Analysis of the Sensitivity of Decision Analysis Results to Errors and Simplifications in Problem Structure: Application to Lake Erie Ecosystem Management

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***Abstract*—In practical decision analyses, the "curse of dimensionality" compels one to make simplifying assumptions that can introduce errors into estimates of various indexes that interest decision-makers. These indexes include the expected performance of optimal and suboptimal strategies, the benefit of explicitly considering uncertainty, and the benefit of additional information. This paper quantifies the effects on these indexes of simplifying assumptions, including discretization of the decision space, omission of some decision variables and uncertainties from the decision tree, and disregarding of risk aversion. To reduce errors arising from discretization of the decision space, we use a multidimensional cubic spline to interpolate the performance of alternatives between a few simulated points. A case study analyzes decisions concerning phosphorus loading, fisheries management, and lower trophic research projects in Lake Erie under multiple criteria and ecological uncertainties. Results show that spline-based solutions often yield potentially superior decisions from those based on discretized decision spaces, but that omitting important uncertainties makes more of a difference in this case study's decisions and indexes than simplifying the decision space. On the other hand, incorrect multicriteria weights affect the case study's outcomes more than incorrect probabilities.**

***Index Terms*—Bayesian decision analysis, decision trees, Lake Erie ecosystem, multidimensional cubic spline, subjective judgments.**

I. Introduction

UNCERTAINTY and multiple objectives make decisions difficult in business, government, and personal decisions. Decision analysis can help people make better decisions by helping them to structure the problem, quantify various risks, and compare alternatives based on outcomes and expressed preferences [1]–[3]. A useful device in structuring problems is the decision tree, a graphical device that portrays the sequence of choices and random or uncertain events as decision and chance nodes, respectively. The tree is used to show the performance of the decision alternatives under alternative scenarios and to facilitate the calculation of optimal strategies. Bayesian analysis, which is a fundamental concept in decision analysis, is a probabilistic approach to uncertainty analysis and can be used to integrate information from experts, models, and observation to quantify the likelihood of alternative "states of nature." In Bayesian analysis, alternatives are chosen to maximize probability weighted "utility" (a measure of desirability), and Bayes' law is used to update probabilities when new information is obtained.

Some indexes of interest that decision analysis yields include the following: 1) the expected performance of optimal and suboptimal strategies on various objectives under uncertainty; 2) the performance penalty that results from disregarding uncertainty, i.e., the "expected cost of ignoring uncertainty" (ECIU); and 3) the expected improvement in performance associated with decreased uncertainty through information acquisition or new research, the "expected value of perfect or imperfect information" (EVPI and EVII, respectively). However, the recommendations and indexes yielded by decision analysis can be distorted by simplifying the following assumptions:

1) reductions in the set of alternatives considered, such as using a small discrete set of alternatives rather than a continuous range, or reducing the number of decision variables considered;
2) limiting the uncertainties considered to only the most important uncertain parameters;
3) assuming risk neutrality instead of risk aversion or seeking in utility functions.

Such simplifications are often necessary because decision tree-based decision analyses are afflicted by the curse of dimensionality, i.e., the number of combinations of decision options and realizations of uncertain parameters grows exponentially with the number of options and variables. For example, if we have four decision variables, each with ten levels, and if we consider ten uncertain parameters, each with three levels, a decision tree to be analyzed will have $10^4 \cdot 3^{10} = 590\,490\,000$ endpoints. If the value of the objective function has to be estimated for each branch of the tree using a computationally intensive simulation model, the computational time can become prohibitive. Therefore, simplifications such as the above must be made.

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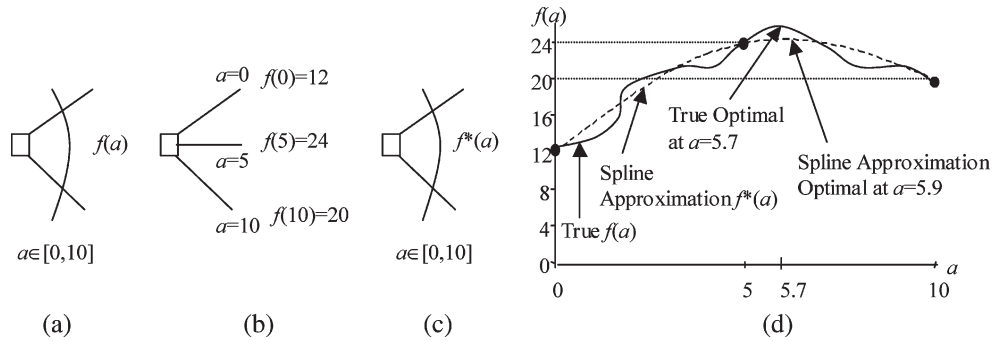

Fig. 1. Example of true function, discrete approximation, and spline approximation. (a) True problem. (b) Discrete approximation. (c) Spline approximation. (d) Optimal of true function, discrete, and spline approximation.

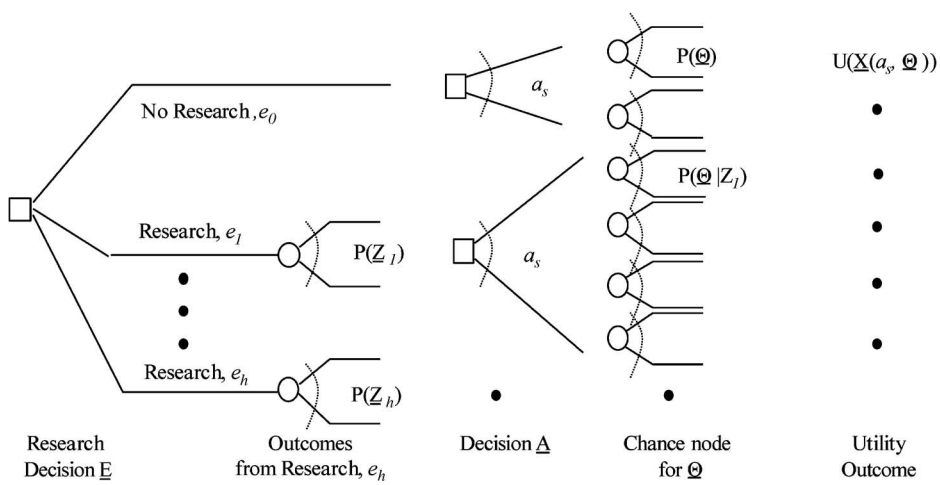

Fig. 2. Decision tree with research options.

But, such simplifications come at a cost [4]. For instance, the true optimum may not lie in the discrete set of alternatives considered, so that a suboptimal choice is recommended. The resulting loss of utility may be significant. Another cost is that errors may be introduced into decision analysis outputs.

To illustrate this point, Fig. 1 shows a simple example of a decision analysis problem. Fig. 1(a) represents an example of a true problem which results in outcomes $f(a)$ given the possible decision a that can be chosen from the continuous range [0, 10]. Because $f(a)$ may be difficult to assess (e.g., because a complex simulation model has to be run for each value of a considered), a discretization such as in Fig. 1(b) is often used. A difficulty is that the true optimum may be missed [see Fig. 1(d)]. If we could estimate the response surface, for example, by fitting a spline function through a few discrete points, we can interpolate and estimate the value of $f(a)$ for all other values of a between the discrete points. For instance, see spline $f^*(a)$ in Fig. 1(c) and (d). We then can optimize $f^*(a)$ over the full range of a, and the resulting decision may be better than if we only choose from the discrete set.

The plan of this paper is as follows. In the method section, we summarize a procedure by the following: 1) approximating the $f(a)$ for a vector of continuous decision variables given discrete data points; 2) combining the approximated response surface with discrete realizations of uncertain parameters; and 3) solving the resulting continuous optimization problem in order to calculate optimal management and research strategies, ECIU, EVPI, and EVII. The application section summarizes an evaluation of nutrient, fisheries, and research management in Lake Erie that uses the proposed procedure, and assesses the relative importance of different simplifying assumptions in decision analysis.

II. Method

A. Formalization of the Decision Problem

Fig. 2 is a schematic of a decision tree in evaluating information gathering options using the Lake Erie management problem as an example. The tree flows from left to right through time and includes many of the features considered in most decision tree-based decision analyses.

For the case in Fig. 2, two decision stages (square nodes) are shown representing the following: 1) what research projects $e_h \in \underline{E}$ can be undertaken and then 2) what fisheries and nutrient management actions $a_s \in \underline{A}$ can be implemented. At the first stage, we decide whether a research project should be undertaken and, if so, which project. If several mutually exclusive research projects are possible, the chosen project(s) should represent the best balance of research cost and the value of reduced uncertainty. Then, at the second decision stage, we identify an optimal management alternative a_s based in part on the knowledge obtained from the research project. Although in reality $\underline{A}$ is a continuous set with infinity of alternatives,

decision analyses generally proceed by discretizing this set. Our main goal in this paper is to use spline methods to allow us to consider a continuum of management alternatives. For simplicity, however, we assume that the research decision set $\underline{E}$ consists of discrete alternatives.

Chance nodes (circles) represent random events, with a branch for each possible realization (an element of either Z or $\underline{\Theta}$). The realization Z from a research project affects the expected performance of a chosen alternative because the research project's outcomes affect (through Bayes' Law) the posterior probabilities of the states of nature. Meanwhile, the second chance nodes $\underline{\Theta}$ affect the actual performance $X(a_s, \underline{\Theta})$ which does not depend on the outcome of a research project. Mathematical models that are used to estimate the performance $X(a_s, \underline{\Theta})$ of each management alternative under each realization are often highly uncertain, and the model we used [the Lake Erie Ecological Model (LEEM)] is no exception. Uncertainties can include structural uncertainty, variability (e.g., spatial or temporal variability), and parameter uncertainty (e.g., due to measurement error or small sample size) [5]. This paper quantifies the implications of uncertainties due to parameter uncertainty and natural variability. Probabilities are attached to each branch, which are based upon subjective probability assessments by fishery managers and modelers.

Here, we have two types of chance nodes: one for model parameters $\underline{\Theta}$ and the other for outcomes of research $\underline{Z}$. The uncertainties are represented here as alternative parameter values; therefore, the uncertainty $\underline{\Theta}$ is a set of parameters that are assumed to be independent of each other, $\underline{\Theta} = \{\theta_n, n = 1, 2, \ldots, N\}$. We discretize the distributions of each θ_n, denoting possible realizations by $\theta_{n1}, \theta_{n2}, \ldots, \theta_{nm}, \ldots, \theta_{nM(n)}$, where $M(n)$ is the number of possible values of θ_n. The multidimensional spline method we use to interpolate between discrete values of a could also be used to interpolate between discrete realizations of θ_n; this will be the subject of future research. The set $\underline{\Theta}$ is divided into two subsets: $\underline{\Theta}^{\mathrm{LTL}}$ and $\underline{\Theta}^{\mathrm{OTH}}$. The first subset $\underline{\Theta}^{\mathrm{LTL}}$ represents those uncertainties that can be reduced through additional research. In our application, these are uncertainties concerning the lower trophic level parameters of LEEM; these parameters are highly uncertain because of the recent invasion of zebra mussels in Lake Erie. (Lower trophic parameters are those that represent energy and nutrient flow in phytoplankton, zooplankton, and zoobenthos populations.) Alternative values of those parameters represent different hypotheses about the mussels' effect on energy and nutrient flow in the lake. Information from the research projects allows us to update the distributions of this subset of parameters according to Bayes' Law. Let the outcome of a research project e_h be designated Z_h, with possible realizations $\{Z_{hk}, k = 1, 2, \ldots, K(h)\}$. Bayes' Law uses outcomes to update prior probabilities $P(\theta_{nm})$ of the parameters, yielding posterior probabilities $P(\theta_{nm}|Z_{hk})$

$$\begin{aligned} P(\theta_{nm}|Z_{hk}) &= P(Z_{hk}|\theta_{nm})P(\theta_{nm}) \Big/ \sum_m P(Z_{hk}|\theta_{nm})P(\theta_{nm}) \\ &= P(Z_{hk}|\theta_{nm})P(\theta_{nm})/P(Z_{hk}) \end{aligned} \tag{1}$$

where $P(Z_{hk}|\theta_{nm})$ is the likelihood of outcome k from project h, given that θ_{nm} is the true state of parameter θ_n, and $P(Z_{hk})$ is the unconditional probability of outcome Z_{hk}.

The second subset of uncertainties $\underline{\Theta}^{\mathrm{OTH}}$ includes uncertainties whose probabilities are not affected by additional research, but can still significantly impact the performance of alternatives due to nonlinearities in system responses and in people's risk attitudes. In our case study, this subset includes uncertainty in upper trophic level (fishery) parameters in LEEM. Multivariate samples of this vector are taken from assumed distributions using Latin Hypercube Sampling (LHS) as a variance reduction technique [6].

Because most decision problems involve several conflicting objectives that are accorded with different priorities by different interests, "utility" is necessarily a multidimensional notion. The outcome for a given management alternative a under a given realization of the uncertainties consists of a vector $\underline{X}(a, \underline{\Theta})$ that includes variables ("attributes") $x_i(a_s, \underline{\Theta})$ describing the performance on each of the multiple objectives (e.g., fish harvests and water quality). There are many methods in translating multiple criteria into a single index of desirability [7]. Here, we choose to apply a multiattribute utility theory [1], [8]. Following traditional practice, we construct multiattribute utility functions as follows. A single attribute utility function u_i is assessed separately for each attribute x_i. Second, the u_i are weighted and aggregated, resulting in a scalar index of desirability. A simple aggregation is the additive utility function

$$U\left(\underline{X}(a_s, \underline{\Theta})\right) = \sum_i w_i u_i\left(x_i(a_s, \underline{\Theta})\right) \tag{2}$$

where w_i represents the weight for criterion $i(\sum_i w_i = 1)$ and $U(\underline{X}(a_s, \underline{\Theta}))$ is the overall utility of alternative a_s, given state of nature $\underline{\Theta}$. The major assumption underlying the additive form is additive independence, which applies if preferences between two distinct alternatives depend only on the marginal probability distributions of the x_i within an alternative and not on their joint distribution [2].

In our case study, the participants (six U.S. and Canadian fishery managers) use the tradeoff method to select the weights, which derives weights from the observed choices among alternatives by users [9].

We next describe how several products of decision analysis are obtained, including identification of optimal decisions and quantification of ECIU, EVII, and EVPI. Finally, some approaches in quantifying the robustness of the analysis are discussed.

B. Optimal Decisions and Other Outputs

Decision analysis can yield several outputs of interest, including optimal strategies (research projects, management alternatives), ECIU, EVPI, and EVII [6]. Detailed procedures in calculating these outputs are presented in [6], and an illustrative application to Lake Erie management is described in [10]

Disregarding the possibility of research, the optimal decision is the one that maximizes expected total utility $E_{\underline{\Theta}}[U(\underline{X}(a_s, \underline{\Theta}))] = \sum_{\underline{\Theta}} P(\underline{\Theta})U(\underline{X}(a_s, \underline{\Theta}))$. The numerical difference in the expected utility between two alternatives (for example, a_1 and a_2) is not directly interpretable, since utility functions are, strictly speaking, only ordinal scales. Fortunately, a difference in the expected utility can be converted to an equivalent difference in one of the criteria x_i [10]. Here, we

use annual walleye sport harvest (x_2) to gauge the difference between alternatives because it is the most important attribute for most of the six fishery managers participating in this study.

The ECIU compares the expected performance of two strategies: 1) a naive strategy $a_{naïve}$ developed assuming that a nominal value for $\underline{\Theta}$ (e.g., its expected value) occurs with probability 1 and 2) an optimal strategy a_{opt} developed considering the full range of possibilities and their probabilities. The ECIU is defined as the improvement in the expected utility if a_{opt} is chosen instead of $a_{naïve}$. This is the expected loss of performance when a decision is made as if there were no risk. Although ECIU can include effects of other uncertainties, we include only the uncertainties that can be reduced by research $\underline{\Theta}^{\text{LTL}}$ so that ECIU can be compared with EVII and EVPI.

The EVII is calculated by considering whether research outcomes could affect decisions [10]. The calculation recognizes that the information obtained is imperfect. For the "no research" alternative e_0, the optimal management decision and the utility for the optimal decision are a_{opt} and $E_{\underline{\Theta}}[U(\underline{X}(a_{\text{opt}}, \underline{\Theta}))]$, respectively. The value of a research project (EVII) is assessed by comparing the expected performance of management developed using the posterior probabilities $P(\theta_{nm}|Z_{hk})$ to the expected performance of the optimal strategy based on just the priors $P(\theta_n)$. Of course, the research also involves cost, and an improved performance must be weighed against this cost.

The EVPI is obtained by assuming that new information $\underline{Z}$ permits the user to estimate $\underline{\Theta}$ without error. As a result, the development in EVII can be simplified as follows. First, define $a_{\text{opt}|\underline{\Theta}}$ as the optimal strategy given parameter values $\underline{\Theta}$. Then, the expected utility over all possible realizations if decisions could be made with perfect information would be $E_{\underline{\Theta}}[U(\underline{X}(a_{\text{opt}|\underline{\Theta}}, \underline{\Theta}))]$. Comparing this to the expected utility if no information is available $E_{\underline{\Theta}}[U(\underline{X}(a_{\text{opt}}, \underline{\Theta}))])$ allows us to gauge the value of perfect information. EVPI is an upper bound to EVII. This upper bound is useful because if EVPI falls short of the cost of a research project, the project cannot be justified by the value of the information it provides. Thus, we can use EVPI to screen research projects—if their cost exceeds their maximum possible benefit, then we do not consider them further.

As mentioned above, those quantifications of the expected penalty depend on the assumptions concerning the sets of alternatives and uncertainties considered. In the next few sections, we describe how we assess the effect of these assumptions on the results of a decision analysis.

C. Approximation of Response Surfaces

In this section, we explain how to construct a spline-based approximated response surface for a multidimensional continuous set of alternatives from a discrete set. Further, because the decision analysis is considering multiple attributes $\underline{X}$ and multiple uncertainties $\underline{\Theta}$, we propose a method to aggregate the surface over these attributes and uncertainties. The three-step procedure is as follows.

1) For each attribute x_i, construct the response surface $\mu_i(x_i(a, \underline{\Theta}))$ that approximates the single attribute utility function values $u_i(x_i(a, \underline{\Theta}))$ as a function of a, given a realization of the uncertainties $\underline{\Theta}$.
2) Aggregate the surfaces over the attribute space by applying weights w_i in (2) to the approximations of $u_i(x_i(a, \underline{\Theta}))$, yielding an approximation of $U(\underline{X}(a, \underline{\Theta}))$ as a function of a, given a realization of $\underline{\Theta}$.
3) Use the probabilities of the realizations of $\underline{\Theta}$ to aggregate the surfaces over uncertainties space $\underline{\Theta}$, yielding an approximation of $E_{\underline{\Theta}}[U(\underline{X}(a, \underline{\Theta}))]$ as a function of a. This function can then be optimized to identify the best value of a.

An alternative approach would have been to approximate the overall utility $U(\underline{X}(a, \underline{\Theta}))$ with one spline rather than each of its constituent $u_i(x_i(a, \underline{\Theta}))$ with separate splines. However, we undertake extensive sensitivity analyses on the weights w_i in (2). As a result, each weight set would result in a different function $U(\underline{X}(a, \underline{\Theta}))$, and thus, a different spline would have to be obtained for each sensitivity case. Approximating each $u_i(x_i(a, \underline{\Theta}))$ just once and then applying multiple weight sets to those splines minimize computational effort.

In the next section, we explain this procedure in more detail. First, we explain it for the 1-D decision variable case associated with multiple attributes and uncertainties. Later, we expand it to the multidimensional case.

Step 1) *One-Dimensional Response Surface μ_i Under Multiple Attributes and Uncertainties*: There are several ways to approximate a function of one or more variables. In this paper, we adopt the interpolation method because there are no random errors associated with our simulation model (LEEM). To interpolate a function between data points, two general methods are used. One is the polynomial approach. However, if there are many data points, such functions may oscillate in an unreasonable manner. An alternative approach is the spline method, which applies lower order polynomials to subsets of adjacent data points [11].

In order to ensure that the nth derivatives are continuous at the discrete points in the spline, the order of the polynomial should be at least $n + 1$. Therefore, cubic or higher order splines can ensure continuous first and second derivatives that yield visually continuous smooth functions. In contrast, first-order splines are not smooth because the slope changes abruptly at the data points. To satisfy this condition with minimal oscillatory behavior, we use the multidimensional cubic spline method [12].

A 1-D spline satisfies the following conditions:

a) continuity at data points (which define the boundaries between the segments of the spline);
b) slope continuity (i.e., first derivatives) at the interior points;
c) curvature continuity (i.e., second derivatives) at the interior points.

Further, we assume a zero rate of change for the slope of the cubic spline at exterior data points.

In actual implementation, a is a vector rather than a scalar; therefore, a multidimensional spline is required. We defer the definition of the multidimensional spline procedure until Section II-E.

Step 2) *Aggregation Over Criteria and Uncertainties*: Consider a vector of splines $\underline{\mu}(a, \underline{\Theta}) = (\mu_1(a, \underline{\Theta}), \ldots, \mu_I(a, \underline{\Theta}))$, where $\mu_i(a, \underline{\Theta})$ represents the spline of attribute i, given the realization of $\underline{\Theta}$.

To aggregate the criteria [Step 2) of the procedure], permitting overall comparison of the alternatives a, we use the additive utility function (2) as follows:

$$M(a, \underline{\Theta}) = \sum_i w_i \mu_i(a, \underline{\Theta}) \tag{3}$$

where $M(a, \underline{\Theta})$ is the overall spline utility function of alternative a_s, given the realization of $\underline{\Theta}$.

Through the above two steps, we create a multidimensional spline that estimates the total utility as a function of a, given a realization of $\underline{\Theta}$. To choose the option that maximizes the expected utility $E_{\Theta}[M(a, \underline{\Theta})] = \sum_{\underline{\Theta}} P(\underline{\Theta}) M(a, \underline{\Theta})$, we first need to aggregate each spline over possible values of $\underline{\Theta}$ using the assumed probabilities [Step 3) of the procedure].

D. Finding the Optimal Solution a^*

The problem of choosing optimal solution a^* at a decision node is a nonlinear optimization problem over the possible values of a. The problem is stated as follows:

$$\begin{aligned} &\text{Max } E_{\underline{\Theta}}\left[M(a, \underline{\Theta})\right] \\ &\text{Subject to } L \le a \le U \end{aligned} \tag{4}$$

where L and U are lower and upper bounds, respectively.

We use a variant on Newton's method to find the optimal solution [13]. Because $E_{\underline{\Theta}}[M(a, \underline{\Theta})]$ consists of piecewise spline regions, we need to search all spline regions to find the optimal solution. For example, if there are n decision variables, each with m different levels, $(m-1)^n$ regions are searched, and their solutions are compared to find the best solution. Since, in general, the objective of (4) can be nonconcave, no algorithm can guarantee a global optimum. Therefore, we repeatedly execute the algorithm with different initial points. In our application, the method always converged to the same solution.

E. Multidimensional Cubic Spline

To extend the 1-D spline approach to the multidimensional case, we approximate the utilities $u_i(a, \underline{\Theta})$, where $\underline{a}$ is now a vector of decision variables. The multidimensional interpolation scheme is based on the study in [12]. For simplicity, we illustrate it with a 2-D case.

Suppose that we have two decision variables, a_1 and a_2 where $o = a_{11} < a_{12} < \cdots < a_{1\chi} = p$ and $q = a_{21} < a_{22} < \cdots < a_{2\eta} = r$. Each subrectangle μ_{idf} of the entire domain is defined as

$$\begin{aligned} \mu_{idf} = \{u_i(a_1, a_2, \underline{\Theta}) \colon\, & a_{1d-1} \le a_1 \le a_{1d},\ a_{2f-1} \le a_2 \le a_{2f}, \\ & \text{where } d = 0, \ldots, D \text{ and } f = 0, \ldots, F\} \end{aligned}$$

for each attribute i, given a realization of parameter values $\underline{\Theta}$. A bicubic spline is fit to these points

$$\mu_{idf}\left(u_i(a_1, a_2, \underline{\Theta})\right) = \sum_{p1=0}^{3} \sum_{p2=0}^{3} C_{idf}^{p1p2} a_1^{p1} a_2^{p2}. \tag{5}$$

As we want the function μ_{idf} to be valid over the subrectangle df, we need to obtain sixteen coefficients C_{idf}^{p1p2} ($p_1 = 0, \ldots, 3; p_2 = 0, \ldots, 3$). In contrast to the 1-D spline, more conditions are needed to solve for these coefficients. These conditions include the following:

1) continuity at each interior discrete point (i.e., any point in two dimensions that is surrounded by four subrectangles);
2) slope continuity at any interior point with respect to both a_1 and a_2 directions and any exterior point adjacent to two rectangles with respect to either a_1 or a_2 direction;
3) curvature (second derivative) continuity at interior points in both the a_1 and a_2 directions.

Furthermore, we assume that the second derivative of the cubic spline is zero at the region's border ($d = 0,\ D;\ f = 0,\ F$), in directions normal to the border.

The mathematical development of n-dimensional cubic splines is available in the literature [14]. Algorithms used to find coefficients c_{idf}^{p1p2} for the 2-D case can be found in [12]. The cases of three or more dimensions are analogous to this 2-D method. For example, as we will present a case study later that considers four decision variables, we create a 4-D cubic spline in interpolating utility outcomes as a function of those variables

$$\begin{aligned} &\mu_{idfhk} u_i(a_1, a_2, a_3, a_4, \underline{\Theta}) \\ &\quad = \sum_{p1=0}^{3} \sum_{p2=0}^{3} \sum_{p3=0}^{3} \sum_{p4=0}^{3} C_{idfhk}^{p1p2p3p4} a_1^{p1} a_2^{p2} a_3^{p3} a_4^{p4} \end{aligned} \tag{6}$$

where d, f, h, and k are the indexes for the 4-D subrectangle.

Equations (5) and (6) are specified for each attribute i, given a fixed realization of the uncertain parameters $\underline{\Theta}$. Therefore, similar to Section II-C, we need to aggregate each μ_{idfhk} over the attributes [Step 2)] and realizations [Step 3)]

$$\begin{aligned} E_{\underline{\Theta}}\left[M(\underline{a}, \underline{\Theta})\right] &= \sum_{\underline{\Theta}} P(\underline{\Theta}) \sum_i w_i \mu_{idfhk}(\underline{a}, \underline{\Theta}) \\ &= \sum_{\underline{\Theta}} P(\underline{\Theta}) \sum_i w_i \\ &\quad \times \sum_{p1=0}^{3} \sum_{p2=0}^{3} \sum_{p3=0}^{3} \sum_{p4=0}^{3} C_{idfhk}^{p1p2p3p4} a_1^{p1} a_2^{p2} a_3^{p3} a_4^{p4}. \end{aligned} \tag{7}$$

III. Illustrative Application

A. Background

Lake Erie is chosen as a case study because of large uncertainties resulting from the various human and other stresses it

has been subjected to and the many tradeoffs involved in its management.

One attempt to restore the ecological integrity of the lake was the 1972 Great Lakes Water Quality Agreement (GLWQA) between the U.S. and Canada. This agreement emphasized the restoration of water quality by reducing nutrient inputs. As a result of the GLWQA, point and nonpoint phosphorus loadings were reduced to the agreement's target of 11 000 tons/year. In addition, fisheries restoration efforts were undertaken. Interagency cooperation between the U.S. and Canada has implemented quota management for fisheries.

Through quota management and phosphorus reduction, the walleye population has recovered significantly from its severely depressed status in the 1960s [15]. Yet, the highly successful recovery of fisheries in the 1980s did not continue in the 1990s. The invasion in the late 1980s of zebra mussels, which fed extensively on phytoplankton, may have contributed to this stagnant production, but the true impact of zebra mussels on the productivity of the system is still subject to extensive debate. To date, researchers have proposed many hypotheses as to how zebra mussels might affect energy and nutrient flows in the lower trophic level. Such scientific debate and the considerable uncertainty regarding the system's response to management hamper managers' efforts to make effective decisions [16].

These hypotheses are summarized in [10]. One hypothesis is that the decline of phytoplankton may decrease the energy flow in higher trophic levels through changes in the availability of algae and other food. In a contrasting hypothesis, zebra mussels are believed to increase internal phosphorus recycling, which could enhance primary productivity and, thus, food availability for higher trophic levels. Such changes can have important implications for ecosystem decisions such as fisheries management.

Alternative hypotheses such as the aforementioned hypotheses are represented as alternative parameterizations of LEEM [17]. LEEM was designed to aid the understanding of ongoing changes and interactions between zebra mussels, fisheries productivity, phosphorus loading, and fisheries management in the Lake Erie system. By using an annual time step, LEEM models the age structure of 15 species and dynamically describes prey–predator relationships and the lower trophic level. The model allows users to explore alternative decisions concerning phosphorus loading and fisheries management while considering the implications of assumptions about the parameters and structure of the system. LEEM was parameterized by a combination of expert judgment, empirical analysis of particular functional relationships, and calibration to historic values of output variables. We will use LEEM to estimate how decisions a affect the managers' criteria $\underline{X}$. (Background on LEEM's formulation and parameterization is in [17].)

Ten fishery biologists from the U.S. and Canadian resource management agencies participated in two workshops held in Cleveland, OH, in early 2000, to assist us in the decision analysis of Lake Erie research, fisheries, and nutrient alternatives. The purpose of the workshops was to ask the participants to identify fishery management objectives, alternatives, and uncertainties to define research projects that could address lower trophic level uncertainties $\underline{\Theta}^{\mathrm{LTL}}$ and to quantify utility functions and probabilities needed by the decision analysis. In this paper, we report results only for the six managers who attended the entirety of both workshops. These six managers represented agencies from both Canada and several states in the U.S. Details concerning the procedures used in the workshop and the basic calculation methods used in the decision analysis are available elsewhere [10].

B. Alternatives a

The decision alternatives a that we considered in this paper involve two general policy levers: phosphorus loading levels and fisheries effort levels. Compared to our previous analysis [10], which did not consider nutrient policies, this allows us to analyze the important interactions of nutrient and fishery management. For simplicity, three levels of phosphorus loading level are considered in constructing the splines: the present GLWQA target of 11 000 tons/year and $\pm 50\%$ deviations from that level. Although a decreased phosphorus loading level will improve water quality (which is desirable), it may also decrease fisheries productivity (which may not be) [9]. In terms of fisheries management, we consider the management of commercial trawling (which targets smelt, *Osmerus mordax*), commercial gill netting (for yellow perch, *Perca flavescens*), and sport harvest (predominantly of walleye, *Stizostedion vitreum*). Management by regulation and quota is modeled as a set of targets for fishing mortality levels, one target for each of the three species. For simplicity, three levels of each targets are considered: medium (M), high (H), and low (L), representing current average mortality, and 50% increase and decrease in mortality, respectively.

C. Ecological Model $\underline{X}\ (a_s, \underline{\Theta})$, Uncertainties $\underline{\Theta}$

Various views among ecologists concerning the impact of zebra mussels upon phosphorus and energy cycling constitute the scientific uncertainty $\underline{\Theta}^{\mathrm{LTL}}$; our decision analysis assesses the implications of that controversy for fishery harvest and phosphorus management. In order to limit the complexity of the probability assessments, we include just five of the lower trophic level LEEM parameters in $\underline{\Theta}^{\mathrm{LTL}}$: zebra mussel recycling (ZM_r), zebra mussel production (Z_{kp}), zooplankton production efficiency (AZP), zoobenthos production efficiency (AZB), and primary productivity (g_0). These parameters were chosen because they represent key responses of lower trophic variables to phosphorus and energy inputs. Given the qualitative characteristics of the hypotheses, we first described whether the hypotheses imply High (H), Medium (M), or Low (L) values of those parameters and, then, selected corresponding numerical values with a large enough range to be informative without causing unrealistic model behavior. Since g_0 had originally been estimated using regression from productivity-nutrient data for a cross section of lakes, the standard error estimated from that regression also informed the chosen range of values for that parameter. There are 3, 2, 3, 2, and 3 hypothesized levels for each parameter, respectively, implying 108 combinations.

The task assigned to each of the fisheries biologists in the workshops was to review and, where appropriate, rephrase the hypotheses and, then, assign a prior probability $P(\theta_{nm})$ for each level m of each parameter θ_n that reflects their

understanding of the hypotheses and their judgments about the hypotheses' relative likelihood. No covariances among the parameters were elicited among the parameters in order to limit the complexity of the assessment, and because there is little basis in quantifying correlations. As explained below, the biologists also chose likelihood probabilities for research outcomes, which we use together with the prior probabilities and Bayes' Law to estimate the value of that research. Values for both prior and likelihood values are given in [10].

The second set of uncertainties $\underline{\Theta}^{\mathrm{OTH}}$ includes nonlower trophic level LEEM parameters for which there is significant uncertainty and which affect the outcomes $\underline{X}$. (This set of uncertainties was not considered in our earlier fisheries decision analysis [10].) The research projects are assumed to yield no information on them. LEEM includes more than 1900 parameters that have been estimated by a combination of expert judgment, laboratory data, and calibration to observation [17]; therefore, we need to determine which uncertain parameters most affect the model output. In our study, we first divided uncertain model parameters into four functional groups representing distinct processes: recruitment, consumption, growth, and survivability. From each category, we chose the parameters $\underline{\Theta}^{\mathrm{OTH}}$ that have the most impact on the model's results. Impact was gauged by an index equaling the product of a parameter's sensitivity (the estimated derivative of an important output—walleye population—with respect to the parameter) and its uncertainty (standard deviation, subjectively assessed by the model creators). As a result, we focused on 60 uncertain parameters (i.e., one parameter from each functional group for each of 15 fish species) that have the most impact on LEEM's results.

Two ecological experts that are familiar with Lake Erie fishery dynamics then chose means and standard deviations of a subjective probability distribution for each of those parameters. For instance, they specified the means and standard deviations for the following walleye parameters: egg survivability (Egg Surv), variability of length (VarPref) and swimming speed (SWS), and slope in the bioenergic model (Rhom) for the recruitment, consumption, survival, and growth categories, respectively. The values were selected based on variation in published and unpublished estimates, and also by considering what values would result in population characteristics that are broadly consistent with historical experience in Lake Erie. Log normal, normal, or right-truncated normal distributions were assumed for each, as judged appropriate by the experts. For all other species, a similar parameter distribution shape as walleye was assumed [i.e., the same coefficient of variation (ratio of mean to standard deviation) and distribution type]. As in the case of the first set of uncertainties, no covariances were assessed.

Because of the curse of dimensionality, we need to limit the number of realizations of $\underline{\Theta}^{\mathrm{OTH}}$ we considered. In our application, this number is five. Those samples are drawn from the assumed distributions of the 60 variables using LHS [6]. Of course, more samples would be desirable to reduce the sampling error, but with 108 realizations of $\underline{\Theta}^{\mathrm{LTL}}$ and 3^4 discrete values of the decision variables a_s, five samples of $\underline{\Theta}^{\mathrm{OTH}}$ yield a total of 43 740 LEEM simulations, which takes approximately 240 h of clock time on an advanced PC. The results of these simulations are the outcomes [i.e., $\underline{X}(a_s, \underline{\Theta})$] for each a_s and $\underline{\Theta}$ that are then input to the spline analysis explained in Section II.

D. Research Projects and Outcome Likelihoods: $\underline{E}$ and $P(Z_{hk}|\theta_{nm})$

The first decision node in Fig. 2 represents possible research projects $\underline{E} = \{e_0, e_1, \ldots, e_H\}$. If a project is undertaken, the prior probabilities $P(\theta_{nm})$ are updated by Bayes' rule (1), yielding posterior probabilities $P(\theta_{nm}|Z_{hk})$. Likelihoods $P(Z_{hk}|\theta_{nm})$ are required by Bayes' Law to describe how the information provided by the potential research projects influences the posterior distribution of LEEM parameters. Because of time limitations, we elicited values of $P(Z_{hk}|\theta_{nm})$ from the workshop participants for four of the projects, representing a cross section of the original 16 [10]. The participants also estimated the likelihoods for these research projects and their cost and time.

E. Selection of Objectives, Weights, and Utility Function: $\underline{X}$, w_i, and u_i

Because the results of a decision analysis—the optimal decision, ECIU, EVPI, and EVII—are sensitive to the people's value judgments, a careful procedure in defining objectives and attributes and in eliciting their single attribute utility functions and weights is required.

The participating fisheries managers were asked to define general objectives along with specific numerical attributes in evaluating fishery management options. Their overall objective was to maximize ecosystem health and human well-being, which was subdivided into three categories: social, ecological, and economic, which were then further divided [9]. A total of ten attributes were then used to quantify the impacts of the phosphorus and fishery policies on the objectives. Participants chose attribute weights by the tradeoff method, a standard approach in which users state how much of one attribute they would be willing to sacrifice for a given improvement in another [10].

For simplicity, we assumed that each single attribute utility function is linear between the worst $(u_i = 0)$ and best $(u_i = 1)$ levels of its attribute, as defined by each participant; if a value of x_i falls outside that range, then it was assigned a utility of 0 or 1, as appropriate.

F. Application of the Multidimensional Spline

For a given research project, the decision tree with discrete alternatives considers three possible outcomes Z of the project, three phosphorus management options, 27 fishery management decisions, 108 lower trophic level scenarios $(\underline{\Theta}^{\mathrm{LTL}})$, and five Latin Hypercube samples for other uncertainties $\underline{\Theta}^{\mathrm{OTH}}$, so that the tree has a total of 43 740 branches, as noted previously. Based on the LEEM simulation results, we constructed the multidimensional cubic splines required by the procedure in Section II. Once the splines were obtained, the computing time of the procedure described in Section II in finding the optimum in the continuous decision space required about 2 min of clock time on an advanced PC.

TABLE I
SUMMARY OF SEVEN DECISION PROBLEM STRUCTURES

Problem Structure	Decision Space	Decision Set	$\underline{\Theta}^{\mathrm{OTH}}$	Risk Attitude
1 Base Case	Continuous	Fish+ Phosphorus	Yes	Risk Neutrality
2 Discrete Decision	Discrete	F+P	Yes	R.N.
3 No Phosphorus Decision	Continuous	F	Yes	R.N.
4 No Error Propagation	Continuous	F+P	No	R.N.
5 Risk Aversion	Continuous	F+P	Yes	Risk Aversion
6 Risk Aversion and No Error Propagation	Continuous	F+P	No	R.A.
7 Simplest Structure	Discrete	F	No	R.N.

TABLE II
AVERAGE OPTIMAL DECISION ACROSS ALL COMBINATIONS OF WEIGHTS AND PROBABILITIES JUDGMENTS

Problem Structure	Optimal Decision			
	P Loading	Trawling	Gill Netting	Sport Harvest
1	0.92	0.72	0.75	0.88
2	0.89	0.74	0.75	0.88
3	N/A	0.79	0.77	0.91
4	1.00	0.97	0.49	0.81
5	0.96	0.45	0.78	0.95
6	1.00	0.93	0.50	0.74
7	N/A	0.88	0.57	0.87

IV. RESULTS

Here, we emphasize that the optimal decision and the value of information can be affected by the structure of the problem, including the decision space and which uncertainties are considered. In order to assess the impact of simplifying the problem, we consider seven different problem structures that differ in terms of the decision space (continuous spline versus discrete, just fisheries management versus fisheries and phosphorus management), uncertainties (just $\underline{\Theta}^{\mathrm{LTL}}$ versus both $\underline{\Theta}^{\mathrm{LTL}}$ and $\underline{\Theta}^{\mathrm{OTH}}$), and risk attitude (risk neutral versus risk averse).[1] Table I summarizes the seven problems.

In Sections IV-A–D below, we consider just a one-stage decision problem, disregarding alternatives to obtain additional information. First, we discuss optimal management strategies a developed without any additional information (Section IV-A), the expected deterioration in performance if we assume the wrong problem structure (Section IV-B), and whether value or probability judgments make more of a difference in optimal decisions (Section IV-C). The effect of disregarding uncertainty upon ECIU results is also described (Section IV-D). In contrast, if research would reduce uncertainty, and its expected benefits (in terms of better management) would exceed its cost, we should consider undertaking it. Section IV-E reviews the EVPI results, along with EVII for the four research projects. In that analysis, we consider 64 combinations of eight sets of value judgments (each of the six manager's value judgments, plus the mean weights and equal weights) and eight sets of prior probability judgments (the six manager's probabilities, along with the mean probabilities and equal probabilities).

A. Single-Stage Optimal Strategies

In this section, we give an example of how optimal strategies depend upon the representations of the decision space and uncertain parameters based on assumptions described in Table I. For example, Table II shows the average of the optimal decisions a_{opt} obtained for each of the 64 combinations of weights and probability judgments. To describe the decisions, the range of each strategic variable is rescaled from 0 to 1. For example, 0, 0.5, and 1 for phosphorus loading are equivalent to 50%, 100%, and 150% of the current GLWQA target, respectively.

From the fisheries managers' point of view, in most cases, alternatives resulting in an increase over the current level of phosphorus are preferred (11 000 tons/year, corresponding to P loading $= 0.5$). This is because the increased phosphorus loading level enhances fisheries harvests, which these managers weigh more heavily than other attributes. Furthermore, in most cases, optimal harvests for all three categories of fisheries tend to be higher than the historical level (0.5).

However, the decisions are not the same under all seven problem structures. This means that simplifications to the problem structure can change the decision. The aspects of problem

[1]Note that we are treating the use of a risk-averse utility function as a "simplification," although, in fact, it is more complicated to apply than a risk-neutral (linear) function. Thus, a comparison of structures 1 and 5 should be viewed as an indication of the cost of using the wrong risk attitude, rather than the cost of "simplification." There are several methods, such as the certainty equivalent and probability equivalent techniques, to elicit risk attitudes when calibrating utility functions [1]. However, due to lack of time in the workshops, we did not ask the participants to apply such a method. Instead, for simplicity, we use a constant risk attitude function, $u_i(x_i(a_s, \underline{\Theta})) = a - b(e^{-cx})$, where a and b are constants to scale each u_i from zero to one (worst to best), respectively, and c, which is called risk tolerance, is positive for increasing utility functions and negative for decreasing utility functions when b is positive. Furthermore, the greater the risk aversion, the greater c becomes in absolute value. Many case studies and empirical research show that the attitude of decision-makers is often risk averse because they want to achieve their objectives with more certainty (e.g., [18] and [19]). In contrasting problem structures 5 and 6, we assume an exponential function in which $x_i = 0.3x_i^* + 0.7x_{i*}$ is indifferent to a 50 : 50 lottery between the best value x_i^* and the worst value x_{i*}. For each single-attribute utility function, we used a different utility function because each attribute has different lowest and highest values.

TABLE III
EXPECTED LOSS OF UTILITY (IN TERMS OF ANNUAL WALLEYE SPORT HARVEST) OF OPTIMAL DECISION FROM EACH PROBLEM STRUCTURE COMPARED TO PROBLEM STRUCTURE 1, AVERAGED ACROSS ALL COMBINATIONS OF WEIGHTS AND PROBABILITIES JUDGMENT (TONS PER YEAR)

Problem Structure	1	2	3	4	5	6	7
Row Average	0	11	1,011	1,573	858	1,878	1,942
# of Assumptions Differing from Structure 1	0	1	1	1	1	2	3

structure that seem to be less important are discretization and consideration of phosphorus levels. The results of problem structure 2 deviate least from problem structure 1, implying that the use of a discrete set of alternatives is the least distorting of the simplifications considered here. This is because the optimal decisions for the continuous case often lie on the upper or lower bounds of the decision variables and so happen to coincide with the discrete alternatives. Meanwhile, on average, omitting phosphorus management alternatives does not appreciably change optimal fishery exploitation. This implies that the interactions between fisheries and nutrient management may not be important, even though, in theory, lower nutrient levels should imply lowered fishery productivity.

In contrast, other changes in the problem structure can matter. For instance, if managers are risk averse (structure 5), optimal smelt mortality (trawling) is reduced, but optimal walleye mortality (sport fishing) is increased, on average, compared to the risk-neutral case (structure 1). As another example, optimal decisions that consider all uncertainties (structures 1, 2, 3, and 5) can differ from optimal decisions that only consider lower trophic uncertainties (structures 4, 6, and 7). In particular, the optimal levels of trawling and gill netting in problem structures 4, 6, and 7 (structures that omit nonlower trophic level uncertainties) diverge greatly from their levels in structure 1. When these other trophic level uncertainties are omitted, trawling for smelt increases, but gill netting of yellow perch is reduced.

B. Expected Loss of Utility Due to Simplified Problem Structure

Decisions can be affected by a problem structure, as a comparison of the different rows in Table II shows. In this section, we estimate the expected deterioration in performance if we assume the wrong problem structure. For this analysis, we assume that problem structure 1 (base case) is the "true" problem structure. This structure considers the interaction between phosphorous loading level and fisheries management, a continuous decision space, neutral attitudes toward risk, and $\underline{\Theta}^{\mathrm{OTH}}$, which are uncertainties other than lower trophic level uncertainties.

The expected loss of utility associated with the optimal decision from an incorrect problem structure S, assuming that the assumptions of problem structure 1 are true, can be calculated as

$$\begin{aligned}&\text{Expected Loss of Utility of Problem}\\&\quad\text{Structure } S\text{'s Optimal Decision}\\&\quad= E\left[U\left(\underline{X}(a_{\mathrm{opt}|\mathrm{Structure}\ 1})|\mathrm{PS}\ 1\right)\right]\\&\qquad- E\left[U\left(\underline{X}(a_{\mathrm{opt}|\mathrm{Structure}\ S})|\mathrm{PS}\ 1\right)\right]\end{aligned} \tag{8}$$

where $S = 2, \ldots, 7$; $E[U(\underline{X}(\underline{a})|\mathrm{PS}\ 1)]$ is the expected utility of alternative a, given the utility function and uncertainties considered in structure 1; and $a_{\mathrm{opt}|\mathrm{PS}\,S}$ is the optimal strategy chosen under structure S.

As an example, the expected loss of utility of problem structure 2's optimal decision (i.e., $S = 2$) represents the expected loss resulting from discretizing the decision space when the true decision space is continuous (i.e., this can be interpreted as "the value of the spline."). If $S = 5$, then this is the expected cost of assuming risk aversion when real risk attitude is neutral.

For ease of comparison, Table III summarizes the expected loss of utility (in terms of walleye sport harvest) of optimal decisions from each problem structure. Differences in expected utility between alternatives can be translated into a change in a single attribute by a standard technique in which the variation in one attribute that yields the same utility difference is calculated [2] (see [10] for the exact formulas used). Because the losses in the table are in terms of walleye sport harvest x_2, they depend on the weight w_2 assigned to that attribute. As a result, a low w_2 will inflate the ECIU in terms of x_2. The values shown for each problem structure represent the average of the 64 expected losses that result from using each of the eight sets of probabilities and value.

To put this value in perspective, the mean walleye sport harvest over 1990–1998 was about 2000 tons/year (2 307 000 fish [20] times a mean size for 1998 of 0.882 kg [21]), which at an estimated value to anglers of US \$12/fish [22] implies an annual value of about \$28 million. This indicates that the loss due to a wrong problem structure assumption can result in a loss of several tens of millions of dollars per year.

As we would expect, the results under problem structure 1 are all zero because the structure 1 is assumed to be the "true" structure [i.e., the two terms of (8) are identical]. Table III shows that the expected loss of utility generally increases as the number of simplifications increases. The exception is problem structure 4, which differs in just one assumption (exclusion of nonlower trophic level uncertainties), but results in almost as much loss as problem structure 7, which differs in three assumptions (by discretizing alternatives, ignoring phosphorus management, and like structure 4, excluding nonlower trophic level uncertainties). Indeed, for some situations (20 cases from the 64 possible combinations of value and probability judgments), structure 4 actually results in more loss than structure 7.

Among the four structures that involve only one simplification (i.e., problem structures 2, 3, 4, and 5 represent the discretization of the decision space, disregarding of nutrient management, exclusion of other uncertainties, and risk aversion, respectively), problem structure 2 (discretization) yields the smallest loss. In contrast, as just noted, problem structure 4

TABLE IV
LOSS FROM USING THE "WRONG" PROBLEM STRUCTURE, ATTRIBUTE WEIGHTS, AND SUBJECTIVE PROBABILITIES (TON WALLEYE PER YEAR)

Loss Category	Person							
	1	2	3	4	5	6	Equal Weights, Probs.	Mean Weights, Probs.
Loss due to wrong problem structure(i)[a]	29	5,232	147	14,492	2,999	1,590	204	1,147
Average loss due to wrong weights(i)	1,918	1,867	963	7,559	1,191	686	3,470	835
Average loss due to wrong probabilities(i)	41	8	10	197	8	11	42	8

a. The average of this row's values (3230) differs from the mean loss resulting from using Problem Structure 7 as shown in Table 2 (1940) because Table 2's average is over all 64 combinations of i's weights and j's probabilities, whereas Table 4 only shows those values for the eight sets of i's weights and probabilities.

TABLE V
AVERAGE ECIU OVER EACH PERSON'S PROBABILITY AND VALUE JUDGMENT IN TERMS OF WALLEYE SPORT HARVEST (TONS PER YEAR)

Problem Structure	1 (Base case)	2	3	4	5	6	7
Average	19 (18)[a]	13 (15)	1,143 (6)	516 (27)	1,506 (129)	1,032 (48)	781 (328)

a. The values in parentheses show the average ECIU in terms of walleye sport harvest x_2 without person 4's ECIU

(exclusion of nonlower trophic level uncertainties) results in the most significant impact. The other simplifications (omission of nutrient management and assumption of risk aversion) result in losses intermediate between these two extremes. Thus, in this case, inclusion of all uncertainties should be the highest priority, followed by capturing risk attitudes and including all decisions, while considering a continuum of alternatives using a cubic spline is least important. However, this is not a general result; the prioritization might well be very different for management problems other than the specific Lake Erie problem considered in this paper.

C. What Makes More of a Difference in Decisions—Value Judgments, Probability Judgments, or Problem Structure?

Optimal decisions are affected by not only the problem structure as shown in the previous section but also the people's judgments concerning probabilities and weights. This section analyzes how the performance under problem structure 1 can be diminished by using incorrect probability and value judgments, and compares that loss of utility to that caused by simplifying the problem structure (in particular, by using the most simplified problem structure 7 rather than structure 1). We consider the loss in the expected utility for person i's values and probabilities if the optimal solution from using person j's values and person k's probabilities in problem structure S is implemented instead of the alternative that is best under i's values and probabilities and problem structure 1. This loss is

$$\text{Loss}(\text{PS } S, j, k|i) = E\left[U\left(\underline{X}(a_{\text{opt}|1,i,i})|i\right)|\text{PS } 1, i\right] - E\left[U\left(\underline{X}(a_{\text{opt}|S,j,k})|i\right)|\text{PS } 1, i\right] \quad (9)$$

where $E[U(X(a_{\text{opt}|\text{PS } S,j,k})|i)|\text{PS } 1, i]$ is the expected utility (given i's weights and probabilities under problem structure 1) of the decision $a_{\text{opt}|S,j,k}$ that is made when, instead optimizing, assuming structure S under j's values and k's probabilities.

To gauge the relative importance of weight judgments, subjective probabilities, and problem structure, we calculate the following three indexes for each person i based on the above loss definition:

$$\text{Loss due to wrong problem structure}(i) = \text{Loss}(\text{PS } 7, i, i|i)$$

$$\text{Average loss due to wrong weights}(i) = \sum_{j=1,\ldots,8;j\neq i} \text{Loss}(\text{PS } 1, j, i|i)/7$$

$$\text{Average loss due to wrong probabilities}(i) = \sum_{k=1,\ldots,8;k\neq i} \text{Loss}(\text{PS } 1, i, k|i)/7.$$

The values are shown in Table IV for all i, which are measured in terms of equivalent tons per year of walleye sport harvest. They show that the loss that results from using someone else's probabilities for the lower trophic level uncertainties rather than one's own is very small compared to the other losses. The loss due to wrong problem structure is higher, when averaged across the eight sets i, than the loss due to using someone else's weights. However, this difference is not statistically significant (when assessed by a paired observation nonparametric test). As mentioned above, these magnitudes of losses (on the order of 10^2–10^4 tons/year) are equivalent to several million dollars/year (when evaluated at \$12/fish).

TABLE VI
MEAN EVPI OVER EACH PERSON'S PROBABILITY JUDGMENT IN TERMS OF WALLEYE SPORT HARVEST (TONS PER YEAR)

Project	Problem Structure						
	1	2	3	4	5	6	7
A& B	486 (36)	460 (43)	702 (18)	334 (47)	1,179 (78)	787 (54)	1,925 (125)
C& D	77 (8)	76 (8)	102 (3)	32 (14)	91 (14)	60 (26)	506 (39)
All	630 (57)	566 (62)	801 (22)	423 (52)	1,496 (106)	987 (78)	2,130 (133)

* *Note*: The values in the parentheses indicate the average EVPIs excluding person 4's EVPI.

D. ECIU

ECIU, the expected cost of disregarding uncertainty when it actually exists, can only be nonzero if $a_{naïve}$ differs from a_{opt}. In Table V, the values (termed "ECIU") shown for each problem structure represent the average ECIU over the ECIUs that result from the 64 combinations of each person's probabilities and weights judgments. ECIU is expressed as the improvement in annual walleye sport harvest needed to equate the expected utility of $a_{naïve}$ and a_{opt}. Our analysis here considers ECIU with respect to just the lower trophic level uncertainties $\underline{\Theta}^{\mathrm{LTL}}$.[2]

Because the ECIUs in the table are expressed in terms of the attribute "walleye sport harvest" x_2, those ECIUs depend on the weight assigned to that attribute (i.e., a low weight on walleye sport harvest will inflate the ECIU). Because person 4's weight on walleye sport harvest is very low compared to the weight other persons assigned that attribute, the table also shows the average ECIU in terms of walleye sport harvest without person 4's ECIU in parentheses.

The results indicate that the cost of ignoring uncertainty in problem structure 2 (i.e., just discrete options) is the smallest because the biggest contributor to average ECIU, person 4's ECIU, happens to be zero in that case. A smaller ECIU for this structure is unsurprising, since consideration of just a small set of discrete alternatives means that decisions are less likely to change if assumptions are changed than if there is a continuum of alternatives.

In contrast, under problem structures 3, 5, and 6, the ECIUs might be as much as half of the annual walleye sport harvest (2000 tons/year). These values are very different from those of structure 1. This indicates that the simplification of disregarding phosphorus decisions and their interaction with fisheries (structure 3) and consideration of risk aversion (structure 5) may matter the most in ECIU.

However, because the ECIUs in the table are in terms of walleye sport harvest x_2, those ECIUs depend on the weight assigned to that attribute. As a result, a low w_2 will inflate the ECIU in terms of x_2. One participant (person 4)'s weight on x_2 is 0.013, which is very low compared to what the other persons gave that attribute. Therefore, Table V also shows the average ECIU in terms of walleye sport harvest x_2 without person 4's ECIU (in parentheses).

[2]A comparison of structures 1 and 4 in Table V represents an ECIU with respect to just the other uncertainties $\underline{\Theta}^{\mathrm{OTH}}$ because we assumed that the decisions based on problem structure 1 (base case) are the optimal decisions, while the values resulting from problem structure 4 in Table V show the expected loss of performance of problem structure 4's optimal decisions which exclude other uncertainties.

Without person 4's ECIU, the smallest cost of ignoring uncertainty is under structure 3 (disregarding phosphorus decisions), and the greatest cost of ignoring uncertainty is for structure 7 (the simplest problem structure). Problem structure 5, which gives the biggest ECIU among problem structures when the person 4's ECIU is included, also results in a relatively large ECIU (the second largest average value) when person 4's ECIU is excluded. These results confirm that our conclusions about the relatively large impact upon ECIU of these simplifications are robust with respect to whether person 4's very high values are included or not.

E. Two-Stage Problem: EVPI and EVII Results

In the previous sections, we considered only a one-stage decision problem: which fishing and phosphorus policies are optimal, given the present state of information? In contrast, if research would reduce uncertainty, and its benefits (in terms of better management) could exceed its cost, we should consider undertaking it. An evaluation of the research requires a two-stage analysis, as in Fig. 2. Information from research generally reduces uncertainty without eliminating it. Thus, as we noted above, the actual value of information (EVII) is equal to or less than EVPI (and often greatly so). Table VI shows mean EVPIs over the 64 combinations of each person's probabilities and weights judgments.

The first of the three EVPIs for each combination is the EVPI associated with the uncertain parameters $\underline{\Theta}^{\mathrm{LTL}}$ for which projects A and B provide information: ZM_r (mussel phosphorus recycling), Z_{kp} (zebra mussel production), AZP (zooplankton production), and AZB (i.e., zoobenthos production). Therefore, the upper bounds of value of information, EVPI, from project A and B are identical because those projects address the same parameters, and the EVPI calculation assumes that the research result is an error-free parameter estimate. The second of the EVPIs is for the two parameters with which projects C and D are concerned: ZM_r and g_0 (the relation between phosphorus loading and primary productivity). Consequently, the EVPIs from projects C and D are the same. The last row's values are the EVPIs for eliminating all five uncertainties (ZM_r, Z_{kp}, AZP, AZB, g_0). We also show the average EVPI without person 4's EVPI in parentheses. As we can see from the mean EVPIs in the rightmost rows, EVPI based on problem structure 7 is the highest for all structures, no matter whether person 4's EVPI is excluded or not. Projects A and B clearly have higher potential benefits than projects C and D for all structures.

To compare the potential value of information for each project with its cost, we need to translate the cost into an

TABLE VII
AVERAGE EVII OVER 64 COMBINATIONS OF EACH PERSON'S PROBABILITY AND VALUE JUDGMENT IN TERMS OF WALLEYE SPORT HARVEST (TONS PER YEAR)

Project	Problem Structure						
	1	2	3	4	5	6	7
A	160 (14)	148 (18)	216 (7)	104 (17)	326 (24)	281 (32)	606 (36)
B	99 (8)	98 (11)	206 (3)	44 (17)	158 (11)	144 (28)	652 (41)
C	13 (2)	14 (2)	25 (1)	9 (5)	19 (3)	24 (23)	81 (7)
D	16 (2)	18 (1)	9 (0)	2 (1)	13 (2)	19 (19)	88 (5)

* *Note*: The values in the parentheses indicate the average EVPIs excluding person 4's EVPI.

equivalent value of x_2. Based on a review of nonmarket valuation of Great Lakes fishes [22], we used a value of \$12 per fish caught as a basis for comparison. Meanwhile, the costs of research projects A, B, C, and D are estimated to be \$194 K, \$932 K, \$813 K, and \$149 K, respectively. As the research cost is a one-time expense, while the ecological benefits of management are ongoing, an interest rate assumption is needed; here, we use 10%/year and a ten-year time horizon for the ecological benefits. As a result, the break-even points at which the EVPI or EVII of a project in terms of x_2 equals its annualized cost are 2.3, 11, 9.7, and 1.7 tons/year of walleye for research projects A, B, C, and D, respectively. Therefore, the EVPIs indicate that the average benefits from any project, no matter which problem structure is considered, are potentially greater than its cost.

However, the actual expected benefits of these projects (EVII) could be much smaller than EVPI, depending on the reliability of their results. Table VII shows the EVII results for the four research projects under each problem structure, average EVII over the EVIIs that result from 64 combinations of each person's probabilities and weights judgments.

The first, second, third, and fourth values in each row indicate the EVII of projects A, B, C, and D under each respective problem structure. The results show that the EVII ranges from 2 to 652 tons of sport walleye harvest. Except for person 4, the values are less than 50 tons/year in every case.

Using the cost/walleye and interest rate assumptions of the previous section, we find that research project A's benefits exceed its cost for all problem structures, regardless of whether the average includes person 4's EVII. However, for projects B, C, and D, their benefits depend on whether the person 4's EVII is included. The last two rows in Table VII indicate that without person 4's EVII averaged in, the expected value of information from projects C and D is very small. Projects A and B tend to have higher benefits than C and D because the former provide information on four of the five uncertain parameters in $\underline{\Theta}^{\mathrm{LTL}}$, whereas the latter projects shed light on just two of the parameters. Furthermore, on average over problem structure, value, and probability judgments, these EVIIs (Table VII), including person 4's EVII, are 31%, 24%, 20%, and 17% of the EVPI values (Table VI) including person 4, for Projects A, B, C, and D, respectively. Therefore, these results indicate the Projects A and B tend to be more reliable than Projects C and D.

V. CONCLUSION

A multicriteria Bayesian decision analysis based on a continuous decision set has been presented that includes multiple decisions (including research), uncertainties, and objectives. Unfortunately, simplification is often necessary in decision analyses because of the curse of dimensionality, but simplification can distort decisions and estimates of the cost of disregarding uncertainty (ECIU) and the value of additional information (EVPI and EVII). We present a spline approach for allowing a continuum of alternatives to be considered in such decision analyses. Such an approach can also be used to interpolate between discrete realizations of uncertain parameters in a decision tree.

We have also undertaken a case study in which the above methods were applied to Lake Erie phosphorus and fishery management alternatives. The fishery managers who provided the utility function weights and subjective probabilities were asked how easy, meaningful, and useful they found each of the workshop exercises. In general, the participants felt that value judgments, including definition of objectives and weight assessment, were easier and more meaningful than probabilistic judgments, including prior probabilities and likelihoods of research outcomes [10], although they felt that more time should be taken for the assessment tasks than what was available in the study. Decision analysis is now being used by Canadian fishery interests to assess walleye management policies in Lake Erie [23]. However, decision analysis is not presently utilized to set fishing limits, which is the responsibility of the Lake Erie Committee of the Great Lakes Fishery Commission.

In this paper, we quantified the impact of problem simplification for this case study. We find that the optimal decisions are not the same in all seven problem structures. This means that simplifications to the problem structure, including discretized fisheries management options, no consideration of phosphorus loading changes, no nonlower trophic-level uncertainties, and alternative risk attitudes, can change the decision. In the present case, considering just discrete options (structure 2) does not often change the solution because the optimal decisions for the continuous case frequently lie on the upper or lower bounds of the variables and so happen to coincide with the discrete alternatives. Likewise, narrowing the class of alternatives considered (disregarding phosphorus management options, structure 3) has relatively little effect on decisions, although the estimated ECIUs, EVPIs, and EVIIs do change considerably. In contrast, the problem structure simplifications that omit nonlower trophic-level uncertainties (structures 4, 6, and 7) result in relatively large changes in both decisions and the ECIU, EVPI, and EVII indexes, as well large expected losses in performance assuming that problem structure 1 (base case) is the "true" problem structure. The assumption of omitting other uncertainties tends to decrease gill netting effort and increase trawling effort. Assuming that the fishery managers

are strongly risk averse rather than risk neutral also causes significant changes in decisions.

Estimates of ECIU, EVPI, and EVII are also affected by simplifications, often being greatly inflated. Under problem structures 3, 5, and 6, the ECIUs might be as much as half of the historical annual walleye sport harvest (2000 tons/year). This indicates that the simplification of disregarding phosphorus decisions and their interaction with fisheries (structure 3) and consideration of risk aversion (structure 5) may significantly affect ECIU. On the other hand, the simplest case, structure 7 (i.e., discretized fisheries management options, no phosphorus loading changes, no nonlower trophic-level uncertainties, and risk attitude) can make the biggest difference in estimates of both EVII and EVPI. Unsurprisingly, EVII is the most sensitive for those projects (A and B) for which the estimated value of information is the highest.

In summary, these analyses show that simplifications in problem structure can significantly distort the results of decision analyses, including decisions and indexes of interest (ECIU, EVPI, and EVII). However, which simplifications have the greatest effect is likely to depend on the specific problem studied. We also find that the resulting loss of expected utility (resulting from selecting suboptimal alternatives) is on the same order of magnitude as the losses resulting from using the "wrong" attribute weights in a multiattribute analysis. The latter is gauged by examining how different fishery managers' weights change decisions. But, in this particular case, the losses from using the "wrong" (i.e., a different manager's) subjective probabilities have much less of an effect on the decision analysis.

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