, and the reason is that, if the response is frequency invariant, then as long as at one single frequency (Ω_r) the designed response is close to the desired one, the whole response will also be close to it. A design example for a frequency-invariant beamformer, over the normalized frequency range $[0.3\pi, \pi]$, based on a uniform linear array (ULA) of 10 sensors and a TDL length of 20 is shown in Figure 1.

The preceding FIB design techniques can be employed to design a FIB network, where multiple frequency-invariant beamformers pointing to different directions are placed in parallel to transform the wideband array signal processing problem into a narrowband one so that traditional narrowband beamforming and DOA estimation solutions can be applied directly to the output of the FIB network [5]; the second part of the cost function in (8) can also be incorporated into the adaptive beamforming process to realize adaptive FIB directly instead of relying on the FIB network [14].

Note that the TDL-based wideband beamforming structure could be replaced by the sensor delay line (SDL)-based structure [5], [15], where multiple sensors are placed behind the original array sensors in place of the delay lines for effective wideband beamforming; such an SDL-based structure may prove to be very important for the coming terahertz (THz) and sub-THz communication systems, where the delays required for effective wideband beamforming/beam steering may be too short to be implemented in practice.

DOA estimation

DOA estimation is another core SAM research area. Originally, it was realized by various beamforming algorithms in its simplest form, such as the Butler matrix, the Capon beamformer, and the LCMV beamformer, and then more advanced super-resolution solutions were developed under the classic subspace framework. In the past 25 years, inspired by developments of compressive sensing (CS) [16], two important advances in this area are the sparsity-based DOA estimation framework [17], [18], which, unlike the subspace-based framework, can deal with coherent sources directly, and the underdetermined DOA estimation approach based on various signal properties (such as noncircularity and non-Gaussianity) and the coarray concept (both sum and difference coarrays) [17], [19], [20], [21]. (Here, "underdetermined" means that the number of signals is larger than or equal to the number of physical sensors.)

Sparsity-based DOA estimation

To introduce the basic idea for sparsity-based DOA estimation, consider the following discrete version of the continuous model in (1):

$$\mathbf{x}[i] = \mathbf{A}\mathbf{s}[i] + \mathbf{n}[i] \tag{9}$$

where $\mathbf{x}[i]$ is the array data vector for the ith snapshot, $\mathbf{s}[i]$ is the source signal vector, and $\mathbf{n}[i]$ is the noise vector.

For the ith snapshot, to exploit the spatial sparsity property of the source signals, a search grid of K_g ($K_g \gg K$) potential incident angles $\theta_{g,0}, \ldots, \theta_{g,K_g-1}$ is first generated, and an overcomplete representation of $\mathbf{A}$ is then constructed, given by

$$\mathbf{A}(\boldsymbol{\theta}_g) = [\mathbf{a}(\theta_{g,0}), \ldots, \mathbf{a}(\theta_{g,K_g-1})]. \tag{10}$$

Here, $\mathbf{A}(\theta_g)$ is independent of the actual source directions θ_k. We also construct a $K_g \times 1$ column vector $\mathbf{s_g}[i]$, with each entry representing a potential source at the corresponding angle. Then, the model, from the perspective of sparse signal reconstruction, becomes

$$\mathbf{x}[i] = \mathbf{A}(\boldsymbol{\theta}_g)\mathbf{s_g}[i] + \mathbf{n}[i]. \tag{11}$$

Now the sparsity-based DOA estimation for a single snapshot can be formulated as

$$\begin{aligned} \min \quad & \|\mathbf{s_g}[i]\|_0 \\ \text{subject to} \quad & \|\mathbf{x}[i] - \mathbf{A}(\boldsymbol{\theta}_g)\mathbf{s_g}[i]\|_2 \le \varepsilon \end{aligned} \tag{12}$$

where $\|\cdot\|_0$ is the ℓ_0 norm to promote sparsity in $\mathbf{s_g}[i]$. Locations of the nonzero entries in the resultant $\mathbf{s_g}[i]$ represent the corresponding DOA estimation results.

Since the ℓ_0 norm is nonconvex, in practice, it is normally replaced by the ℓ_1 norm as an approximation. Finally, the sparsity-based DOA estimation for a single snapshot is formulated as

$$\begin{aligned} \min \quad & \|\mathbf{s_g}[i]\|_1 \\ \text{subject to} \quad & \|\mathbf{x}[i] - \mathbf{A}(\boldsymbol{\theta}_g)\mathbf{s_g}[i]\|_2 \le \varepsilon \end{aligned} \tag{13}$$

where $\|\cdot\|_1$ is the ℓ_1 norm.

When multiple data snapshots are available, we could perform DOA estimation by (12) for each snapshot i separately. However, a more effective approach is to jointly estimate the DOAs of the impinging signals across multiple snapshots by employing the group sparsity concept since they all have the same spatial support.

Denote $\mathbf{X} = [\mathbf{x}[0], \ldots, \mathbf{x}[P-1]]$, where P is the number of snapshots. Similarly, we can define $\mathbf{S} = [\mathbf{s}[0], \ldots, \mathbf{s}[P-1]]$ and $\mathbf{N} = [\mathbf{n}[0], \ldots, \mathbf{n}[P-1]]$. Then, the signal model for multiple snapshots can be obtained by

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N}. \tag{14}$$

To introduce spatial sparsity, similar to the single-snapshot case, we construct $\mathbf{S_g} = [\mathbf{s_g}[0], \ldots, \mathbf{s_g}[P-1]]$ and use the row vector $\mathbf{s}_{\mathbf{g},k_g}, 0 \le k_g \le K_g - 1$ to represent the k_gth row of the matrix $\mathbf{S_g}$:

$$\mathbf{X} = \mathbf{A}(\boldsymbol{\theta}_g)\mathbf{S_g} + \bar{\mathbf{N}}. \tag{15}$$

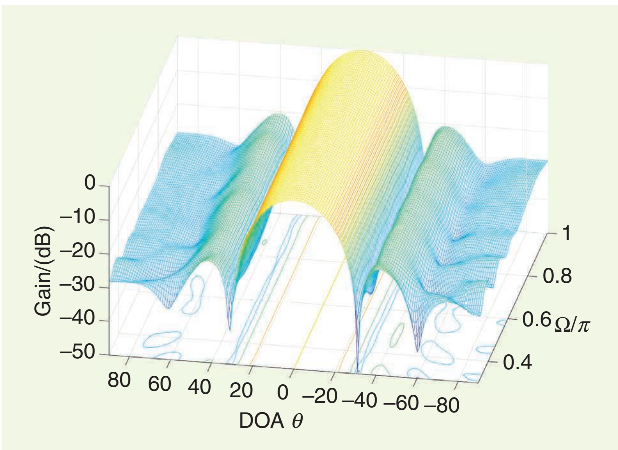

FIGURE 1. A frequency-invariant beamformer.

Then, a new $K_g \times 1$ column vector is generated by computing the ℓ_2 norm of each row in $\mathbf{S_g}$, expressed as

$$\hat{\mathbf{s}}_{\mathbf{g}} = [\|\mathbf{s}_{\mathbf{g},0}\|_2, \|\mathbf{s}_{\mathbf{g},1}\|_2, \ldots, \|\mathbf{s}_{\mathbf{g},K_g-1}\|_2]^T. \quad (16)$$

Finally, the problem for multiple snapshots can be formulated as

$$\begin{aligned} \min_{\mathbf{S_g}} \quad & \|\hat{\mathbf{s}}_{\mathbf{g}}\|_1 \\ \text{subject to} \quad & \|\mathbf{X} - \mathbf{A}(\boldsymbol{\theta}_g)\mathbf{S_g}\|_F \le \varepsilon \end{aligned} \quad (17)$$

where $\|\cdot\|_F$ represents the Frobenius norm and $\|\hat{\mathbf{s}}_{\mathbf{g}}\|_1$ is also called the $\ell_{2,1}$ *norm* of the matrix $\mathbf{S_g}$. Locations of the nonzero entries of the resultant column vector $\hat{\mathbf{s}}_{\mathbf{g}}$ are, then, the corresponding estimation results.

One problem with the preceding group sparsity-based formulation is its high computational complexity, especially when a large number of snapshots P is available. To reduce the complexity, we can perform singular value decomposition (SVD) to $\mathbf{X}$ and project the data to a lower-dimension signal space, leading to the so-called ℓ_1-SVD method [22], or use the covariance matrix of the data to form a virtual array directly [23].

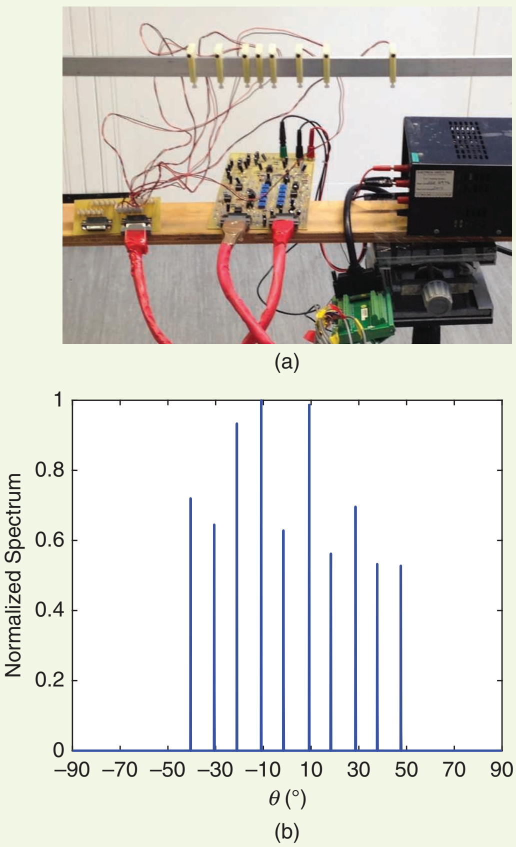

FIGURE 2. Group sparsity-based underdetermined wideband DOA estimation, where 10 uncorrelated acoustic source signals are distributed from around −40 to 50°, with an approximate step size of 10° [27]. (a) The coprime microphone array system. (b) The estimation result for the 10 sources.

Underdetermined DOA estimation

For underdetermined DOA estimation, although it can be achieved by exploiting the non-Gaussianity, noncircularity, and nonstationarity of the signals, the most important development is through constructing various sparse array structures for virtual coarray generation, such as coprime arrays, nested arrays, and their numerous extensions [24], [25], [26].

For second-order statistics-based coarray generation, one common step is to vectorize the covariance matrix of the physical sparse array. Consider the covariance matrix

$$\mathbf{R_{xx}} = \mathrm{E}\{\mathbf{x}[i]\mathbf{x}^H[i]\} = \sum_{k=1}^{K} \sigma_k^2 \mathbf{a}(\theta_k)\mathbf{a}^H(\theta_k) + \sigma_n^2 \mathbf{I}_N \quad (18)$$

where σ_k^2 is the power of the kth impinging signal and θ_k is its AOA.

By vectorizing $\mathbf{R_{xx}}$, we obtain a virtual array model

$$\mathbf{z} = \mathrm{vec}\{\mathbf{R_{xx}}\} = \tilde{\mathbf{A}}(\boldsymbol{\theta})\tilde{\mathbf{s}} + \sigma_n^2 \tilde{\mathbf{i}}_{N^2} \quad (19)$$

where $\tilde{\mathbf{A}}(\boldsymbol{\theta}) = [\tilde{\mathbf{a}}(\theta_1), \ldots, \tilde{\mathbf{a}}(\theta_K)]$ is the equivalent virtual steering matrix, with $\tilde{\mathbf{a}}(\theta_k) = \mathbf{a}^*(\theta_k) \otimes \mathbf{a}(\theta_k)$ being the corresponding steering vector ($\otimes$ denotes the Kronecker product); $\tilde{\mathbf{s}} = [\sigma_1^2, \ldots, \sigma_K^2]^T$ is the equivalent source signals; and $\tilde{\mathbf{i}}_{N^2}$ is obtained by vectorizing $\mathbf{I}_N$.

In the preceding virtual array model, although there are repeated entries in $\mathbf{R_{xx}}$, the number of virtual sensors corresponding to the difference coarray is much more than that of the physical sensors, and the equivalent source signals share the same spatial support with the original impinging signals. The virtual model in (19) is similar to the single-snapshot array model, and sparsity-based DOA estimation methods such as that in (13) can be applied here.

Instead of employing a sparse array, it is possible to extend the coarray concept to different frequencies, where a single ULA can be used with two continuous-wave signals of coprime or other different frequencies, and to the wideband case through frequency decomposition and employing multiple frequency pairs [17].

The group sparsity concept employed for the multiple-snapshot case can be applied to general underdetermined and overdetermined wideband DOA estimation [17]; as in traditional wideband DOA estimation, focusing can also be employed for sparsity-based wideband DOA estimation to simplify the problem to a single reference frequency. One interesting observation about the wideband case is that the sensor spacing can be larger than half the wavelength corresponding to the highest frequency of the signal, without causing the spatial aliasing problem; on the contrary, an improved estimation performance can be achieved for a larger spacing, due to an increased aperture.

Figure 2 gives a real experimental result based on an eight-microphone coprime array for estimating the directions of 10 speech signals, with a bandwidth from 5 to 10 kHz and sampling frequency of 20 kHz [27].

Sensor location optimization

In many applications, the array's geometrical layout is assumed to be fixed and given in advance. However, it is possible to change the geometrical layout of the array, including the adjacent sensor spacing, and these additional spatial degrees of freedom (DOF) can be exploited to improve the performance in terms of beamforming, DOA estimation, or both. For the beamforming side, given the nonconvex nature of the optimization problem, traditionally, it is solved by genetic algorithms, simulated annealing, and similar approaches [28]. With the development of CS and the sparsity maximization framework, a new CS-based framework with a theoretically optimum solution (given the convex nature of the formulated problem) has been developed for sensor location optimization for fixed beamforming [29], followed by further work in adaptive beamforming [30], [31], with robustness against various array model errors considered, too. For the DOA estimation side, the main efforts have been focused on the coarray design to increase the DOF for underdetermined DOA estimation. As mentioned in the previous section, coprime arrays and nested arrays are two representative array structures [24], [25], based on which numerous second-order and fourth-order (and even higher) coarray construction methods have been developed. In this part, we focus on the sparse array design problem for beamforming.

To illustrate how the design works, consider a narrowband linear array structure consisting of M omnidirectional sensors, where the distance from the first sensor to subsequent sensors is denoted as d_m, for $m = 1, 2, \ldots, M$, with $d_1 = 0$, i.e., the distance from the first sensor to itself. The output of the beamformer is a weighted sum of the received signals, and the weighting coefficients are denoted by w_m and $m = 1, 2, \ldots, M$, which are placed together into the weighting vector $\mathbf{w}$. Then, the sparsity-based design for sensor location optimization can be described as follows.

First, consider the array geometry being a grid of potential active antenna locations. In this instance, d_M is the maximum aperture of the array, and the values of d_m, for $m = 1, 2, \ldots, M-1$, are selected to give a uniform grid, with M being a very large number so that the spacing between adjacent antennas is very small. Through selecting the minimum number of nonzero-valued weight coefficients to generate a beam response close to the desired one, a sparse array design result is obtained. In other words, if a weight coefficient is zero valued, the corresponding sensor will be inactive and therefore can be removed, leading to a sparse or nonuniformly spaced sensor array.

Mathematically, it is formulated as a constrained ℓ_1-norm minimization problem

$$\min \quad \|\mathbf{w}\|_1 \tag{20}$$

$$\text{subject to} \quad \|\mathbf{p}_r - \mathbf{w}^H \mathbf{A}\|_2 \leq \varepsilon \tag{21}$$

where $\mathbf{p}_r$ is the vector holding the desired beam responses at the sampled angular range of interest; $\mathbf{A}$ is the steering matrix corresponding to those angles, with $\mathbf{w}^H \mathbf{A}$ representing the designed beam responses; and ε is the allowed error between the designed and desired beam responses. The minimization of the ℓ_1 norm of the weight vector helps to promote sparsity in the weight vector, and the reweighted ℓ_1-norm minimization could be used instead to have a closer approximation to the ideal ℓ_0-norm minimization problem, where smaller weighting terms are added to the larger elements of the weight vector $\mathbf{w}$ so that smaller values in $\mathbf{w}$ are penalized more and become closer to zero after minimization [32].

A broadside main beam design example is provided in Figure 3, where the sensor locations are optimized over an overall aperture of $d_M = 10\lambda$, which is split into 181 potential sensor locations ($M = 181$). It can be seen that the resultant weight vector is sparse, with only 12 nonzero-valued coefficients, leading to a sparse array of 12 sensors, and compared to the beam pattern of a standard 12-sensor ULA with

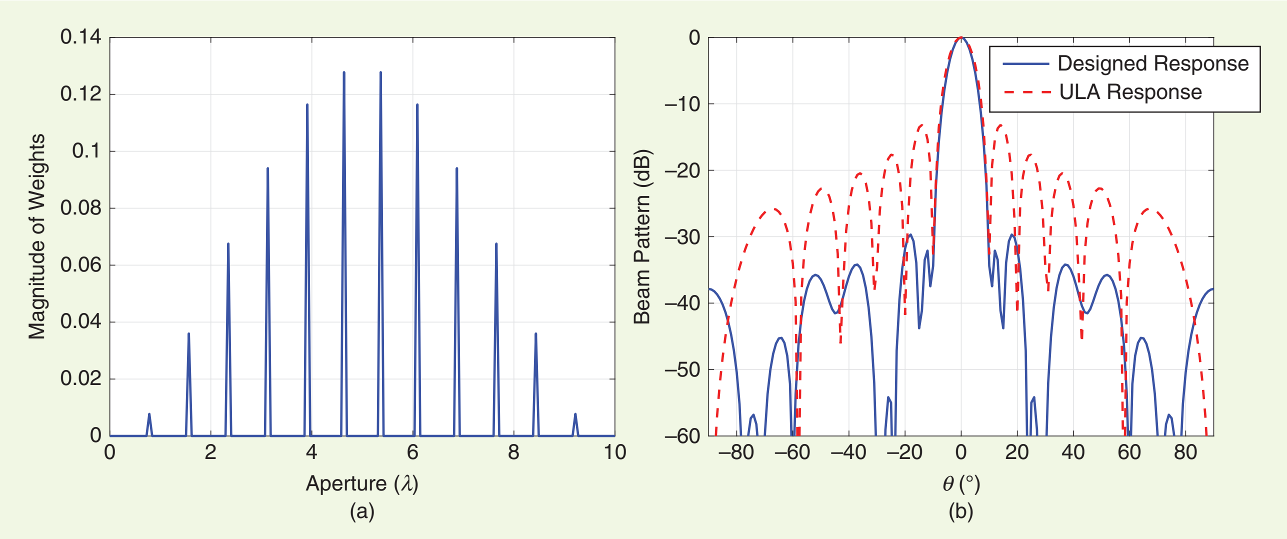

FIGURE 3. The location optimization result and comparison with the ULA. (a) The magnitude of the weights. (b) The array beam patterns.

half-wavelength spacing, the sparse array has a similar main beamwidth but a much lower sidelobe level.

Various constraints can be added to the preceding formulation to deal with more complicated application scenarios. For example, in the preceding formulation, the steering vector of the array is assumed to be known exactly, which may not be true due to various possible model perturbations, such as errors in sensor locations, mutual coupling, and discrepancies in individual sensor responses; then, robust designs can be achieved by applying a norm-bounded error constraint to the weight vector. In another case, it has been assumed that the sensors in the array are of zero size; however, this is not true in the real world, and various size constraints can be added to the design, and some postprocessing methods can be introduced to make sure the minimum spacing between adjacent sensors in the result is larger than the size of the sensor. Based on the concept of group sparsity, the design can also be extended to the wideband case with TDLs [29].

One unique problem for wideband beamforming is how to design a beamformer with a frequency-invariant beam response or beam pattern.

Target/source localization based on sensor arrays

This is another important problem in array signal processing, and significant progress has been made in this area in the past 25 years. Typical solutions include those based on the received signal strength [33]; those based on distance-related measurements, such as the time of arrival [34]; and those based on the AOA/DOA [35], [36]. The last is also called *bearing-only localization*, and it is an attractive candidate since synchronization among the distributed platforms is not required, and it can be used in both active and passive sensing networks and adopted in a wide range of applications, including multistatic radar, distributed massive MIMO, and wireless sensor networks. There are normally two steps in this bearing-only localization: the first is applying existing DOA estimation methods to find the AOAs at all distributed sensor arrays, while the second is to find intersections of those estimated AOAs to localize the sources, and the maximum likelihood estimator has been adopted to minimize the total least-squares errors of the noise-corrupted angle measurements among all distributed sensor arrays. However, the performance of such a two-step localization approach is dependent on the accuracies of angle measurements obtained at all platforms, and even one bad AOA estimation result can lead to a serious performance degradation.

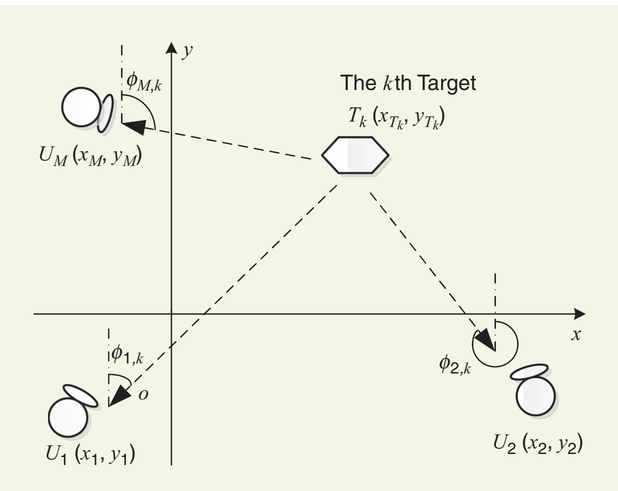

FIGURE 4. A general target/source localization model based on distributed sensor arrays [37].

To tackle the shortcomings of the two-step approach, we could jointly process the collected information across the observation platforms in lieu of fusing the separate angle estimation results at all platforms. One recent advance in this direction is a group sparsity-based one-step approach [37], where a common spatial sparsity support corresponding to all distributed sensor arrays is enforced, leading to a better estimation performance, which also avoids the possible pairing and ambiguity problems associated with the two-step AOA-based solution.

To show how this idea works, we consider a distributed narrowband sensor array network with M subarrays and K targets, as illustrated in Figure 4, where $U_m(x_m, y_m)$ and $T_k(x_{T_k}, y_{T_k})$ represent locations of the receiver platform and the kth target, respectively. For each receiver, a linear subarray with L_m sensors is employed.

For each target located at $T_k(x_{T_k}, y_{T_k})$, a unique incident angle $\theta_{m,k}$ relative to the mth subarray can be obtained. Without loss of generality, a square area of interest in the Cartesian coordinate system is divided into a $K_x \times K_y$ grid, with K_x and K_y being the number of grid points along the x-axis and the y-axis, respectively. Here, $G(x_{k_x}, y_{k_y})$ represents the location of the (k_x, k_y)th search grid, and the signal originating from the possible source located at $G(x_{k_x}, y_{k_y})$ will arrive at the mth subarray, with a DOA angle $\theta_m^g(k_x, k_y)$. Since (x_{k_x}, y_{k_y}) is common to all subarrays and a source located at $G(x_{k_x}, y_{k_y})$ will appear to come from the same location with respect to all subarrays, we can apply the group sparsity concept to all subarrays' source data.

For example, for the mth subarray, corresponding to the data model in (14), we can have the multiple-snapshot model as

$$\mathbf{X}_m = \mathbf{A}_m \mathbf{S}_m + \mathbf{N}_m \tag{22}$$

with $m = 1, 2, \ldots, M$. Applying the sparsity-based approach, we can construct the following overcomplete data model:

$$\mathbf{X}_m = \mathbf{A}_m^g \mathbf{S}_m^g + \mathbf{N}_m \tag{23}$$

where $\mathbf{A}_m^g$ is the overcomplete steering matrix corresponding to the $K_x K_y$ potential signal directions $\theta_m^g(k_x, k_y)$ and $\mathbf{S}_m^g$ is the potential source matrix. If there is no source located at a particular position $G(x_{k_x}, y_{k_y})$, then the corresponding row of $\mathbf{S}_m^g$ will be zero valued for all $m = 1, 2, \ldots, M$. We can place all the matrices $\mathbf{S}_m^g$ together to form a new matrix $\mathbf{S}^g$, as follows:

$$\mathbf{S}^g = [\mathbf{S}_1^g, \mathbf{S}_2^g, \ldots, \mathbf{S}_M^g]. \tag{24}$$

Then, the group sparsity-based localization problem can be formulated by minimizing the $\ell_{2,1}$ norm of $\mathbf{S}^g$, subject to limiting the overall reconstruction error for all subarrays to a small value. One main advantage of the group sparsity-based approach for direct target localization is that the different subarrays are not required to be synchronized and can work on different frequencies, the statistical properties of the sources can be different for different subarrays, and sensor numbers, rotation angles, and corresponding source signals of different subarrays do not need to be the same (as long as they come from the same set of target locations). This group sparsity-based one-step direct localization idea can be extended to the wideband, the underdetermined case, or both without difficulty [37].

Figure 5 displays a simulation result for underdetermined wideband localization, where the normalized signal frequency band is from 0.75π to π, and there are six subarrays and five targets, with each subarray being a four-sensor minimum redundancy array [4].

MIMO arrays

MIMO, which is, by its multichannel implementation at both the transmitter and the receiver, a natural fit within the SAM portfolio, represents another significant development in array signal processing in the past 25 years. There are mainly two totally different directions. One is MIMO radar, which exploits the orthogonality of the transmitted waveforms to increase the DOF of the system to improve the resolution and capacity of the array [38], [39], [40], which will play an important part in 4D auto radar imaging in addition to traditional radar detection applications. Note that nonorthogonal waveforms can also be employed for MIMO radar [41]. The other one is MIMO for wireless communications to exploit the spatial diversity of the channel to improve the performance and, in particular, the capacity of the communication system [42]. While MIMO has already been in use for both Wi-Fi and 4G communication systems, its new evolution, the so-called massive MIMO, or ultramassive MIMO (UM-MIMO), will play a crucial role in next-generation communication systems and beyond [43].

MIMO radar

In a MIMO radar, multiple transmit antennas emit orthogonal waveforms and multiple receive antennas, then receive the echoes reflected by the targets. Antennas of the MIMO radar can be widely separated [38] and colocated [39], [40], with the latter more widely studied. For the case with colocated antennas, the transmitting side and the receiving side can be located either at the same site or far away from each other.

Consider a colocated narrowband MIMO radar system where the transmit and receive antennas are located at the same place. The transmitted multiple orthogonal waveforms are then reflected back by K present targets and received by the receive array. After matched filter processing, the output signal vector $\mathbf{x}[i]$ at the receiver at the ith snapshot can be expressed as

$$\begin{aligned}\mathbf{x}[i] &= \sum_{k=1}^{K} \mathbf{a}_t(\theta_k) \otimes \mathbf{a}_r(\theta_k) b_k[i] + \mathbf{n}[i] \\ &= [\mathbf{a}_t(\theta_1) \otimes \mathbf{a}_r(\theta_1), \ldots, \mathbf{a}_t(\theta_K) \otimes \mathbf{a}_r(\theta_K)]\mathbf{b}[i] + \mathbf{n}[i] \end{aligned} \tag{25}$$

where θ_k is the DOA of the kth target; $\mathbf{a}_t(\theta_k)$ and $\mathbf{a}_r(\theta_k)$ are the steering vectors of the transmit and receive arrays, respectively; and $b_k[i] = \gamma_k e^{j2\pi f_k i}$, with γ_k being the complex-valued reflection coefficient of the kth target and f_k being the Doppler frequency for moving targets.

It can be seen that with the MIMO radar configuration, a virtual array with a significantly increased aperture has been created due to the effect of the Kronecker product in (25). For example, if both the transmit array and receive array are three-sensor ULAs with a spacing of d and $3d$, respectively, the newly generated virtual ULA will consist of nine virtual sensors. In this way, by exploiting waveform diversity, a virtual array with a much larger aperture and significantly increased DOF is formed using a small number of physical sensors, providing enhanced spatial resolution, higher target detection capacity, and better performance.

MIMO for wireless communications

On the other hand, MIMO for wireless communications is a huge research area, and numerous techniques have been developed centered around this concept, such as space-time coding, MIMO beamforming, spatial multiplexing, and spatial modulation. Today, an element of MIMO can be found in most of the publications in wireless communications. It is impossible to list all the important advances in the area, and in this section, we focus only on MIMO beamforming, which is playing an increasingly important role in the implementation of MIMO communication systems.

As well known by the array signal processing community and also presented in the "Beamforming" section, traditionally, beamforming is designed for line-of-sight (LOS) transmission

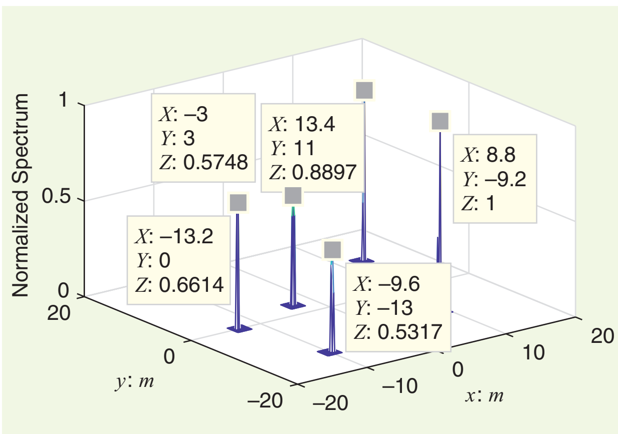

FIGURE 5. The localization results for five targets with six subarrays (20-dB signal-to-noise ratio, 500 snapshots).

and reception, and physically, a beam will be formed in the process, pointing to different directions around the array system. However, in MIMO beamforming, due to a very strong multipath effect, the result of beamforming between the transmitter and receiver will not necessarily form a beam in space but, rather, an overall enhanced signal transmission link between them. Decades of research in MIMO beamforming have pushed the boundaries of beamforming well beyond the technology's traditional meaning, and today, any process achieving enhancement of the desired signal while reducing the effect of interference can be considered beamforming. However, with the introduction of massive MIMO and millimeter-wave (mmwave) communications in 5G and beyond, the LOS case is becoming more and more important again in MIMO beamforming, and one interesting development in this context is the hybrid beamforming structure proposed for massive MIMO systems [8].

Today, any process achieving enhancement of the desired signal while reducing the effect of interference can be considered beamforming.

Hybrid beamforming is a combination of analog beamforming and digital beamforming. Ideally, beamforming could be implemented completely in the digital domain for maximum flexibility and adaptability; however, for extremely large arrays, as in the case of massive/UM-MIMO, the extremely high cost associated with the large number of high-speed analog-to-digital converters (ADCs)/digital-to-analog converters (DACs) and the high-level power consumption will render it practically infeasible. For hybrid beamforming in the receive mode, analog beamforming is performed first to reduce the number of analog channels, which are then converted into digital via a reduced number of ADCs, and after that, digital beamforming can be performed; for the transmit mode, the process is simply reversed. There are many hybrid beamforming structures proposed in the literature, and one representative is the subaperture-based hybrid beamformer. An interesting recent development in this area is a new class of multibeam multiplexing designs, where the number of analog coefficients is the same as the number of antennas, independent of the number of parallel independent user beams generated, while the number of subarrays is the same as the number of beams; interested readers can refer to [44] and [45] for details.

New developments in the SAM area

In the era of AI, multi-sensor-based systems and techniques are ubiquitous and will play an even greater role in the future. As a result, there has been an exponential increase in research activities in the SAM area in the past few years, and in the following, we introduce some new developments that may well indicate promising future research directions.

GSP for sensor networks

GSP is an emerging new mathematical tool for analysis of data resident on a largely irregular network of either physical or virtual sensor nodes, where the regular network can be considered a special case [46], [47]. Examples for the physical sensor network include traffic networks, brain neural networks, and energy consumption sensor networks, while for the virtual one, a good example is social networks. In connection with classic signal processing, basic concepts, such as frequency, and operations, such as shift/delay and filtering, have been introduced. However, there is still no unified framework for GSP, and it is still an open problem to find the best representations of a graph signal. However, this has not stopped the wide application of GSP, and it has been shown to be a powerful data analysis tool providing new insights into the studied problems; for example, brain signals can be mapped to a graph network to analyze cognitive behavior of the brain. Application of GSP to traditional sensor array signal processing problems, such as direction finding and target localization, is an emerging but somewhat open area, as traditional sensor arrays and networks normally have a regular structure, and traditional sensor array signal processing tools have been extremely successful in tackling those associated problems. It is not clear yet whether GSP can bring any advantage to the traditional sensor array signal processing problems or not.

Tensor-based array signal processing

Tensors are extensions of matrices to higher dimensions and have been widely employed for multidimensional data analysis and processing with the aid of tensor decomposition tools and algorithms. Many sensor array signals and data can be transformed into a multidimensional form and viewed directly as a multidimensional structure [48], [49]. For example, the narrowband data received by a rectangular array and multiple subarrays are 3D, the data received by a wideband linear array can be transformed into the 3D space–time–frequency domain, and the data received by vector sensor arrays are naturally higher dimensional. For MIMO communication systems, the data can be placed into a tensor form by accounting for diversities in space, time, frequency (including Doppler frequency), and polarization. As a result, tensor processing can be applied to solve many array signal processing problems directly without much adaptation. However, although it is recognized that tensors can keep the inherent data structures and therefore have the potential to provide improved performance compared to classic array signal processing methods and algorithms, further research is needed to demonstrate the clear benefits of tensor processing and fully realize its potential.

Quaternion-valued array signal processing

As a higher-dimensional extension of complex numbers, a quaternion has one real part and three imaginary parts, and quaternion calculus has been applied to a range of signal processing problems related to 3D and 4D signals, such as color image processing, wind profile prediction, vector sensor array processing, and quaternion-valued wireless communications [50], [51]. In addition to solving the classic array signal processing problems, such as DOA estimation and beamforming, one important development is the quaternion-valued MIMO array, where pairs of antennas with orthogonal polarization directions are employed at both the transmitter and receiver sides and a 4D modulation scheme across the two polarization diversity channels using a quaternion-valued representation is employed. Although the polarization states will change during transmission through the channel, and there may be interferences between these two states, we can employ a quaternion-valued adaptive algorithm to recover the original 4D signal, which inherently also performs an interference suppression operation to separate the original two 2D signals. For the MIMO array, reference signal-based and blind quaternion-valued adaptive algorithms can be employed for both channel estimation and beamforming. Signal processing has experienced a revolutionary change from real-valued processing to complex-valued processing, and we may be at the doorstep of a quaternion-valued world, and increasing interest in quaternion-valued sensor array signal processing is expected in the near future.

Signal processing has experienced a revolutionary change from real-valued processing to complex-valued processing, and we may be at the doorstep of a quaternion-valued world.

One-bit and noncoherent sensor array signal processing

Given the extremely high data rate and storage requirements for a fully digital large sensor array system, there has been significant work aimed at achieving a reasonable sensor array processing performance with 1-bit representation of the array signals; i.e., only signs of the data samples are reserved, while the magnitude information is removed [52]. This problem can be simply considered the normal case but with extremely high quantization noise, and we can perform normal array processing irrespective of the number of bits per data sample; however, a more effective way is to try to achieve effective estimation of the statistics of the signals using the 1-bit data samples and then, based on the newly obtained statistics information, perform the corresponding tasks. Contrary to 1-bit array processing, the signs of the data samples are removed, and only the magnitude information is kept, which leads to the so-called noncoherent sensor array signal processing problem, with the advantage of being robust against array phase errors. One representative example is noncoherent DOA estimation and target localization [53], [54], [55], which can be cast into a phase retrieval problem; however, the difference is that there is usually only one snapshot in phase retrieval, while in array signal processing, multiple snapshots are available, which can be exploited by applying group sparsity to existing phase retrieval algorithms, such as the ToyBar and modified GESPAR algorithms [53], [54].

Machine learning and AI for sensor arrays

Machine learning and AI have been applied to almost all areas of research in the signal processing community, and the SAM area is no exception. For example, machine learning and AI have been applied to DOA estimation, beamforming, and source separation successfully [56]. There are strong topical connections among sparsity-inspired array processing (see the "DOA Estimation" section), compressed sensing (see the "Sensor Location Optimization" section), and machine learning. Unlike in traditional machine learning and AI applications, where it is a challenge to acquire sufficient training data, in most of the array signal processing applications, the required training data can be obtained easily by simulation. Nonetheless, their application to array signal processing also faces some similar issues. For example, after training, the system may work very well for the targeted scenario, but it may struggle if there is change to the system and the environment, while the traditional array signal processing methods and algorithms can cope with such changes well. Another challenge is how to apply machine learning and AI to distributed sensor arrays and networks effectively. As a hot topic, federated learning may prove to be a promising direction of research for the SAM community [57].

Array signal processing for next-generation communication systems

Antenna array design and signal processing is one of the fundamental techniques in 5G (and beyond) wireless communication systems since the two underpinning 5G/6G technologies—massive MIMO/UM-MIMO and mm-wave/sub-THz/THz communications—are all based on antenna arrays [58]. It will continue to play a significant role in many other aspects in the future, such as the Internet of Things and integrated sensing and communication, both of which are hot topics for 6G wireless communications research, with extensive research activities in the community. Moreover, beamforming is essential to achieve effective communication over the THz and sub-THz frequency bands, as it is necessary to employ a large number of antennas for such high frequencies, while the widely studied reconfigurable intelligent surfaces can be considered semipassive antenna array systems [59]. To a great degree, array signal processing will be a main focus of research for next-generation communication systems and for the integration of sensing and communications, particularly at mm-waves [60].

Concluding remarks

Accompanied by intensive research activities and the significant progress made in signal processing, the world now has stepped into the new era of AI, where multi-sensor-based systems and techniques have become ubiquitous and indispensable to our daily life and will play an even greater role in our society in the very near future. This is an exciting time for the SAM community, and we welcome new members at different levels to join the TC and work together to promote its activities, make a more extensive and deeper impact in the real world, and further enhance its standing in our wider society.

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Authors

Wei Liu (w.liu@sheffield.ac.uk) received his Ph.D. degree from the University of Southampton, U.K., in 2003. He is currently with the Department of Electronic and Electrical Engineering, University of Sheffield, S1 3JD Sheffield, U.K. He has published 210+ journal papers, 170+ conference papers, and two research monographs [*Wideband Beamforming: Concepts and Techniques* (Wiley, 2010) and *Low-Cost Smart Antennas* (Wiley, 2019)] in the area of signal processing, with a particular focus on sensor array signal processing and its various applications, such as robotics and autonomous systems, radar, sonar, and wireless communications. He is a member of the IEEE Signal Processing Society Sensor Array and Multichannel Technical Committee (2021–2022 chair) and the IEEE Circuits and Systems Society Digital Signal Processing Technical Committee (2022–2024 chair), and he is an IEEE Aerospace and Electronic Systems Society Distinguished Lecturer for 2023–2024.

Martin Haardt (martin.haardt@tu-ilmenau.de) received his Ph.D. degree from Munich University of Technology in 1996. He is currently a full professor and the head of the Communications Research Laboratory, Ilmenau University of Technology, D-98684 Ilmenau, Germany. He chaired the IEEE Signal Processing Society (SPS) Sensor Array and Multichannel Technical Committee (TC) during 2017–2018, and he has been an elected member of the SPS Signal Processing Theory and Methods TC since 2020. He received a 2009 SPS Best Paper Award; the Vodafone Innovations Award for outstanding research in mobile communications; the Association of Electrical Engineering, Electronics, and Information Technology ITG Best Paper Award; and the Rohde & Schwarz Outstanding Dissertation Award. His research interests include wireless communications, array signal processing, high-resolution parameter estimation, and numerical linear and multilinear algebra. He is a Fellow of IEEE.

Maria S. Greco (maria.greco@unipi.it) received her Ph.D. degree from University of Pisa in 1998. She is a full professor in the Department of Information Engineering, University of Pisa, 56122 Pisa, Italy. She has coauthored many book chapters and more than 220 journal and conference papers and is the editor-in-chief of *EURASIP Journal of Advances in Signal Processing*. She is the president-elect of the IEEE Aerospace and Electronics Systems Society (AESS) for 2022–2023. Previously, she was the IEEE SPS director at large for Region 8 (2021-22), a member of the SPS Board of Governors (2015–2017), an SPS Distinguished Lecturer (2014–2015), and an AESS Distinguished Lecturer (2015–2020). She was a corecipient of the 2001 and 2012 AESS Barry Carlton Award, the 2008 AESS Fred Nathanson Young Engineer of the Year award, and the AESS Board of Governors Exceptional Service Award. Her research interests include clutter models, cognitive radars, and the integration of sensing and communications.

Christoph F. Mecklenbräuker (cfm@tuwien.ac.at) received his Dr.-Ing. degree (with honors) from Ruhr University Bochum, Germany, in 1998. He is currently a full professor at the Institute of Telecommunications, TU Wien, 1040 Vienna, Austria. He chaired the IEEE Signal Processing Society Sensor Array and Multichannel Technical Committee during 2019–2020. His doctoral dissertation received the 1998 Gert-Massenberg Prize, and he has authored approximately 300 journal and conference papers and was granted several patents in the field of mobile cellular networks. His research interests include 5G/6G radio interfaces (vehicular connectivity and sensor networks) and antennas and propagation.

Peter Willett (peter.willett@uconn.edu) received his Ph.D. degree from Princeton University in 1986. He is currently a professor in the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, CT 06269 USA. He chaired the IEEE Signal Processing Society Sensor Array and Multichannel Technical Committee during 2015–2016. His research interests include statistical signal processing, detection, machine learning, communications, data fusion, and tracking. He is a Fellow of IEEE.

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