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SIMO High Mobility Wireless Communications with Doppler Diversity: Fundamental Performance Limits

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***Abstract*—This paper studies the fundamental tradeoff between maximum Doppler diversity and channel estimation errors of single-input multiple-output (SIMO) high mobility communication systems. In high mobility systems, channel estimation errors are usually inevitable due to severe Doppler effects caused by fast time-varying fading, and this might have significant adverse impacts on system performance. On the other hand, Doppler effect provides potential Doppler diversity which can be exploited to improve system system performance.. It aims to quantify the fundamental tradeoff between Doppler diversity and channel estimation errors in high mobility systems. The fast time-varying fading in SIMO high mobility systems is tracked with pilot-assisted minimum mean squared error(MMSE) channel estimation with the aid the simple repetition codes at the transmitter. In high mobility systems, the accurate estimation and tracking of the fast time-varying channel are vital to fast and reliable wireless communications. However, in high mobility systems, these tasks are challenging thus channel estimation errors are inevitable and might have significant adverse impact on system performance when the Doppler spread is high. On the other hand , Doppler effect induced by temporal fading variation provides potential Doppler diversity which can be exploited to improve system system performance, The tradeoff relationship between Doppler diversity and channel estimation errors is studied by deriving the exact analytical asymptotic expressions of two performance metrics:maximum Doppler diversity order per unit time with imperfect channel state information (CSI) and lost in signal-to-noise ration (SNR) due to channel estimation errors through asymptotic analysis. It is discovered that, due to the fact that, channel estimation for each channel at the pilot symbols locations in SIMO high mobility are independent, the asymptotic mean squared error (MSE) of the estimation of the channel coefficient at the pilot symbols locations in the SIMO high mobility system is the same as that of singe-input single-output (SISO) system. Based on the studies of the statistical properties of channel estimation errors a new optimum diversity receiver for SIMO high mobility employing Doppler diversity is proposed. Using the analytical results, an optimum allocation of transmission energy between pilot symbols and data symbols is done to simultaneously maximize Doppler diversity order and minimize the loss in SNR thereby achieving the optimum tradeoff between the two metrics. The results reveal the fundamental performance limits of Doppler diversity SIMO mobility systems operating in the presence imperfect CSI.The results can serve as a guide in the design of practical SIMO high mobility systems.**

***Index Terms*—High Mobility wireless communications, Single-input multiple-output (SIMO) systems, Doppler diversity, SNR loss, channel estimation error, imperfect channel state information**

I. INTRODUCTION

WITH the surging in demand for wireless applications on high speed trains and aircrafts, high mobility wireless communications has attracted considerable increasing attention in the past recent years. Fast time-varying fading which induces Doppler spread in the received signal is one of the principal performance inhibiting factors for reliable wireless communications in the high speed environment. Fast fading variation in the time domain or a large Doppler spread in the frequency domain can adversely impact on system performance. However, if appropriately handled, fast time-varying fading can be employed to benefit system performance; since Doppler effect induced by temporal fading variation provides potential doppler diversity which can be explored to improve systems performance.

A body of works exploring Doppler diversity present in fast-time-varying fading channels to combat the detrimental effects of multipath fading for reliable wireless communications are discussed in [1]–[7] In [1], a joint time-frequency RAKE receiver is proposed based on canonical time-frequency decomposition of the mobile channel into a series of independent fading channels to explore multipath-Doppler diversity. A Doppler domain multiplexing (DDM) communication structure and a linear precoder are designed in [2] and [3] to explore maximum Doppler diversity. The works in [1]–[3] assume that channel state information (CSI) is present in the receiver and hence didn't consider how imperfect CSI impacts on system performance in high mobility systems.

In [4]–[6]. the designs of high mobility systems in the presence of imperfect CSI using Doppler diversity gain to achieve reliable wireless communication is discussed. In [4], it is discovered that, high mobility systems a proposed Doppler diversity receiver, in the presence of imperfect CSI can achieve maximum Doppler at the expense of low spectral efficiency. An energy and spectral efficient Doppler diversity receiver is proposed in [5]. which achieves a balanced tradeoff between energy and spectral efficiencies. The fundamental performance limits of high mobility wireless communications with Doppler diversity in the presence of imperfect CSI is studied in [6] and an optimum allocation of transmission energy between

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pilot symbols and data symbols scheme is proposed to simultaneously maximize Doppler diversity order and minimize the loss in signal-to-noise ratio (SNR)to reveal the fundamental performance limits of high mobility systems with Doppler diversity. A single-input single-output (SISO) is employed in the works of [4]–[6].

A single-input multiple-output (SIMO) with identically and independently distributed (i.i.d.) Rician fading channels is discussed in [7] and it is shown that, the conventional maximal ratio combining (MRC) receiver is no longer optimism in the presence of imperfect CSI hence new diversity receivers are designed by utilizing the statistics of the channel estimation errors. The results of [7] are not applicable to studies in the high mobility environment because quasi-static channels are employed in the study.

In this paper, the ideas generated in [6] are extended to the context of multiple-input multiple-output (MIMO) high mobility wireless communications systems focusing on single-input multiple output (SIMO) high mobility wireless communications systems by developing a new SIMO Doppler diversity receiver in the presence of imperfect CSI. Our work is motivated by the fact that, MIMO channels are know to provide a number of advantages such as spectral efficiency, substantial increase in capacity and an improvement in the reliability of wireless communications through space diversity over SISO channels in a rich multipath environment. We therefore intend to take advantage of the space diversity inherent in MIMO channels in a rich multipath environment to further enhance the reliability of the SIMO wireless communications structure.

As in [6], we investigate the maximum Doppler diversity order achievable by a high mobility communications system employing a SIMO pilot-assisted transmission (called SIMO high mobility system in this paper) operating in the presence of imperfect CSI and the minimum performance gap between such a system with imperfect and one with perfect CSI. These investigations will not only quantitatively identify the tradeoff between Doppler diversity and channel estimation errors at various Doppler spreads but also show the fundamental performance limits of SIMO high mobility systems with Doppler diversity.

The fast time-varying channels are estimated and tracked through pilot-assisted minimum mean squared error (MMSE) estimation. A new optimum diversity receiver for SIMO high mobility employing Doppler diversity transmissions is proposed through the studies of the statistical properties of the channel estimation errors. An exact error probability expression for the proposed optimum diversity receiver which quantifies the effects of both Doppler diversity and channel estimation error is derived. The asymptotic expressions of two performance metrics: maximum Doppler diversity order per unit time with imperfect CSI and the loss in signal-to-noise ratio due to channel estimation errors are then derived using the analytical error probability expression through asymptotic analysis.

It is discovered that, the asymptotic MSE of the estimation of the channel coefficients at the pilot symbols locations in the SIMO high mobility is the same as that of the SISO high mobility system. This is attributed to the fact that, the channel estimation for each channel at the pilot symbols locations are independent in the SIMO high mobility system. It is also discovered through theoretical analysis that, both Doppler diversity order and loss in SNR are increasing functions in maximum Doppler spread and number of diversity receivers. The obtained analytical results are used to balance the tradeoff between Doppler diversity and channel estimation errors by identifying the optimum energy allocation between pilot and data symbols such that the Doppler diversity is maximized whilst loss in SNR due to channel estimation errors is simultaneously minimized. This energy allocation scheme gives the optimum tradeoff between Doppler diversity and channel estimation errors ib SIMO high mobility systems. The aforementioned results provide solutions to the investigations and they as well show the fundamental performance limits of SIMO high mobility systems with Doppler diversity.

The rest of this paper is organized as follows. The system model is presented in II. Section III studies the statistical properties of the estimated channels. A proposed optimum diversity receiver for a SIMO high mobility in the presence of imperfect CSI and the derivation of its exact error probability is presented in IV. Sections V and VI respectively studies the maximum Doppler diversity order and loss in SNR due to channel estimation errors. Numerical and simulation results are given in Section VII and Section VIII concludes the paper.

$$\overbrace{\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}\hline s_1 & \cdots & s_K & p_1 & s_1 & \cdots & s_K & p_2 & \cdots & s_1 & \cdots & s_K & p_N \\ \hline\end{array}}^{N_{\mathrm{sym}}}$$

Fig. 1. The slot structure after precoding and insertion of pilots.

II. System Model

Consider a SIMO communication system with one transmitter and N_R diversity receivers operating in a wireless environment with fast time-varying fading as shown in Figure ... The system employs pilot-assisted channel estimation for the channels between the transmitter and each of the N_R diversity receivers. At the transmitter,the modulated data symbols are precoded to achieve Doppler diversity transmission, and pilot symbols are then inserted for estimating and tracking the fast time-varying fading channels between the transmitter and each of the N_R diversity receivers.

The data symbols to be transmitted from the transmitter to each of N_R diversity receivers is divided into slots. As depicted Figure 1, each slot contains K unique modulated data symbols $\mathbf{s} = [s_1, \cdots, s_K]^T \in \mathcal{S}^{K\times 1}$, where $\mathcal{S}$ is the modulation alphabet set, and the superscript $(\cdot)^T$ represents the matrix transpose. To achieve Doppler diversity transmission, a simple repetition code is employed at the transmitter, where each modulated data symbol is repeated N times. Such a repetition precoding scheme ensures that the maximum Doppler diversity can be achieved, at the expense of a lower spectral efficiency. The error probability performance of the system with the repetition code can serve as a lower bound for systems employing spectral-efficient precoding schemes [4].

After precoding, equally-spaced pilot which are used to track the fast time-varying channels are then inserted among the data symbols.

The precoded data vectors are transmitted slot by slot. Each slot can be denoted as $\mathbf{x} = [\mathbf{s}, p_1, \mathbf{s}, p_2, \cdots, \mathbf{s}, p_N]^T$, where p_k, for $k = 1, \ldots, N$, are N pilot symbols, and the data symbol vector $\mathbf{s}$ is repeated N times. Without loss of generality, it is assumed that the pilot symbols are from constant amplitude modulation, such as M-ary phase shift keying (MPSK). There are totally $N_{\text{sym}} = (K+1)N$ symbols in one slot. With such a slot structure, the time duration between two adjacent pilot symbols is $T_p = (K+1)T_s$, where T_s is the symbol period. Thus the pilot symbols sample the channel at a rate $R_p = \frac{1}{(K+1)T_s}$.

The function of the pilot symbols is to estimate and track the fast time-varying fading channels between the transmitter and each of the N_R diversity receivers whilst the data symbols are for data transmission and detection. Due to the difference in the functions of the pilot and the coded data symbols, different amounts of energy are allocated to them. Denote the energy for each pilot and coded data symbol as E_p and E_c, respectively. The total energy in one slot is thus $E_pN + E_cKN$, and the energy per uncoded information bit can be calculated as $E_b = \frac{E_pN+E_cKN}{K\log_2 M}$, where $M = |\mathcal{S}|$ is the cardinality of the modulation constellation set. It is assumed the channels between between transmitter and each of the N_R diversity receivers are identically independently distributed (i.i.d.). Based on this assumption, the received signal can be interpreted as a stack of N_R copies.

A. Pilot Symbols

The pilot and coded data symbols are transmitted over the time-varying fading channel. The index of the k-th pilot symbol p_k, is denoted as $i_k = k(K+1)$, where $k = 1, \cdots, N$. Then the pilot symbols observed at the n_r-th receive antenna can be represented by

$$\mathbf{y}_{n_r,p} = \sqrt{E_p}\mathbf{X}_p\mathbf{h}_{n_r,p} + \mathbf{z}_{n_r,p}, \tag{1}$$

where $\mathbf{y}_{n_r,p} = [y(n_r, i_1), \cdots, y(n_r, i_N)]^T \in \mathcal{C}^{N\times 1}$ and $\mathbf{z}_{n_r,p} = [z(n_r, i_1), \cdots, z(n_r, i_N)]^T \in \mathcal{C}^{N\times 1}$ are the received pilot vector and additive white Gaussian noise (AWGN) vector, respectively, with $\mathcal{C}$ denoting the set of complex numbers,$\mathbf{X}_p = \text{diag}([p_1, \cdots, p_N])$ is a diagonal matrix with the N pilot symbols on its main diagonal,and $\mathbf{h}_{n_r,p} = [h(n_r, i_1), \ldots, h(n_r, i_N)]^T \in \mathcal{C}^{N\times 1}$ is the discrete-time channel fading vector sampled at the pilot locations for the n_r-th antenna. The AWGN vector is a zero-mean symmetric complex Gaussian random vector (CGRV) with covariance matrix $\sigma_z^2\mathbf{I}_N$, where σ_z^2 is the noise variance and $\mathbf{I}_N$ is a size N identity matrix.

Thus, the equivalent receive signals can be denoted as,

$$\overline{\mathbf{y}}_p = \sqrt{E_p}\overline{\mathbf{X}}_p\overline{\mathbf{h}}_p + \overline{\mathbf{z}}_p, \tag{2}$$

where $\overline{\mathbf{y}}_p = [\mathbf{y}_{1,p}^T, \cdots, \mathbf{y}_{N_R,p}^T]^T \in \mathcal{C}^{N_RN\times 1}$ and $\overline{\mathbf{z}}_p = [\mathbf{z}_{1,p}^T, \cdots, \mathbf{z}_{N_R,p}^T]^T \in \mathcal{C}^{N_RN\times 1}$ are the equivalent received pilot vector and additive white Gaussian noise (AWGN) vector, respectively, $\overline{\mathbf{X}}_p = \mathbf{I}_{N_R} \odot \mathbf{X}_p$ is a diagonal matrix with the N_R repeated N pilot symbols on its main diagonal, and $\overline{\mathbf{h}}_p = [\mathbf{h}_{1,p}^T, \cdots, \mathbf{h}_{N_R,p}^T]^T \in \mathcal{C}^{N_RN\times 1}$ is the equivalent channel fading vector. It should be noted that $\mathbf{h}_{n_r,p}$ for $n_r = 1, \cdots, N_R$ are i.i.d variables.

B. Data Symbols

With the repetition code, each modulated data symbol is transmitted N times. The k-th data symbol s_k is transmitted over symbol indices $k_n = (n-1)(K+1)+k$, for $n = 1, \cdots, N$ in a slot. The received sample vector corresponding to the k-th data symbol s_k at the n_r-th antenna can then be expressed as

$$\mathbf{y}_{n_r,k} = \sqrt{E_c}\mathbf{h}_{n_r,k}s_k + \mathbf{z}_{n_r,k}, \tag{3}$$

where $\mathbf{y}_{n_r,k} = [y(n_r, k_1), \cdots, y(n_r, k_N)]^T \in \mathcal{C}^{N\times 1}$, the fading and AWGN vectors are denoted as $\mathbf{h}_{n_r,k} = [h(n_r, k_1), \cdots, h(n_r, k_N)]^T \in \mathcal{C}^{N\times 1}$ and $\mathbf{z}_{n_r,k} = [z(n_r, k_1), \cdots, z(n_r, k_N)]^T \in \mathcal{C}^{N\times 1}$, respectively.

The equivalent receive signals related to the k-th data symbol can then be denoted as,

$$\overline{\mathbf{y}}_k = \sqrt{E_c}\overline{\mathbf{h}}_k s_k + \overline{\mathbf{z}}_k, \tag{4}$$

where $\overline{\mathbf{y}}_k = [\mathbf{y}_{1,k}^T, \cdots, \mathbf{y}_{N_R,k}^T]^T \in \mathcal{C}^{N_RN\times 1}$, the fading and AWGN vectors are denoted as $\overline{\mathbf{h}}_k = [\mathbf{h}_{1,k}^T, \cdots, \mathbf{h}_{N_R,k}^T]^T \in \mathcal{C}^{N_RN\times 1}$ and $\overline{\mathbf{z}}_k = [\mathbf{z}_{1,k}^T, \cdots, \mathbf{z}_{N_R,k}^T]^T \in \mathcal{C}^{N_RN\times 1}$, respectively.

The channel for each transmit-receive antenna pair is assumed to experience wide sense stationary uncorrelated scattering. Also based on the above assumption about the SIMO high mobility system, the channels between the different transmit-receive antenna pair are assumed to be independent, thus the correlation between these channel coefficients is equal to 0. Hence $h(n_r, n)$ is a zero-mean symmetric complex Gaussian random process with the covariance function

$$\mathbb{E}[h(n_r1, m)h^*(n_r2, n)] = \begin{cases} J_0(2\pi f_{\text{D}}|m-n|T_s), & n_{r1} = n_{r2} \\ 0, & n_{r1} \neq n_{r2}. \end{cases} \tag{5}$$

where $\mathbb{E}(\cdot)$ is the mathematical expectation operator, the superscript $(\cdot)^*$ denotes complex conjugate, f_{D} is the maximum Doppler spread of the fading channel, $|\cdot|$ returns the absolute value, T_s is the symbol period, and $J_0(x)$ is the zero-order Bessel function of the first kind.

At the receivers, we can estimate the channel coefficients $\mathbf{h}_k$ of the coded data symbols by using the distorted observations of the pilot symbols $\overline{\mathbf{y}}_p$. The estimated channel coefficients will then be used to detect the data symbols.

III. Studies of Statistical Properties of the Estimated Channel

This section studies the statistical properties of the estimated CSI with the aid of MMSE channel estimations.

A. MMSE Channel Estimation

Due to the temporal channel correlation, we can directly estimate the channel coefficients of the coded data symbols by using the received pilot symbols. The linear MMSE estimation of the channel coefficients corresponding to the k-th data symbol is

$$\hat{\overline{\mathbf{h}}}_k = \overline{\mathbf{W}}_k^H \overline{\mathbf{y}}_p, \tag{6}$$

where $\overline{\mathbf{W}}_k \in \mathcal{C}^{N \times N}$ is the MMSE estimation matrix and $(\cdot)^H$ denotes the matrix Hermitian operation. Define the error vector $\overline{\mathbf{e}}_k = \overline{\mathbf{h}}_k - \hat{\overline{\mathbf{h}}}_k$. Based on the orthogonal principle, $\mathbb{E}[\overline{\mathbf{e}}_k \overline{\mathbf{y}}_p^H] = \mathbf{0}$, the MMSE estimation matrix can be solved as

$$\overline{\mathbf{W}}_k^H = \sqrt{E_p} \overline{\mathbf{R}}_{kp} \overline{\mathbf{X}}_p^H (E_p \overline{\mathbf{X}}_p \overline{\mathbf{R}}_{pp} \overline{\mathbf{X}}_p^H + \sigma_z^2 \mathbf{I}_{N_R N})^{-1}, \tag{7}$$

where $\overline{\mathbf{R}}_{kp} = \mathbb{E}[\overline{\mathbf{h}}_k \overline{\mathbf{h}}_p^H] \in \mathcal{R}^{N_R N \times N_R N} = \mathbf{I}_{N_R} \odot \mathbf{R}_{kp}$ and $\overline{\mathbf{R}}_{pp} = \mathbb{E}[\overline{\mathbf{h}}_p \overline{\mathbf{h}}_p^H] = \mathbf{I}_{N_R} \odot \mathbf{R}_{pp} \in \mathcal{C}^{N_R N \times N_R N}$ with their elements defined in (5), and $\mathcal{R}$ denotes the set of real numbers.

The matrix $\mathbf{R}_{pp}$ is a symmetric Toeplitz matrix with the first row and column being $[\rho_0, \rho_1, \cdots, \rho_{N-1}]^T$, where $\rho_n = J_0(2\pi f_{\text{D}} |n| T_p)$. The matrix $\mathbf{R}_{kp}$ is a Toeplitz matrix, with the first row being $[\tau_{-1}, \tau_{-2}, \cdots, \tau_{-N}]$, and the first column is $[\tau_{-1}, \tau_0, \cdots, \tau_{N-2}]^T$, where $\tau_u = J_0\left(2\pi f_{\text{D}}(uT_p + kT_s)\right)$.

The correlation of the error vector, $\overline{\mathbf{R}}_{ee} = \mathbb{E}[\overline{\mathbf{e}}_k \overline{\mathbf{e}}_k^H] \in \mathcal{R}^{N \times N}$, can be calculated as

$$\overline{\mathbf{R}}_{ee} = \overline{\mathbf{R}}_{kk} - \overline{\mathbf{R}}_{kp} \left(\overline{\mathbf{R}}_{pp} + \frac{1}{\gamma_p} \mathbf{I}_{N_R N} \right)^{-1} \overline{\mathbf{R}}_{kp}^H, \tag{8}$$

where $\gamma_p = \frac{E_p}{\sigma_z^2}$ is the SNR of the pilot symbols, $\overline{\mathbf{R}}_{kk} = \mathbb{E}[\overline{\mathbf{h}}_k \overline{\mathbf{h}}_k^H] = \mathbf{I}_{N_R} \odot \mathbf{R}_{kk} \in \mathcal{R}^{N_R N \times N_R N}$, and $\overline{\mathbf{X}}_p \overline{\mathbf{X}}_p^H = \mathbf{I}_{N_R N}$ are used in the derivation of the above equation. It should be noted that $\overline{\mathbf{R}}_{kk} = \overline{\mathbf{R}}_{pp}$, $\forall\, k$. It is interesting to note that (8) can be written as $\overline{\mathbf{R}}_{ee} = \mathbf{I}_{N_R} \odot \mathbf{R}_{ee}$, where $\mathbf{R}_{ee}$ is denoted as

$$\mathbf{R}_{ee} = \mathbf{R}_{kk} - \mathbf{R}_{kp} \left(\mathbf{R}_{pp} + \frac{1}{\gamma_p} \mathbf{I}_N \right)^{-1} \mathbf{R}_{kp}^H. \tag{9}$$

B. Statistical Properties of the Estimated Channel

In this subsection, the statistical properties of the estimated CSI vector, $\hat{\overline{\mathbf{h}}}_k$, are studied. The results of this study will be used to develop the optimum SIMO high mobility receiver in the presence of imperfect CSI. Given the pilot symbols, the vector $\overline{\mathbf{y}}_p$ is zero-mean Gaussian distributed with covariance matrix $\mathbb{E}[\overline{\mathbf{y}}_p \overline{\mathbf{y}}_p^H] = E_p \overline{\mathbf{X}}_p \overline{\mathbf{R}}_{pp} \overline{\mathbf{X}}_p^H + \sigma_z^2 \mathbf{I}_{N_R N}$. From (6), the estimated channel coefficient vector is a linear transformation of a zero-mean Gaussian vector. Therefore $\hat{\overline{\mathbf{h}}}_k$ is zero mean Gaussian distributed with the covariance matrix

$$\overline{\mathbf{R}}_{\hat{k}\hat{k}} = \overline{\mathbf{R}}_{kp} \left(\overline{\mathbf{R}}_{pp} + \frac{1}{\gamma_p} \mathbf{I}_{N_R N} \right)^{-1} \overline{\mathbf{R}}_{kp}^H = \overline{\mathbf{R}}_{kk} - \overline{\mathbf{R}}_{ee}. \tag{10}$$

Define the channel estimation MSE as $\overline{\sigma}_{e,N}^2 = \frac{1}{N_R N} \text{trace}(\overline{\mathbf{R}}_{ee})$, where $\text{trace}(\cdot)$ is the matrix trace operator. From (8), the MSE for a block with N pilot symbols can be expressed as

$$\overline{\sigma}_{e,N}^2 = \frac{1}{N_R N} \text{trace} \left(\overline{\mathbf{R}}_{kk} - \overline{\mathbf{R}}_{kp} \left(\overline{\mathbf{R}}_{pp} + \frac{1}{\gamma_p} \mathbf{I}_{N_R N} \right)^{-1} \overline{\mathbf{R}}_{kp}^H \right). \tag{11}$$

The channel estimation MSE is a function of the SNR of the pilot symbols, and the channel correlation matrices $\overline{\mathbf{R}}_{kp}$ and $\overline{\mathbf{R}}_{pp}$, which in turn depend on the Doppler spread and the space between two adjacent pilot symbols T_p. From (11), the computation of the MSE involves a matrix inversion and a trace operation. Hence to explicitly identify the impacts of the various system parameters and the channel estimation quality on the MSE, we resort to asymptotic analysis by letting $N \to \infty$ while keeping a finite T_p. The results are presented in Theorem 1.

Theorem 1: When $N \to \infty$ while keeping a finite T_p, the asymptotic MSE, $\overline{\sigma}_e^2 = \lim_{N \to \infty} \overline{\sigma}_{e,N}^2$, of channel estimation is

$$\overline{\sigma}_e^2 = 1 - \frac{8\gamma_p \arctan\left(\sqrt{\frac{2\gamma_p - \omega_{\text{D}} T_p}{2\gamma_p + \omega_{\text{D}} T_p}} \right)}{\pi \sqrt{(2\gamma_p)^2 - (\omega_{\text{D}} T_p)^2}}, \text{ for } \frac{1}{T_p} \geq 2 f_{\text{D}}, \tag{12}$$

where $\omega_{\text{D}} = 2\pi f_{\text{D}}$, and $T_p = (K+1) T_s$ is the time duration between two adjacent pilot symbols.

Proof: Intuitively, channel estimation for each channel is independent so that the estimation MSE is consistent with the SISO case. Simply, based on the property $\text{trace}(\overline{\mathbf{R}}_{ee}) = \text{trace}(N_R \mathbf{R}_{ee}) = N_R \times \text{trace}(\mathbf{R}_{ee})$, it can be proved that $\overline{\sigma}_e^2 = \lim_{N \to \infty} \frac{1}{N_R N} \text{trace}(\overline{\mathbf{R}}_{ee}) = \frac{N_R}{N_R} \lim_{N \to \infty} \text{trace}(\mathbf{R}_{ee}) = \sigma_e^2$ which is derived in [8]. A strict and complete proof is given by proof A. ■

IV. Optimum Diversity Receiver for a SIMO High Mobility System in the Presence of Imperfect CSI

In this section, the optimum diversity receiver for the SIMO high mobility system with imperfect CSI is studied and thereafter, theoretical error probability of the diversity receiver is derived using the statistical properties of the estimated CSI.

With the above analysis, the diversity receiver can be easily obtained from the previous results, through Kronecker product with a identity matrix $\mathbf{I}_N$.

A. The SIMO Optimum Diversity Receiver in the Presence of Imperfect CSI

The receiver performs detection of s_k in (4) based on the knowledge of received data vector $\overline{\mathbf{y}}_k$ and the estimated CSI vector $\hat{\overline{\mathbf{h}}}_k$. Conditioned on $\hat{\overline{\mathbf{h}}}_k$, $\overline{\mathbf{h}}_k$ in (4) is Gaussian distributed with mean $\mathbf{u}_k | \hat{\overline{\mathbf{h}}}_k = \mathbb{E}[\overline{\mathbf{h}}_k | \hat{\overline{\mathbf{h}}}_k] = \hat{\overline{\mathbf{h}}}_k$ and covariance matrix $\mathbf{R}_{kk} | \hat{\overline{\mathbf{h}}}_k) = \mathbb{E}[(\overline{\mathbf{h}}_k - \mathbf{u}_k | \hat{\overline{\mathbf{h}}}_k)[(\overline{\mathbf{h}}_k - \mathbf{u}_k | \hat{\overline{\mathbf{h}}}_k)^H | \hat{\overline{\mathbf{h}}}_k)] = \overline{\mathbf{R}}_{ee}$.

We have the following the proposition regarding the optimum decision rule for SIMO diversity system in (4).

Proposition 1: With the estimated CSI obtained from (6), the optimum detection rule for the diversity system in (4) with MPSK modulation is

$$\hat{s}_k = \underset{s_k \in \mathcal{S}}{\arg\min} \left\{ |\overline{\alpha}_k - s_k|^2 \right\}, \tag{13}$$

where $\mathcal{S}$ is the MPSK modulation set and the decision variable $\overline{\alpha}_k$ is

$$\overline{\alpha}_k = \sqrt{E_c}\hat{\overline{\mathbf{h}}}_k^H \left(E_c\overline{\mathbf{R}}_{ee} + \sigma_z^2\mathbf{I}_{N_RN}\right)^{-1}\overline{\mathbf{y}}_k. \tag{14}$$

Proof: The proof is in Appendix B. ■

Remarks: If we consider the scenario where the SIMO diversity receiver has perfect knowledge of the channel (i.e. perfect CSI), then $\overline{\mathbf{R}}_{ee} = \mathbf{0}$ in (14). The decision variable in (14) then becomes $\overline{\alpha}_k = \frac{\sqrt{E_c}}{\sigma_z^2}\hat{\overline{\mathbf{h}}}_k^H\overline{\mathbf{y}}_k$. In this case, the optimum decision rule in Proposition 1 converges to the conventional maximal ratio combining receiver (MRC) receiver. However, in the scenario where the SIMO diversity receiver has imperfect CSI, the result in Proposition 1 shows that MRC is no longer optimal. The presence of channel estimation errors quantified in the matrix $\overline{\mathbf{R}}_{ee}$ in (14) has an impact on the decision process. Hence, the new optimum decision rule has to take into account the statistical properties of the channel estimation errors.

B. Error Probability in the Presence of imperfect CSI

In this subsection, the error performance of SIMO systems with *M-ary* phase shift keying (MPSK) modulations operating in presence of imperfect CSI is studied. The error probability is derived by using the new optimum decision rule in Proposition 1 and the statistical properties of the estimated CSI $\hat{\overline{\mathbf{h}}}_k$.

From (14)

Since $\mathbf{y}_k|(\hat{\mathbf{h}}_k, s_k)$ is Gaussian distributed, from (14), the decision variable α_k conditioned on $\hat{\mathbf{h}}_k$ and s_k is also Gaussian distributed, *i.e.*, $\alpha_k|(\hat{\mathbf{h}}_k, s_k) \sim \mathcal{CN}(u_{\alpha_k|\hat{h}_k,s}, \sigma^2_{\alpha_k|\hat{h}_k,s})$. The conditional mean and variance are

$$u_{\alpha_k|\hat{h}_k,s_k} = \hat{\mathbf{h}}_k^H\left(\mathbf{R}_{ee} + \frac{1}{\gamma_c}\mathbf{I}_N\right)^{-1}\hat{\mathbf{h}}_k s_k, \tag{15a}$$

$$\sigma^2_{\alpha_k|\hat{h}_k,s_k} = \hat{\mathbf{h}}_k^H\left(\mathbf{R}_{ee} + \frac{1}{\gamma_c}\mathbf{I}_N\right)^{-1}\hat{\mathbf{h}}_k, \tag{15b}$$

where $\gamma_c = \frac{E_c}{\sigma_z^2}$.

Proposition 2: With the optimum receiver described in Proposition 1, the SER of Doppler diversity systems with MPSK modulation and operating with imperfect CSI is

$$P(E) = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}}\left[\det\left(\mathbf{I}_{N_RN} + \frac{\sin^2(\frac{\pi}{M})}{\sin^2(\phi)}\overline{\mathbf{A}}\right)\right]^{-1} d\phi, \tag{16}$$

where $\det(\cdot)$ is the matrix determinant operator, and

$$\begin{aligned}\overline{\mathbf{A}} =& \overline{\mathbf{R}}_{\hat{k}\hat{k}}\left(\overline{\mathbf{R}}_{ee} + \frac{1}{\gamma_c}\mathbf{I}_{N_RN}\right)^{-1} = \overline{\mathbf{R}}_{kp}\left(\overline{\mathbf{R}}_{pp} + \frac{1}{\gamma_p}\mathbf{I}_{N_RN}\right)^{-1}\overline{\mathbf{R}}_{kp}^H \\ &\times\left[\overline{\mathbf{R}}_{pp} - \overline{\mathbf{R}}_{kp}\left(\overline{\mathbf{R}}_{pp} + \frac{1}{\gamma_p}\mathbf{I}_{N_RN}\right)^{-1}\overline{\mathbf{R}}_{kp}^H + \frac{1}{\gamma_c}\mathbf{I}_{N_RN}\right]^{-1}.\end{aligned} \tag{17}$$

Proof: The proof is in Appendix C. ■

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