Gain Compression and Linewidth Enhancement Factor in Mid-IR Quantum Cascade Lasers

Andreas Hangauer, *Member, IEEE*, and Gerard Wysocki

***Abstract*—We have observed and quantified the adiabatic and transient chirp in a directly modulated quantum cascade laser (QCL). Those wavelength tuning effects are well-characterized in diode lasers, and the rate equation model that successfully describe the diode laser behavior also provides an excellent fit for the QCL data. In this study, we have extracted the linewidth enhancement factor (α_H) from the transient chirp and the gain compression factor from the adiabatic chirp. We postulate that the extraction of the α_H from the transient chirp is valid for the QCL case, despite additional tuning effects in QCLs (e.g., voltage tuning) that are negligible in diode lasers. Also in the QCL the adiabatic chirp coefficient strongly increases with laser output power but still stays an order of magnitude below typical values known from diode lasers. We hypotesize possibility of the adiabatic chirp to be connected to the $\chi^{(3)}$ nonlinearity (responsible for four-wave mixing that was recently attributed to the FM self-locking in QCLs), but the exact origin remains to be experimentally confirmed in future work.**

***Index Terms*—Quantum cascade lasers, laser modulation, gain compression, laser chirp.**

I. Introduction

When the injection current of diode lasers is modulated in the 100 MHz to several GHz range a characteristic frequency modulation (FM) (or chirping) is observed. There are extensive studies both experimental and theoretical for diode lasers [1] since laser chirp usually degrades performance [2], [3] of high-speed fiber communication systems (due to the fiber dispersion). Equivalent experimental data for mid-IR QCLs are very sparse, but with emerging spectroscopic applications that require high frequency laser modulation [4], the RF modulation properties of QCLs become increasingly important. We have recently proposed a set of experimental methods to perform measurements of high frequency (up to 1.7 GHz) modulation capabilities of QCLs [5]. In this work we are focusing on modeling and retrieval of specific QCL parameters from the high frequency measurement data.

The primary laser parameters associated with the chirp in diode lasers that play a significant role in describing laser behavior in various applications are the alpha factor (α_H) [6], [7] and the gain compression coefficient (ε) [8]. These parameters are essential in describing the semiconductor laser linewidth [6] (thus α_H is also called the linewidth enhancement factor), laser behavior with respect to optical feed-back [9], [10], injection locking [11], as well as far- and near-field emission characteristic [12]. It also has been found to be crucial in explaining filamentation or self-focusing effects in broad stripe lasers [13] and in predicting the correlation between intensity and frequency noise [14].

The gain compression is very important in correctly describing the RF modulation performance of diode lasers and is the dominant effect that introduces the damping in the intensity modulation (IM) response (first reported in [15], and now found in textbooks e.g. [14]). In the FM response the gain compression introduces the adiabatic chirp [1]—one of the two components of the important (non-thermal) electronic laser wavelength tuning. Gain compression is responsible for non-linear effects at modulation frequencies above thermal cutoff and below relaxation frequency which includes second- [16], [17] and third-order harmonic distortion [18], [17] as well as intermodulation distortion [19], [17] in both the FM and the IM. It has also been shown that injection locked operation can reduce the nonlinear distortion [20] due to interdependence of both effects. Gain compression can also strongly increase relative intensity noise in lasers with low side-mode suppression ratio [21]. Hence, the alpha factor and gain compression extracted from the QCL chirping may play an important role in similar applications of QCLs. As an example the gain compression effect is not included in many theoretical studies on QCLs such as RF modulation [22], feed-back behavior [23], or optical injection locking [24]–[26] where it could potentially improve the models. Therefore in this paper we focus on RF modulation experiments, which allow for very precise determination of these parameters in QCLs.

Measurements of the α_H in QCLs are up to now based on measurement of the line-width broadening compared to the Schawlow-Towns limit [27], [28], (sub-threshold) gain measurements [29]–[32], high speed dual modulation [33] or techniques relying on optical feedback [34]–[37]. While the latter method has the advantage that no photodetector is needed, all these techniques yield relatively high errors in α_H value (with the exception of [37]). Since the alpha factor is very small in QCLs (absolute value typically $|\alpha_H| < 1$, when the laser operates close to the gain peak) the error bars can easily mask the actual value. For the direct linewidth observation this is especially critical as the linewidth broadening factor of $1 + \alpha_H^2$ is very close to 1, which make α_H determination challenging. A precision of α_H determination presented in this paper allows for very small relative error as discussed below.

Manuscript received February 7, 2015; revised April 6, 2015; accepted April 8, 2015. Date of publication April 10, 2015; date of current version May 21, 2015. This work was supported by the U.S. Environmental Protection Agency under Grant RD-83513701-0, and by the NSF ERC MIRTHE Award EEC-0540832.

A. Hangauer was with the Electrical Engineering Department, Princeton University, Princeton, NJ 08544 USA. He is now at Siemens AG, Corporate Technology, 80200 Munich, Germany (e-mail: hangauer@princeton.edu).

G. Wysocki is with the Electrical Engineering Department, Princeton University, Princeton, NJ 08544 USA (e-mail: gwysocki@princeton.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JSTQE.2015.2422073

1077-260X © 2015 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission.
See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

Observing the ratio of the frequency to intensity modulation is one of the most precise methods to determine the linewidth enhancement factor in diode lasers [38]–[40]. For lasers operating in the telecom wavelength range this is usually implemented exploiting the dispersion of fibers [41]. Unfortunately, due to technical limitations (e.g. availability of optical fibers in the mid-IR) this method cannot be adopted easily in the mid-IR. Recently we have presented measurement method for determination of the RF modulation behavior of QCLs, which circumvented the above problem by using the optical dispersion associated with a gas absorption line to implement the FM-IM ratio measurement [5]. This method is used to collect high quality measurement data, which serve as the basis for this analysis paper. The QCL used in this work is a commercial, continuous wave, room-temperature distributed feed-back laser operating at 9.6 μm (Hamamatsu, Type: LC0318, S/N: 3249 -1282 -2Be).

It should be noted that QCLs based on intersubband transitions have somewhat different properties than diode lasers (based on inter-band transitions). In particular QCLs can have a linewidth enhancement factor significantly smaller than 1 and also of both signs [42], [37]. Diode lasers on the other hand only have positive α_H with values typically above 3. Only active media based on a zero dimensional quantization (e.g, "quantum dots"), to which the QCLs intersubband transitions have similar behavior, allow for near zero and even negative alpha factor [43]. Since the gain curve in QCLs is nearly symmetric, the alpha factor in these lasers nulls at the gain peak and increases (with both signs) when the operating wavelength is detuned from the gain peak [42]. Since the gain curve and the lasing wavelength have different temperature tuning coefficients it is important to record the measurements at constant *internal* temperature, which eliminates ambiguity of the results.

In diode lasers one usually observes an adiabatic chirp and a transient chirp (these terms are well established in the field of diode lasers [17]), which can be distinguished by their different frequency behavior [1]. This fast wavelength tuning effects are caused by non-thermal refractive index change of the active medium (e.g. due to carrier density), and in applications that require high-FM such chirp can be performance limiting (see e.g. [3] describing impact of the laser chirp on fast fiber-based communication). The transient chirp describes the variation of the laser frequency proportional to the derivative of the instantaneous laser power, while the adiabatic chirp is proportional to the instantaneous laser power. Since we find a good qualitative match of the experimental data for a QCL with this phenomenological description of chirping we adopted these already well established terms for QCLs as well.

The transient chirp is associated with the linewidth enhancement factor α_H and is present in all lasers for which $\alpha_H \neq 0$ (see the appendix I for theoretical details). The adiabatic chirp is caused by an effect referred to as the gain compression. Therefore by modeling the laser chirp one can determine the related parameters. The experimental data acquired for the QCL also clearly show regions, which are identified as transient and adiabatic chirp. In this work we show that even in the QCL case the transient chirp data is suitable to extract the α_H.

II. Modeling of the FM-IM Ratio

If a laser's injection current is sinusoidally modulated at frequency f it generates an intensity variation with amplitude ΔP and an instantaneous optical frequency variation of $\Delta\nu$, i.e.

$$P(t) = P_0(1 + m\cos(2\pi f t)) \tag{1}$$

$$\nu(t) = \nu_0 + \Delta\nu\cos(2\pi f t - \theta) \tag{2}$$

with $m = \Delta P/P_0$ the IM index. Similarly one defines the FM index $\beta = \Delta\nu/f$ and the FM-IM phase-shift θ [44].

Under very general assumptions (see Appendix I, Eq. (27)) one can derive that the complex FM to IM ratio of a laser can be expressed by

$$\frac{\beta}{m}e^{i\theta} = \frac{\alpha_H}{2}\left(i + \frac{\kappa P_0}{2\pi f}\right) + \frac{P_0\frac{\partial\nu_0}{\partial P_0}}{f(1 + if/f_{\mathrm{th}})} \tag{3}$$

with adiabatic chirp parameter κ, the linewidth enhancement factor α_H, the thermal tuning parameters f_{th} and $\partial\nu_0/\partial P_0$ which quantify the dynamic and static behavior.

This expression consists of three terms with characteristic frequency dependence by which they can be well discriminated in experimental data. First the $\mathrm{i}\alpha_H/2$ term describes the transient chirp that is constant with respect to modulation frequency and with +90° phase-shift; second, the adiabatic chirp with 1/*f* behavior and zero phase-shift; and third, the thermal tuning with asymptotic $1/f^2$ behavior and also +90° phase-shift. Note that $\partial\nu_0/\partial P_0$ is negative. The last term quantifying the thermal tuning is only asymptotically valid (at frequencies well above the thermal cutoff) because QCLs and diode lasers typically show more complex behavior [45]–[47] than the first order low-pass which the last term of (3) describes. Expression (3) (without the thermal tuning) and the terminology of transient and adiabatic chirp was introduced in [1] and is found throughout the literature for diode lasers (e.g. [17]). The meaning of the alpha-factor is well known and can be extracted with high accuracy from the transient chirp (it is also shown in appendix I, that the effect of voltage tuning in QCLs will only affect the κ and does not disturb the α_H measurement).

The adiabatic chirp coefficient κ can be related to the nonlinear refractive index n_2, by (see Appendix I.D)

$$n_2 = -\frac{\lambda_0}{4\pi v_g(1-R)A}\alpha_H\kappa \tag{4}$$

with λ_o being the operating wavelength, *R* the laser facet reflectivity, *A* the effective mode area and v_g the group velocity. Throughout this work we assumed $R = 0.5$ and $\mathrm{A} = 9\,\mu\mathrm{m} \times 2.2\,\mu\mathrm{m}$ and estimated a value of $v_g = c/3.5$.

Fig. 1 shows the measurement data and the corresponding fit to model in Eq. (3). The least squares curve fit varies the three parameters α_H, κ and f_{th} to produce an optimum match to the β/m ratio and the FM-IM phase-shift θ. As expected from [5] the data clearly show the three regions with slopes $1/f^2$ (thermal effect), 1/*f* (adiabatic chirp) and constant region (transient chirp).

The phase-shift data very sensitively indicates the presence of the adiabatic chirp, i.e., when it deviates significantly from 90° as e.g. in the black trace. If there is no adiabatic chirp, the thermal tuning (with asymptotic phase shift 90°) directly leads

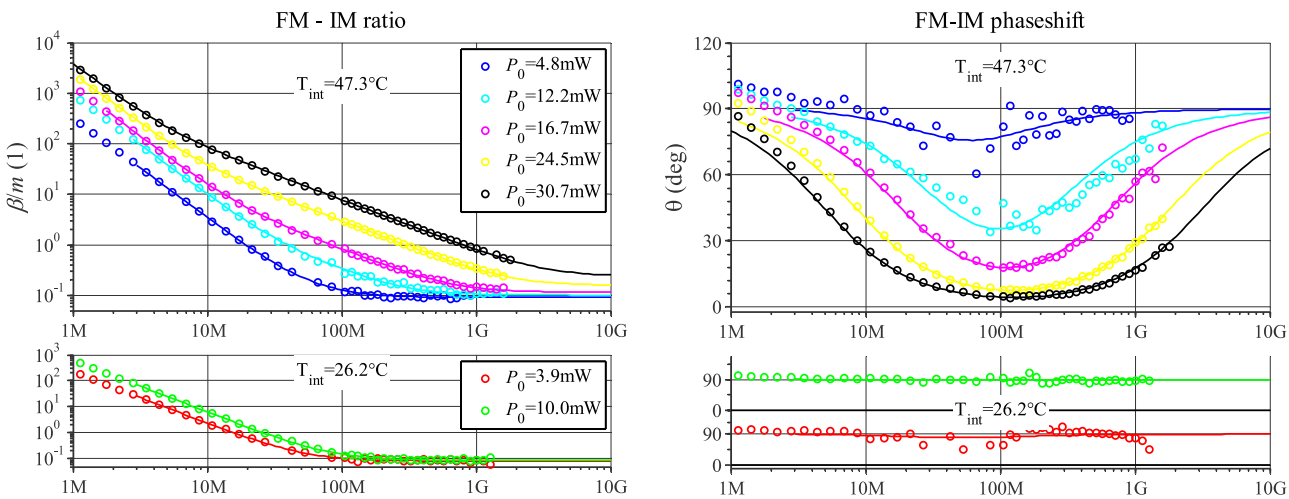

Fig. 1. Measurement data (dots) and theory (solid, parameters obtained using least squares fitting to Eq. (3)). The curves in the upper and lower panel correspond to different bias points (reference by optical power) and at constant *internal* temperature $T_{\rm int}$ (to exclude self-heating as the origin of variation). The laser parameters at each bias point other than optical power (such as injection current, tuning coefficient, etc.) are listed in Table I.

to the transient chirp with also 90° phase shift. This is evident in the 10 mW curve aquired at $T_{\rm int}$ of 26.2 °C, in which the FM-IM phase-shift is constant at 90°.

Fig. 2 shows the extracted parameters and Table I summarizes the obtained fit parameters and the laser parameters that were used to generate the data.

III. Gain Compression Parameter in QCL

In diode lasers the adiabatic chirp parameter κ is typically found to be independent of laser power [48], [49]. As shown in Fig. 2 for the QCL case discussed here this is not the case. Also in case of the QCL linewidth enhancement factor a significant broadening is observed, which is an effect usually referred to as "power broadening" because the laser linewidth deviates from the expected behavior ("constant α_H").

Since we use the measurment technique from [5], which assures constant internal temperature of the QCL, the observed effects can be considered free from temperature effects due to self-heating. Based on this assuption we can show that the gain compression effect is consistent with the bending of the laser power versus current (*L–I*) characteristic which is also related to the second harmonic distortion mentioned earlier in the introduction.

For analysis we use a QCL rate equation model with an added gain compression term. The difference to the established rate equation model for QCLs [42] is that we assume a gain of the form

$$g(N,S) = \frac{\partial g}{\partial N}(N - N_{\rm bf})f(S) \tag{5}$$

where g denotes the modal gain, N the carrier density quantifying the population inversion, S the number of photons in the laser cavity, $\partial g/\partial N$ the gain coefficient and $N_{\rm bf}$ the thermal backfilling parameter. The gain compression is modeled thorugh the function $f(S)$. Previously most of the theoretical work on QCLs utilizing the rate equation model assumed $f(S) = 1$, i.e. ignored the gain compression. In case of diode lasers the best phenomenological forms of $f(S)$ were found to be: $f(S) = 1 - \varepsilon S$ or $f(S) = 1/(1 + \varepsilon S)$,(with ε being the gain compression parameter), which are commonly implemented in the rate equation based theoretical work. With gain compression and asuming that the active region refractive index is only dependent on N the adiabatic chirp parameter becomes (see Eq. (29) in the appendix):

$$\kappa = -\frac{1}{P_0} S_0 v_g \frac{\partial g}{\partial S} = -\frac{1}{P_0 \tau_p} \frac{S_0 f'(S_0)}{f(S_0)}. \tag{6}$$

For the common choice for diode lasers of $f(S) = 1 - \varepsilon S$ the κ parameter is almost independent of a bias point. However, this is inadequate to fit the QCL experimental data (cf. Fig. 2). Therefore we have also chosen to introduce a function $f(S)$ which phenomenologically can match the experimental QCL data.

As outlined in the appendix I the rate equation model allows for derivation of a static solution for the $S(I)$ characteristics, which is essentially the *L–I* curve (since photon number S is proportional to the emitted power P). We obtain

$$S(I) = \frac{\eta \tau_p}{e}(I - I_{\rm th}(S)) \tag{7a}$$

with an intensity-dependent threshold current (see the appendix for details on derivation):

$$I_{\rm th} = I_{\rm th}(S) = \frac{e}{\tau_n \eta} \frac{1}{\tau_p v_g \frac{\partial g}{\partial N}} \frac{1}{f(S)} + I_{\rm ther} \tag{7b}$$

where $I_{\rm ther} = \frac{e}{\tau_n \eta} N_{\rm bf}$ is the current needed to compensate thermal backfilling, which lowers the population inversion. Without gain compression the intensity dependence would not be present and for $S = 0$ we obtain the same result as in the standard QCL literature [42]. Here we will use the model from Eq. (7a) and (7b) to determine gain compression in the QCL device under study.

In the following sections we will use photon number S and output power P interchangeably since both are proportional thorugh $P = h\nu v_g \alpha_m S$ with α_m the mirror loss. So $I_{\rm th}(S)$ or

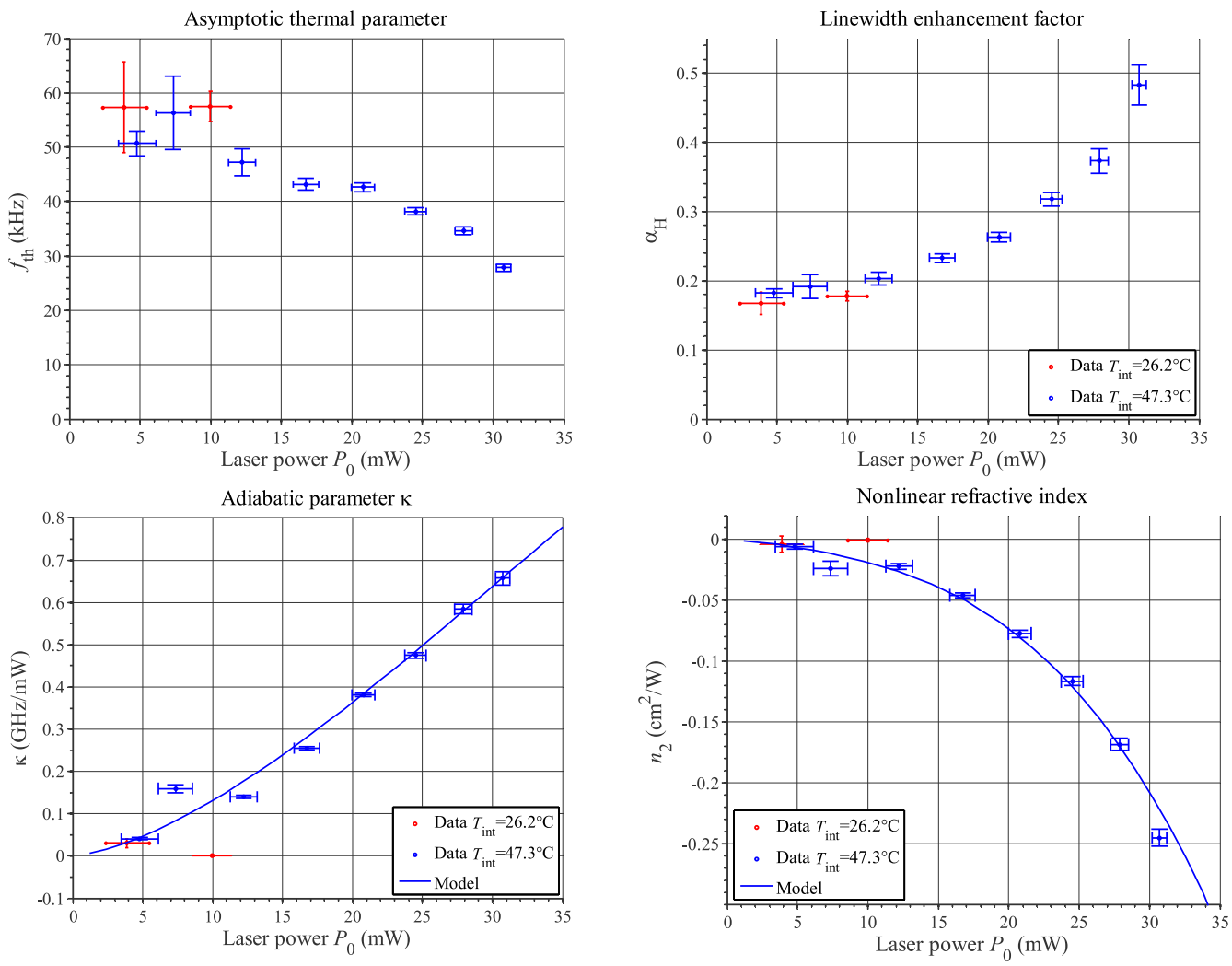

Fig. 2. The obtained laser parameters from fitting the data in Fig. 1, their variation with laser operation point (denoted here by output power P_0; the other parameters can be found in Table I) and at constant internal laser temperature $T_{\rm int}$.

$I_{\rm th}(P)$ refer to the same relative quantity but in different "units" (i.e. photon number or actual optical power). Morover, since we will also operate with relative quatities any signals proportional to P or S (e.g. arbitrary detector signal units) will be sufficient to obtain the results.

Given that $f(0) = 1$, by definition, we can extract the gain compression $f(S)$ from a measurement of the threshold according to Eq. (7):

$$f(S) = \frac{I_{\rm th}(0) - I_{\rm ther}}{I_{\rm th}(S) - I_{\rm ther}} \approx \frac{I_{\rm th}(0)}{I_{\rm th}(S)}. \tag{8}$$

The $I_{\rm ther}$ is not known exaclty, but it will be considered constants for all our measurments performed at constant internal temperature. With an assumption that $I_{\rm ther}$ is small compared to the threshold current, i.e. $I_{\rm th} \gg I_{\rm ther}$, $f(S)$ in Eq. (8) can be approximated with a simple ratio of threshold currents. This approximation should yield satisfactory results even if $I_{\rm ther}$ is <20% of $I_{\rm th}$ (which is true for most of the current designs of QCL gain regions operating cw), because the relative error on the amount of compression $1-f(S)$ should also remain <20%.

The intensity dependent threshold can be extracted from experimental data with the assumption of $f'(0) = \frac{{\rm d}f}{{\rm d}S}\Big|_{S=0} = 0$, because then

$$I_{\rm th}(S) = I - S \times \frac{{\rm d}I}{{\rm d}S}\Big|_{S=0}. \tag{9}$$

In case $f'(0) \neq 0$ the measured slope efficiency at zero power $\frac{{\rm d}S}{{\rm d}I}\Big|_{S=0}$ is not equal to the intrinsic slope efficiency $\eta\tau_p/e$, which is the reason why for diode lasers (which have $f'(0) \neq 0$) this approach is not feasible. For QCLs the measurement data of κ indicates that the gain compression effect is non constant but continuously rises to higher values as S increases. This suggests that $f'(0) = 0$ is a reasonable assumption, and the approximation made here for the $I_{\rm th}(S)$ is valid. Note, that for evaluation of Eq. (9) the unit of S does not matter and it can be replaced by optical power P in either absolute or arbitrary units.

If we apply Eq. (9) to the experimental data and evaluate $f(S)$ with Eq. (8) we obtain curves as shown in Fig. 3.

It is important to emphasize that this approach is only possible because the measurement data is recorded at constant internal

TABLE I
LASER PARAMETERS AND FIT RESULTS DESCRIBING THE CHIRP PARAMETERS AND LINEWIDTH ENHANCEMENT FACTOR

Set		Laser parameter							Fit result			
#	T_S °C	I_0 mA	I_0–I_{th} mA	P_0 mW	U_0 V	λ_0 μm	T_{int} °C	$\partial\nu_0/\partial P_0$ GHz/mW	f_{th} kHz	α_H 1	κ GHz/mW	n_2 1
1	−4	527.5	22.2	3.9	6.95	9.607	26.2	1.0	57.0(170)	0.167(32)	0.031(21)	−0.004(13)
2	−6	562.5	57.1	10.0	7.09	9.607	26.2	1.1	57.5(56)	0.178(14)	0.0015(41)	−0.0002(28)
3	7	572.9	27.4	4.8	7.034	9.618	47.3	1.4	50.7(46)	0.182(13)	0.0405(61)	−0.0057(44)
4	6	587.7	42.1	7.3	7.094	9.618	47.3	1.4	56.0(130)	0.191(34)	0.160(19)	−0.024(12)
5	4	619.1	73.6	12.2	7.222	9.618	47.3	1.7	47.2(50)	0.203(18)	0.1402(73)	−0.0220(48)
6	2	648.4	102.8	16.7	7.345	9.618	47.3	2.0	43.2(22)	0.233(13)	0.2546(65)	−0.0459(45)
7	0	677.7	132.2	20.8	7.470	9.618	47.3	2.2	42.6(16)	0.263(14)	0.3808(88)	−0.0775(56)
8	−2	704.1	158.6	24.5	7.587	9.618	47.3	2.6	38.2(14)	0.318(21)	0.474(13)	−0.1165(72)
9	−4	731.6	186.0	27.9	7.712	9.618	47.3	3.2	34.6(14)	0.373(35)	0.584(23)	−0.1685(99)
10	−6	758.0	212.4	30.7	7.838	9.618	47.3	4.5	27.8(13)	0.483(57)	0.657(32)	−0.245(14)

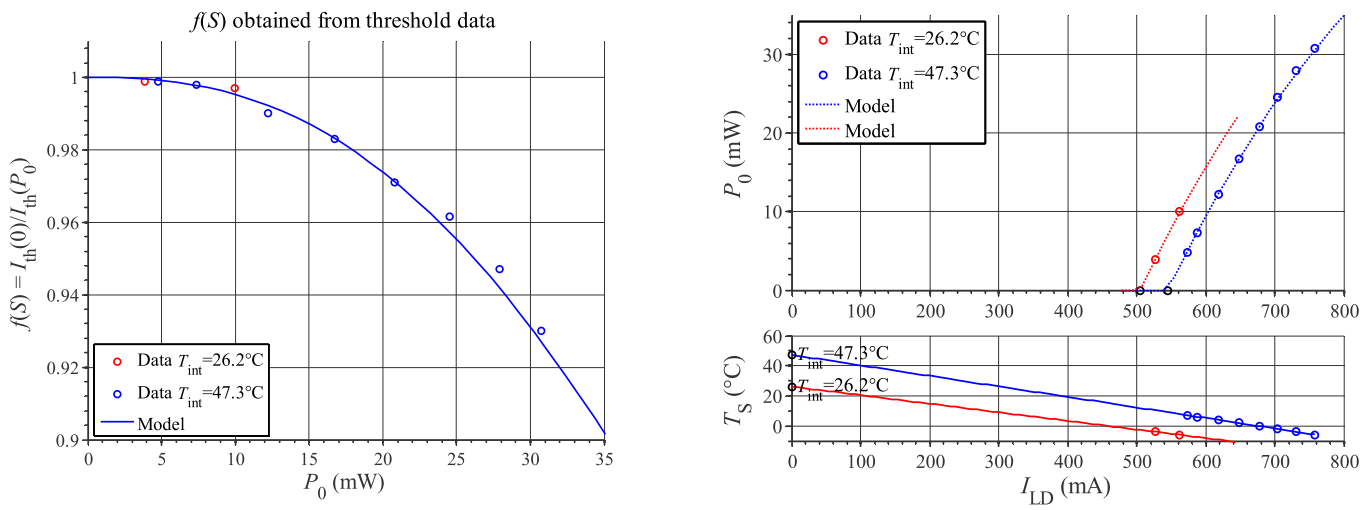

Fig. 3. Gain compression obtained from the intensity dependent threshold current according to Eq. (8) and Eq. (9) (left), which is essentially the residual bending of the *L–I* characteristic at constant internal temperature (right). To both the model with empirical gain compression relation according to Eq. (10) matches the experiment. All the data is recorded at constant internal temperature (left, bottom), because otherwise the self-heating effect would strongly dominate the gain compression and make the discrimination impossible.

laser temperature and, hence, we do not include effects due to self-heating. If one aquires the *L–I* curve without adjusting for internal temperature (see [5]), self-heating is a dominant effect on I_{th} and separation of both effects is not possible. In other words, the observed bending of the ordinary *L–I* characteristic would mostly be due to self-heating and not the intrinsic nonlinearity from gain compression.

To fit the experimental data, $f(S)$ is modeled with

$$f(S) = \frac{1}{1+(S/S_{sat})^n} = \frac{1}{1+(P/P_{sat})^n} \tag{10}$$

with $P_{sat} = 85$ mW and $n = 2.5$ selected as the best phenomenological fit of the data.

Using the rate equation model in the appendix with gain compression given by Eq. (10) one obtains the following expression for κ from (6)

$$\kappa = \frac{1}{\tau_p P_0}\frac{n(S/S_{sat})^n}{1+(S/S_{sat})^n}. \tag{11}$$

Using the above saturation parameters and assuming a typical value for the photon lifetime of $\tau_p = 9$ ps the model traces in Fig. 2 are calculated. This proves the consistency of the gain compression model from Eq. (10) for both the static nonlinearity of the *L–I* characteristic and the RF adiabatic chirp.

IV. DISCUSSION

A. Adiabatic Chirp in Diode Lasers Versus QCLs

The adiabatic chirp parameter κ for different MQW diode lasers is in the order of 5–10 GHz/mW [48], [50] and, hence, compared to the QCL here more than an order of magnitude higher. On the other hand, the parameter is typically constant in diode lasers but we observe a strong dependence on bias in QCLs.

If QCLs would be available at the telecommunication wavelength of 1.55 μm they would have significant advantages in optical fiber communication. Directly modulated QCLs could theoretically enable much longer transmission ranges due to

reduced chirp, and spare the external modulators. Note that besides the low κ the chirp is further reduced by the low alpha factor (cf. (3)).

B. Gain Compression

The phenomenological model of gain compression presented here is not sufficient to link this phenomenon to any particular physical mechanism in QCLs. Several possibilities have been taken into account such as: change of the refractive index due to gain compression, voltage tuning of the gain curve, or the the intrinsic nonlinearity of the gain region. However none of the considered effects could provide conclusive explanation of the observed QCL behavior. Our discussions and reasoning with respect to several different physical phenomena we considered are presented in the appendix II. We believe that this additional discussion, despite being inconclusive, will trigger further investigations of this topic in the scientific community.

C. Evaluation of the Phenomenological Adiabatic Chirp Model

The adiabatic chirp model is based on three assumptions. First a photon lifetime of 9 ps, and second a thermal backfilling current of smaller than 20% of I_{th} were chosen to be typical and plausible values. The third assumption was made by considering a negligible direct influence of gain compression on the refractive index (i.e., see Eq. (28) in the appendix with $\alpha_{\mathrm{S}} = 0$).

The model is purely phenomenological, and the model justification comes solely from experimental evidence. This is essentially the observation that the variation of the adiabatic chirp with bias point as well as the intensity dependent threshold turned out to have the same power dependency (i.e. with the odd exponent 2.5). Thus the three assumptions are only necessary to realize the quantitative match (in addition to matching the exponent, which was evident from the data). While the first two assumptions were made based on typical and plausible values, the third assumption is more critical. Although it is a common approach in diode lasers, it is not expected to be exactly fulfilled for the voltage tuning, spatial hole burning and possibly also the intrinsic nonlinearity that are expected in QCLs. Whether the effect is negligible or just counteracted by some other mechanisms is unknown since the physical mechanism is not fully understood yet. In any case the observed variation of the adiabatic chirp parameters with power is independent of these assumptions and matches the experimental data quite well.

V. Conclusion

The presence of the adiabatic chirp and gain compression may influence some laser properties besides chirping and theoretical models based on rate equations should be carefully evaluated whether this effect is really negligible. Although the full density matrix models (e.g. [51]) predict the intrinsic optical nonlinearities very well, they may also be inadequate since it is unclear whether the observed strong variation and sign of κ and α_{H} can be explained by these models.

On the other hand there does not seem to be a simple, monocausal answer to the origin of the adiabatic chirp. The intrinsic nonlinearity, voltage tuning, lateral and longitudinal spatial hole burning and carrier heating should all be present in the laser and can potentially affect the gain compression and adiabatic chirping. The voltage tuning of the gain curve is the most likely process, but it alone cannot be responsible for all observed variations of retrieved parameters. For example the observed variation with bias point (with non-even exponent of 2.5) indicates a combination of several effects possibly with a mutual influence. This is yet not understood and will be subject to further investigations of the fundamental processes involved.

Nevertheless, for single mode lasers the phenomenological model (with a suitable $f(S)$ and α_S) is very useful for future theoretical investigations of static and dynamic laser behavior as it completely captures all effects irrespective of their origin. Therefore this phenomenological model can be reliably used to describe the RF modulation performance of QCLs in applications that rely on fast modulation of these sources.

Appendix I
Derivation

A. Rate Equations

Starting from the fundamental rate equation for photons the electronic part of the chirp behavior can be derived. The QCL rate equations read:

$$\dot{N}/\chi = \frac{I}{e}\eta - \frac{N}{\tau_n} - v_g g(N,S)S, \tag{12}$$

$$\dot{S} = v_g g(N,S)S - \frac{S}{\tau_p} \tag{13}$$

with N and S being the population inversion and total number of photons, respectively (both unit 1). g is the gain coefficient (unit 1/cm). In the nomenclature of [37] we have

$$N = N_p A_{act}(n_3 - n_2 + n_2^{\mathrm{therm}}) \tag{14a}$$

$$S = N_p A_{act} S \tag{14b}$$

$$v_g g(N,S) = g_c(N - N_{\mathrm{bf}})f(S) \tag{14c}$$

$$N_{\mathrm{bf}} = N_p A_{act} n_2^{\mathrm{therm}} \tag{14d}$$

$$v_g \frac{\partial g}{\partial N} = g_c \tag{14e}$$

$$\tau_n = \tau_{eff} + \tau_2 = \tau_3\left(1 - \frac{\tau_2}{\tau_{32}}\right) + \tau_2 \tag{14f}$$

$$\eta = N_p \frac{\tau_{eff}}{\tau_{eff} + \tau_2} (\approx N_p) \tag{14g}$$

$$\chi = 1 + \frac{\tau_2}{\tau_3(1 - \frac{\tau_2}{\tau_{32}})} (\approx 1). \tag{14h}$$

To derive the above two rate equation system from [42] we assumed $\tau_2 \ll \tau_3$ (or $\mathrm{d}n_2/\mathrm{d}t \ll \mathrm{d}n_3/\mathrm{d}t$) and the resulting second rate equation for n_2 was used to rewrite the rate equation for n_3 in terms of $n_3 - n_2$. For the work here the validity of

this assumption is not required. Since the electron lifetimes in QCLs are extremely short, the term $\mathrm{d}n_i/\mathrm{d}t$ terms are always negligible compared to n_i/τ_n under operation conditions up to at least several GHz. This can be seen when n_i is assumed to be sinusoidally varying: then it is obvious that $\mathrm{d}n_i/\mathrm{d}t$ is much smaller than n_i/τ_n when $2\pi f \ll 1/\tau_n$ (which is $f \ll 160\,\mathrm{GHz}$). This is consistent with [22] where an influence of the third rate equation (which makes the system of third order, and not second order as (12) and (13)) has an effect only at extremely high frequencies well above 2 GHz.

The major difference of (12) and (13) to [42] is in the gain model where we assume

$$g(N,S) = \frac{\partial g}{\partial N}(N - N_{\mathrm{bf}})f(S) \tag{15}$$

which deviates from [37] in the way that the gain compression term $f(S)$ is added.

B. Static Solution

The static solution (i.e., setting $\mathrm{d}N/\mathrm{d}t = \mathrm{d}S/\mathrm{d}t = 0$) of the rate equations is given by

$$N = N(S) = \frac{1}{v_g \frac{\partial g}{\partial N}\tau_p f(S)} + N_{\mathrm{bf}} \tag{16}$$

$$S = \frac{\eta\tau_p}{e}(I - I_{\mathrm{th}}(S)) \tag{17}$$

with

$$I_{\mathrm{th}} = I_{\mathrm{th}}(S) = \frac{e}{\tau_n \eta}\left(\frac{1}{\tau_p v_g \frac{\partial g}{\partial N}}\frac{1}{f(S)} + N_{\mathrm{bf}}\right). \tag{18}$$

Hence we have two contributions to the total current, one from optical losses and one from thermal backfilling. Since I_{th} current is intensity dependent it rises when the laser is above threshold and causes the sub-linear bending of the *L–I* characteristic. The value $I_{\mathrm{th}}(0)$ is equal to the classical threshold current.

If one defines the thermal backfilling current as $I_{\mathrm{therm}} = \frac{e}{\tau_n \eta}N_{\mathrm{bf}}$ with the normalization $\mathrm{f}(0) = 1$ we can also write

$$I_{\mathrm{th}}(S) = (I_{\mathrm{th}}(0) - I_{\mathrm{therm}})/f(S) + I_{\mathrm{therm}}. \tag{19}$$

The internal $I_{\mathrm{th}}(S)$ with the assumption of $f'(0) = 0$, can be aproximated with

$$I_{\mathrm{th}}(S) = I - S \times \left.\frac{\mathrm{d}I}{\mathrm{d}S}\right|_{S=0}. \tag{20}$$

This implicitly uses the fact the the slope efficiency is not dependent on the absolute light power. The assumption $f'(S = 0) = 0$ is supported by the experimental data, because the slope of κ value is very close to zero at $S = 0$ (cf. Fig. 2 and $\kappa \propto f'(S)$ as it is shown in Section C. Note, that for the last equation the unit of *S* does not matter, it can be replaced by *P* in absolute units or arbitrary units.

C. Dynamic FM-IM Ratio (Adiabatic and Transient Chirp)

From the photon rate equation (13), which is applicable to any single mode laser, one can derive a relationship between the gain change and the output intensity.

For small-signal changes around the equilibrium, we obtain after linearization $\mathrm{d}S/\mathrm{d}t = v_g \Delta g S_0$. The symbols S_0 and g_0 denote the steady state values whereas the quantities with delta refer to the small, time-dependent changes around the static values with zero subscript $(S(t) = S_0 + \Delta S(t))$ and $g(t) = g_0 + \Delta g(t))$. When solving Eq. (13) for Δg we obtain

$$\Delta g(t) = \frac{1}{v_g}\frac{\dot{\Delta S}(t)}{S_0}. \tag{21}$$

In other words, the relative power variation equals the net rate of increased or decreased stimulated emission. This is consistent with the phenomenon known as gain clamping. Under static conditions ($\mathrm{d}\Delta S/\mathrm{d}t = 0$), the modal gain is constant ($\Delta g = 0$, "gain clamping"). Only if the laser output power is about to rise or fall (e.g. by injecting more or less current), the stimulated emission rate has to rise (i.e., $\Delta g > 0$) or fall (i.e., $\Delta g < 0$) from the momentary level to create or destroy photons to adapt to the new situation. This is a basic effect present in all lasers, and Eq. (21) is valid even in presence of other effects such as spatial hole burning, carrier heating, and so on. In any semiconductor laser, including QCLs, this gain change is accompanied with a change of the population inversion ΔN providing the gain.

Crucial to deriving a expression for the frequency tuning behavior is the α_{H} factor which relates the gain change to the refractive index change due to a change in the carrier population inversion. It is defined by [7]

$$\alpha_H = -\frac{4\pi}{\lambda_0}\frac{\partial n/\partial N}{\partial g/\partial N} \tag{22}$$

with *n* the refractive index and λ_0 the free space wavelength. Note that the refractive index spectrum is related to the gain spectrum by the Kramers-Kronig relations, so knowledge of the gain spectrum and its dependency on *N* completely defines the alpha factor. Note that Eq. (22) is with respect to the effective refractive index and modal gain for a specific mode of the laser waveguide.

From the resonance shift of a basic Fabry Perot resonator $\Delta\nu = -\Delta n/n_g\nu_0$ with Δn effective index change we obtain for the laser frequency change

$$\Delta\nu = \frac{v_g}{4\pi}\alpha_H\frac{\partial g}{\partial N}\Delta N + \frac{v_g}{4\pi}\alpha_S\frac{\partial g}{\partial S}\Delta S \tag{23}$$

with

$$\alpha_S = -\frac{4\pi}{\lambda_0}\frac{\partial n/\partial S}{\partial g/\partial S} \tag{24}$$

the factor describing the coupling of amplitude and phase from the gain compression effect (in the analog way to the α_H factor). In diode lasers this is usually neglected and, hence, α_S is assumed to be zero. However, since the physical effects causing gain compression in QCLs are unknown and can differ substantially from those in diode lasers, we include α_S to see its effect on chirp parameters.

Expanding

$$\Delta g = \frac{\partial g}{\partial N}\Delta N + \frac{\partial g}{\partial S}\Delta S \tag{25}$$

and substituting Eq. (21) we obtain from Eq. (23)

$$\Delta\nu = \frac{\alpha_H}{4\pi}\left(\frac{\dot{\Delta S}}{S_0} - \left(1 - \frac{\alpha_S}{\alpha_H}\right) v_g \frac{\partial g}{\partial S}\Delta S\right). \quad (26)$$

To describe a sinusoidal variation of frequency *f* the quantities $\Delta\nu$ and ΔS can be replaced by complex numbers containing the amplitude and phase of the sine wave. Formally, this corresponds to a Fourier transform of (26) with the time dependent quantitites $\Delta S = \Delta S(t)$ and $\Delta\nu = \Delta\nu(t)$. After this Fourier transform and with addition of a term describing the residual thermal tuning at high frequencies we finally obtain after normalization

$$\frac{\Delta\nu(f)}{\Delta S(f)} S_0 = \frac{\alpha_H}{4\pi}\left(\mathrm{i}2\pi f + \kappa P_0\right) + \frac{P_0 \frac{\Delta\nu}{\Delta P}\big|_{\mathrm{DC}}}{1 + \mathrm{i} f/f_{\mathrm{th}}} \quad (27)$$

with adiabatic chirp parameter κ defined by

$$\begin{aligned}\kappa &= -\left(1 - \frac{\alpha_S}{\alpha_H}\right)\frac{S_0}{P_0} v_g \frac{\partial g}{\partial S} \\ &= -\left(1 - \frac{\alpha_S}{\alpha_H}\right)\frac{S_0 f'(S_0)}{f(S_0)}\frac{1}{P_0\tau_p}\end{aligned} \quad (28)$$

which with the assumption of $\alpha_S = 0$ becomes

$$\kappa = -\frac{S_0 f'(S_0)}{f(S_0)}\frac{1}{P_0\tau_p}. \quad (29)$$

It is worth noting that by using the form of (27) with the given normalization, there is a significant advantage that it becomes independent of the unit of *S* and the left hand side of (27) can be directly obtained from experimenal data. Note that $m = |\Delta S|/S_0 = |\Delta P|/P_0$ is the IM index, $\beta = |\Delta\nu|/f$ the FM index and the phase differene between $\Delta\nu$ and ΔS is the FM-IM phase-shift θ, i.e. we can replace the complex quantitites in the left hand side of (27) with the more familiar symbols to obtain $(\Delta\nu/\Delta S)S_0 = (\beta/m)e^{i\theta} f$, which is what is used in (3).

The last parameter in (27) describes thermal tuning and is only asymptotically valid. The thermal tuning at low frequencies is generally much more complicated than the simple first order lowpass and is only valid for frequencies well above the thermal cutoff (e.g., $f \gg 1\,\mathrm{MHz}$). In the general case, special modeling or the utilization of several time constants is essential to correctly describe the thermal tuning in quantum cascade lasers [45], edge-emitting diode lasers [46], and vertical cavity surface emitting lasers [47].

D. Relation to Nonlinear Index

The adiabatic chirp is related to the refractive index change by $\Delta n = -\lambda_0/v_g\Delta\nu$ and so also the nonlinear refractive index n_2, which is is defined as $n_2 = \frac{\partial n}{\partial I} = \frac{\Delta n}{\Delta P}\frac{P_0}{I_0}$.

With I_0 the optical intensity (unit W/cm^2). This gives after simplification

$$n_2 = -\frac{\lambda_0}{4\pi v_{\mathrm{g}}(1-R)A}\alpha_H\kappa \quad (30)$$

with $P_0/I_0 = (1-R)A$ the ratio of emitted power and average intensity (unit cm^2) inside the laser cavity, *R* the laser outcoupling mirror reflectivity and *A* the effective area of the light mode.

Appendix II
Potential Physical Mechanisms of Gain Compression

A. Interdependence of Gain Compression and Adiabatic Chirp

For proper interpretation of results it has to be noted that under certain circumstances the gain compression effect does not cause adiabatic chirping. This is for example the case for certain types of spatial hole-burning. This may be because the gain compression induced refractive index variation due to the population inversion change may be exactly compensated by a direct change of the refractive index due to gain compression. The theory in appendix I includes this effect with the α_S parameter. From (28) it is evident that if $\alpha_S = \alpha_H$ the adiabatic chirp coefficient κ is ideally zero. This happens for example if the gain compression effect is acting in the same way on the refractive index as it does for the gain (cf. (5)). This may be the case if in addition to the gain model (5) we have a refractive model of

$$n(N,S) = \frac{\partial n}{\partial N}(N - N_{\mathrm{bf}})f(S) \Rightarrow \kappa = 0. \quad (31)$$

In which case both effects exactly cancel and there is no net refractive index change and also no adiabatic chirp ($\kappa = 0$). The gain compression causes a saturation of the gain coefficient which requires the population inversion to rise. This increase in carriers is typically responsible for the electronic tuning, i.e. the change in refractive index with population inversion. If however we have a relation (31) this will not cause a change in *n* because the rise in *N* is exactly compensated by the presence of *f*(*S*) in (31). However, such a behavior is never exactly fulfilled which is why $\alpha_S = 0$ (or $\partial \mathrm{n}/\partial \mathrm{S} = 0$) is often assumed in the theory for diode lasers. Besides that, the direct influence of gain compression on parameters such as the relaxation damping is still measurable in diode lasers. Even for spatial hole burning (which should have a behavior according to (31)) an adiabatic chirping would still be observed if for example the α_{H} factor is carrier density dependent [52]. In that case we have $\alpha_S \neq \alpha_H$ which then causes an adiabatic chirp.

B. Voltage Tuning of the Gain in QCLs

It is known that in QCLs a change of electric field across the active region (i.e. the external laser voltage minus the voltage drop on the ohmic shunt resistance) causes a shift of the gain spectrum [53]. Since the internal laser voltage well correlates with the optical intensity during direct modulation it is expected that a shifting of the gain spectrum during each modulation cycle causes an effective gain compression. This is expected to be stronger the higher the slope of the gain spectrum is. To investigate the effect we assume a simple Lorentzian gain spectrum [42], i.e.,

$$g(\nu) = \frac{\frac{\partial g}{\partial N}N}{\Delta^2 + 1} \quad n(\nu) = \frac{c}{4\pi\upsilon}\frac{\frac{\partial g}{\partial N}N\Delta}{\Delta^2 + 1} \quad (32)$$

with $\Delta = (\nu - \nu_{\mathrm{C}})/g$ the detuning from gain peak ν_{C} with half-width of γ. In this model the alpha factor is given

as [54], [42]

$$\alpha_H = -\Delta. \tag{33}$$

From (33) we can infer that the positive alpha factor in the experiment (cf. Fig. 2) indicates that the laser is operating at negative detuning, i.e. "left of" the gain peak in our experiment (this was verified with the manufacturer of the laser). Note, that the extraction of the alpha factor is not affected by presence of the mentioned effect. This is because the alpha factor is taken from the transient chirp, which rises linearly with operation frequency. Such a differentiating (or non-adiabatic) behavior is not expected for voltage tuning (or any other gain compression effect) so it is very reliable that Fig. 2 shows the alpha factor as it is defined in (22).

We found that in the experimental data the observed increase in alpha factor can be well correlated with the observed rise of internal voltage. In this data (the laser terminal voltage is given in Table I)[1], the necessary tuning coefficient has the expected sign (blue-shifting) but is rather high in the order of $\partial\Delta/\partial U_{\rm int} = -4.5$ HWHM/V which exceeds the values in [53].

If only tuning is concerned the alpha factor should also decrease with increasing temperature due to the red-shifting of the gain spectrum with temperature. This is not observed in the data, rather the contrary is the case with slight increase of the alpha factor with temperature (see Fig. 2). However blue-shifting of the gain curve caused by voltage tuning (higher threshold voltage is observed at higher internal temperature) may be responsible.

The gain shift model also overestimates the gain compression observed from the bending of the L-I characteristic (see Fig. 3). At highest power $P_{\max}$ we have $\alpha_H \approx 0.48$ and $\alpha_H \approx 0.18$ near threshold, which due to (33) and (32) should correspond to a gain compression of $\sim$16% $(f(S_{\max}) = (0.18^2+1)/(0.48^2+1) \approx 0.84)$. This is more than 50% more then $\sim$10% compression effect as observed from the L-I bending in Fig. 3 (i.e. $f(S_{\max}) =\sim 0.9$).

In addition, the adiabatic chirp theoretically predicted from voltage tuning should be much higher than experimentally found. This can be determined from (28) when the correct α_S is inserted. To calculate α_S we have to recall that in (24) the derivatives with respect to S can be replaced with derivative with respect to $\nu_{\rm C.}$ Evaluation of this with the Lorentzian model (32) yields for the additional $(1+\alpha_S/\alpha_H)$ factor in expression for κ (28):

$$\left(1-\frac{\alpha_S}{\alpha_H}\right) = \frac{1}{2}\left(1+\frac{1}{\alpha_H^2}\right) \quad \gg 1 \quad \text{for} \quad (|\alpha_H| \ll 1). \tag{34}$$

When this factor is evaluated for the alpha factors observed in the experiment (cf. Fig. 2) we obtain values between 16 and 2.7, which would make the model for κ in Fig. 2 larger by that amount, or the amount of gain compression shown in the model in Fig. 3 lower by that amount. Neither fits the experimental data.

[1]For that we assumed a shunt resistance of 4 Ω, which is the differential laser resistance at laser threshold. For higher bias points the differential resistance is higher, indicating $\Delta S/\Delta U > 0$.

C. Physical Effects Causing Gain Compression in Diode Lasers

Originally the gain compression effect referred to the nonlinearity of the active medium, i.e., the dependence of the gain g on the photon number S which is proportional to the light intensity (i.e. $\partial g/\partial S$). However, Channin [15] recognized the strength of a phenemonological approach of adding a gain compression in the form of $g(N,S) = g(N)/(1+\varepsilon S)$ to explain the observed damping in the IM response of diode lasers. The idea was, that although formally this describes an intrinsic saturation non-linearity of a two level system, many effects can cause the observed damping and are not distinbguishable from the device response under standard operation conditions. The identified physical effects are spectral hole burning [55], [56] (the "true" intrinsic optical nonlinearity), carrier heating [57] (similar to the self-heating thermal effect but effective at much higher frequencies), lateral [58] and longitudinal spatial hole burning [59] (the current injection has different distribution as the stimulated emission, causing saturation behavior) and spontaneous emission [1]. All these effects should yield, to a first approximation, a constant gain compression term.

It should be noted that the normal self-heating causes behavior similar to gain compression but only at low frequencies ($<$ 10 MHz), which is why this effect is explicitly excluded. It was found that carrier heating can create a negative gain compression effect [60], when lasing occurs on a particular side of the gain spectrum. Experimental discrimination between spectral hole burning and carrier heating is usually done by their different dynamics, which are revealed in the time domain (by pump probe experiments [61]) or in the frequency domain (by four-wave mixing [62], [63]).

D. Influence of Intrinsic Nonlinearity in QCLs

The most striking result is the observed negative sign of the nonlinear refractive index (see Fig. 2). It is easily verified that this is not a mistake: The original data clearly shows that adiabatic chirp has a 180 degree phase-shift relative to the thermal tuning, i.e. it clearly realizes a blue-shift when the injection current has its positive cycle or, equivalently, the output intensity is at maximum. Since a blue shift corresponds to a decrease in refractive index, the nonlinear index (i.e. refractive index divided by power) is negative. Since the four-wave mixing experiment [64] does not seem to be able to reproduce the sign of the real and imaginary parts of the $\chi^{(3)}$ the explanation by the intrinsic nonlinearity may still be feasible.

However it remains to investigate where the strong variation of the adiabatic chirp parameter comes from. Also one needs to quantify the direct influence of the intrinsic nonlinearity on the refractive index (by α_S) as this may (partly) compensate the adiabatic chirp.

ACKNOWLEDGMENT

The authors would like to thank Dr. M. Belkin for useful discussions on voltage tunability in QCLs, Dr. N. Akikusa at Hamamatsu Photonics for providing gain curve data and Genevieve Plant for assisting with QCL characterization.

REFERENCES

[1] T. L. Koch and R. A. Linke, "Effect of nonlinear gain reduction on semiconductor laser wavelength chirping," *Appl. Phys. Lett.*, vol. 48, no. 10, pp. 613–615, Mar. 1986.

[2] R. Linke, "Modulation induced transient chirping in single frequency lasers," *IEEE J. Quantum Electron.*, vol. 21, no. 6, pp. 593–597, Jun. 1985.

[3] J. Cartledge and G. Burley, "The effect of laser chirping on lightwave system performance," *J. Lightw.Technol.*, vol. 7, no. 3, pp. 568–573, Mar. 1989.

[4] A. Hangauer, G. Spinner, M. Nikodem, and G. Wysocki, "Chirped laser dispersion spectroscopy with directly modulated quantum cascade laser," *Appl. Phys. Lett.*, vol. 103, pp. 1911071–1911074, Jun. 2013.

[5] A. Hangauer, G. Spinner, M. Nikodem, and G. Wysocki, "High frequency tuning capabilities and single-sideband emission of a quantum cascade laser," *Opt. Exp.*, vol. 22, pp. 23439–23455, 2014.

[6] C. Henry, "Theory of the linewidth of semiconductor lasers," *IEEE J. Quantum Electron.*, vol. 18, no. 2, pp. 259–264, Feb. 1982.

[7] M. Osinski and J. Buus, "Linewidth broadening factor in semiconductor lasers—An overview," *IEEE J. Quantum Electron.*, vol. 23, no. 1, pp. 9–29, Jun. 1987.

[8] J. Huang and L. W. Casperson, "Gain and saturation in semiconductor lasers," *Opt. Quantum. Electron.*, vol. 25, no. 6, pp. 369–390, Jun. 1993.

[9] R. Lang and K. Kobayashi, "External optical feedback effects on semiconductor injection laser properties," *IEEE J. Quantum Electron.*, vol. 16, no. 3, pp. 347–355, Mar. 1980.

[10] R. Tkach and A. Chraplyvy, "Regimes of feedback effects in 1.5-μm distributed feedback lasers," *J. Lightw. Technol.*, vol. 4, no. 11, pp. 1655–1661, Nov. 1986.

[11] R. Hui, A. Mecozzi, A. D'Ottavi, and P. Spano, "Novel measurement technique of alpha factor in DFB semiconductor lasers by injection locking," *Electron. Lett.*, vol. 26, no. 14, pp. 997–998, 1990.

[12] R. Pagano, J. Mukherjee, P. Sajewicz, and B. Corbett, "Above threshold estimation of alpha (Henry) parameter in stripe lasers using near- and far-field intensity measurements," *IEEE J. Quantum Electron.*, vol. 47, no. 4, pp. 439–446, Apr. 2011.

[13] P. Kirkby, A. R. Goodwin, G. H. B. Thompson, and P. Selway, "Observations of self-focusing in stripe geometry semiconductor lasers and the development of a comprehensive model of their operation," *IEEE J. Quantum Electron.*, vol. 13, no. 8, pp. 705–719, Aug. 1977.

[14] K. Petermann, *Laser Diode Modulation and Noise*. Norwell, MA, USA: Kluwer, 1988.

[15] D. J. Channin, "Effect of gain saturation on injection laser switching," *J. Appl. Phys.*, vol. 50, no. 6, pp. 3858–3860, 1979.

[16] G. Morthier, F. Libbrecht, K. David, P. Vankwikelberge, and R. Baets, "Theoretical investigation of the second-order harmonic distortion in the AM response of 1.55 μm F-P and DFB lasers," *IEEE J. Quantum Electron.*, vol. 27, no. 8, pp. 1990–2002, Aug. 1991.

[17] G. Morthier and P. Vankwikelberge, *Handbook of Distributed Feedback Laser Diodes*. Norwood, MA, USA: Artech House, Mar. 1997.

[18] J. Chen, R. Ram, and R. Helkey, "Linearity and third-order intermodulation distortion in DFB semiconductor lasers," *IEEE J. Quantum Electron.*, vol. 35, no. 8, pp. 1231–1237, Aug. 1999.

[19] J. Le Bihan and G. Yabre, "FM and IM intermodulation distortions in directly modulated single-mode semiconductor lasers," *IEEE J. Quantum Electron.*, vol. 30, no. 4, pp. 899–904, Apr. 1994.

[20] G. Yabre and J. Le Bihan, "Reduction of nonlinear distortion in directly modulated semiconductor lasers by coherent light injection," *IEEE J. Quantum Electron.*, vol. 33, no. 7, pp. 1132–1140, Jul. 1997.

[21] C. B. Su, J. Schlafer, and R. B. Lauer, "Explanation of low-frequency relative intensity noise in semiconductor lasers," *Appl. Phys. Lett.*, vol. 57, no. 9, pp. 849–851, 1990.

[22] M. Haldar, "A simplified analysis of direct intensity modulation of quantum cascade lasers," *IEEE J. Quantum Electron.*, vol. 41, no. 11, pp. 1349–1355, Nov. 2005.

[23] L. L. Columbo and M. Brambilla, "Multimode regimes in quantum cascade lasers with optical feedback," *Opt. Exp.*, vol. 22, no. 9, pp. 10105–10118, May 2014.

[24] B. Meng and Q. J. Wang, "Theoretical investigation of injection-locked high modulation bandwidth quantum cascade lasers," *Opt. Exp.*, vol. 20, no. 2, pp. 1450–1464, Jan. 2012.

[25] C. Wang, F. Grillot, V. I. Kovanis, J. D. Bodyfelt, and J. Evenon, "Rate equation analysis of frequency chirp in optically injection-locked quantum cascade lasers," *Proc. SPIE*, vol. 898014, pp. 8980141–8980148, Mar. 2014.

[26] C. Wang, F. Grillot, and J. Even, "Intensity modulation response of injection-locked quantum cascade lasers," *Proc. SPIE*, vol. 8619, pp. 86191Q–1–86191Q-9, 2013.

[27] S. Bartalini *et al.*, "Observing the intrinsic linewidth of a quantum-cascade laser: Beyond the Schawlow-Townes limit," *Phys. Rev. Let.*, vol. 104, no. 8, pp. 0839041–0839044, 2010.

[28] L. Tombez *et al.*, "Frequency noise of free-running 4.6 μm distributed feedback quantum cascade lasers near room temperature," *Opt. Lett.*, vol. 36, no. 16, pp. 3109–3111, 2011.

[29] J. Faist *et al.*, "Continuous wave operation of a vertical transition quantum cascade laser above T = 80 K," *Appl. Phys. Lett.*, vol. 67, no. 21, pp. 3057–3059, 1995.

[30] M. Lerttamrab *et al.*, "Linewidth enhancement factor of a type-I quantum-cascade laser," *J. Appl. Phys.*, vol. 94, no. 8, pp. 5426–5428, 2003.

[31] M. Lerttamrab, S. Chuang, R. Yang, and C. Hill, "Linewidth enhancement factor of a type-II quantum-cascade laser," in *Proc. Conf. Lasers Electro-Opt.*, paper CMR5, pp. 1–3, 2004.

[32] J. Kim *et al.*, "Theoretical and experimental study of optical gain and linewidth enhancement factor of type-I quantum-cascade lasers," *IEEE J. Quantum Electron.*, vol. 40, no. 12, pp. 1663–1674, Dec. 2004.

[33] T. Aellen *et al.*, "Direct measurement of the linewidth enhancement factor by optical heterodyning of an amplitude-modulated quantum cascade laser," *Appl. Phys. Lett.*, vol. 89, no. 9, pp. 0911211–0911213, 2006.

[34] M. Ishihara *et al.*, "Linewidth enhancement factor of quantum cascade lasers with single phonon resonance-continuum depopulation structure on Peltier cooler," *Electron. Lett.*, vol. 45, no. 23, pp. 1168–1169, 2009.

[35] N. Kumazaki *et al.*, "Detuning characteristics of the linewidth enhancement factor of a midinfrared quantum cascade laser," *Appl. Phys. Lett.*, vol. 92, no. 12, pp. 1211041–1211043, 2008.

[36] R. P. Green *et al.*, "Linewidth enhancement factor of terahertz quantum cascade lasers," *Appl. Phys. Lett.*, vol. 92, no. 7, pp. 0711061–0711063, 2008.

[37] J. von Staden, T. Gensty, W. Elsäßer, G. Giuliani, and C. Mann, "Measurements of the α factor of a distributed-feedback quantum cascade laser by an optical feedback self-mixing technique," *Opt. Lett.*, vol. 31, no. 17, pp. 2574–2576, Sep. 2006.

[38] G. Giuliani, "The linewidth enhancement factor of semiconductor lasers: Usefulness, limitations, and measurements," in *Proc. 23rd Annual Meeting IEEE Photon. Soc.*, 2010, pp. 423–424.

[39] C. Harder, K. Vahala, and A. Yariv, "Measurement of the linewidth enhancement factor alpha of semiconductor lasers," *Appl. Phys. Lett.*, vol. 42, no. 4, pp. 328–330, 1983.

[40] J. Provost and F. Grillot, "Measuring the chirp and the linewidth enhancement factor of optoelectronic devices with a Mach-Zehnder interferometer," *IEEE Photon. J.*, vol. 3, no. 3, pp. 476–488, Jun. 2011.

[41] R. Srinivasan and J. Cartledge, "On using fiber transfer functions to characterize laser chirp and fiber dispersion," *IEEE Photon. Technol. Lett.*, vol. 7, no. 11, pp. 1327–1329, Nov. 1995.

[42] J. Faist, *Quantum Cascade Lasers*. London, U.K.: Oxford Univ. Press, May 2013.

[43] Y. Miyake and M. Asada, "Spectral characteristics of linewidth enhancement factor α of multidimensional quantum wells," *Jpn. J. Appl. Phys.*, vol. 28, no. 7R, pp. 1280–1281, 1989.

[44] H. Olesen and G. Jacobsen, "A theoretical and experimental analysis of modulated laser fields and power spectra," *IEEE J. Quantum Electron.*, vol. 18, no. 12, pp. 2069–2080, Dec. 1982.

[45] L. Tombez *et al.*, "Wavelength tuning and thermal dynamics of continuous-wave mid-infrared distributed feedback quantum cascade lasers," *Appl. Phys. Lett.*, vol. 103, no. 3, pp. 0311111–0311115, Jul. 2013.

[46] H. Shalom *et al.*, "On the various time constants of wavelength changes of a DFB laser under direct modulation," *IEEE J. Quantum Electron.*, vol. 34, no. 10, pp. 1816–1822, Oct. 1998.

[47] A. Hangauer, J. Chen, R. Strzoda, and M.-C. Amann, "The frequency modulation response of vertical-cavity surface-emitting lasers: Experiment and theory," *IEEE J. Sel. Topics Quantum Electron.*, vol. 17, no. 6, pp. 1584–1593, Nov./Dec. 2011.

[48] A. Villafranca, J. Lasobras, and I. Garces, "Precise characterization of the frequency chirp in directly modulated DFB lasers," in *Proc. Spanish Conf. Electron Devices*, Feb. 2007, pp. 173–176.

[49] K. Sato, S. Kuwahara, and Y. Miyamoto, "Chirp characteristics of 40-Gb/s directly modulated distributed-feedback laser diodes," *J. Lightw. Technol.*, vol. 23, no. 11, pp. 3790–3797, Nov. 2005.

[50] P. Krehlik, "Characterization of semiconductor laser frequency chirp based on signal distortion in dispersive optical fiber," *Opto-Electron. Rev.*, vol. 14, pp. 119–124, Jun. 2006.

[51] A. Gordon *et al.*, "Multimode regimes in quantum cascade lasers: From coherent instabilities to spatial hole burning," *Phys. Rev. A*, vol. 77, pp. 053804-1–053804-18, May 2008.

[52] H. Wenzel, H. J. Wunsche, and U. Bandelow, "Linewidth rebroadening in semiconductor lasers due to lateral spatial holeburning," *Electron. Lett.*, vol. 27, no. 25, pp. 2301–2302, Dec. 1991.

[53] Y. Yao, Z. Liu, A. Hoffman, K. Franz, and C. Gmachl, "Voltage tunability of quantum cascade lasers," *IEEE J. Quantum Electron.*, vol. 45, no. 6, pp. 730–736, Jun. 2009.

[54] R. Maulini *et al.*, "Continuous-wave operation of a broadly tunable thermoelectrically cooled external cavity quantum-cascade laser," *Opt. Lett.*, vol. 30, no. 19, pp. 2584–2586, 2005.

[55] G. P. Agrawal, "Gain nonlinearities in semiconductor lasers: Theory and application to distributed feedback lasers," *IEEE J. Quantum Electron.*, vol. QE-23, no. 6, pp. 860–868, Jun. 1987.

[56] R. F. Kazarinov, C. H. Henry, and R. A. Logan, "Longitudinal mode self-stabilization in semiconductor lasers," *J. Appl. Phys.*, vol. 53, no. 7, pp. 4631–4644, 1982.

[57] G. Shtengel, R. Kazarinov, G. Belenky, and C. Reynolds, "Wavelength chirp and dependence of carrier temperature on current in MQW InGaAsP-InP lasers," *IEEE J. Quantum Electron.*, vol. 33, no. 8, pp. 1396–1402, Aug. 1997.

[58] R. S. Tucker and D. Pope, "Circuit modeling of the effect of diffusion on damping in a narrow-stripe semiconductor laser," *IEEE J. Quantum Electron.*, vol. 19, no. 7, pp. 1179–1183, Jul. 1983.

[59] J. Whiteaway *et al.*, "The static and dynamic characteristics of single and multiple phase-shifted DFB laser structures," *IEEE J. Quantum Electron.*, vol. 28, no. 5, pp. 1277–1293, May 1992.

[60] G. Wang, R. Nagarajan, D. Tauber, and J. Bowers, "Reduction of damping in high-speed semiconductor lasers," *IEEE Photon. Technol. Lett.*, vol. 5, no. 6, pp. 642–645, Jun. 1993.

[61] C. T. Hultgren and E. P. Ippen, "Ultrafast refractive index dynamics in AlGaAs diode laser amplifiers," *Appl. Phys. Lett.*, vol. 59, no. 6, pp. 635–637, 1991.

[62] K. Kikuchi, M. Kakui, C. en Zah, and T.-P. Lee, "Observation of highly nondegenerate four-wave mixing in 1.5 μm traveling-wave semiconductor optical amplifiers and estimation of nonlinear gain coefficient," *IEEE J. Quantum Electron.*, vol. 28, no. 1, pp. 151–156, Jan. 1992.

[63] A. D'Ottavi *et al.*, "Investigation of carrier heating and spectral hole burning in semiconductor amplifiers by highly nondegenerate four-wave mixing," *Appl. Phys. Lett.*, vol. 64, no. 19, pp. 2492–2494, 1994.

[64] P. Friedli *et al.*, "Four-wave mixing in a quantum cascade laser amplifier," *Appl. Phys. Lett.*, vol. 102, no. 22, pp. 2221041–2221044, 2013.

Andreas Hangauer (M'08) was born in Aachen, Germany. He received the Diploma degree in electrical engineering from the University of Karlsruhe, Karlsruhe, Germany, in 2006, and the Diploma degree in mathematics from the Munich University of Technology (TUM), Munich, Germany, in 2010. He received the Ph.D. degree in electrical engineering at the Munich University of Technology, in 2013.

After receiving the Ph.D. degree, he was a Postdoctoral Research Associate at Princeton University, Princeton, NJ, USA, until 2015 where he was researching novel laser spectroscopic methods. He is currently working at Siemens, Corporate Technology in Munich, Germany in the field of laser spectroscopic sensing. He is interested in photonic sensing for advanced detection of chemicals and modeling of photonic devices to enable new spectroscopic techniques or new applications.

Dr. Hangauer received the Kaiser-Friedrich Research Award with coworkers in 2009 and the award of Association for Electrical, Electronic and Information Technologies in Germany for the best dissertation in 2013.

Gerard Wysocki received the Ph.D. degree in physics from Johannes Kepler University in Linz, Linz, Austria, in 2003, and the M.S. degree in electronics from the Wroclaw University of Technology in Poland, Wroclaw, Poland, in 1999.

He is an Assistant Professor of electrical engineering at Princeton University, Princeton, NJ, USA. He conducts research focused on the development of mid-infrared laser spectroscopic instrumentation for applications in trace gas detection and chemical sensing. His current research interests include development and applications of new molecular dispersion spectroscopy techniques for *in-situ* and remote chemical detection, new concepts of broadband mid-infrared lasers, and distributed wide area spectroscopic sensor networks. He published more than 70 peer-reviewed publications and coauthored more than 180 papers and seminars in the international conferences and meetings.

Dr. Wysocki is a Member of the OSA and SPIE. For his scientific contributions, and technical innovations he has received multiple awards including among others the NSF CAREER Award, the Masao Horiba Award for contributions to analytical science, Finalist of the 2011 Blavatnik Award by the New York Academy of Sciences, and 2014 Peter Werle Early Career Scientist Award.