The 2.5D Pseudospectral Time-Domain (PSTD) Algorithm With PML Absorbing Boundary Condition

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I. Introduction

In many applications, such as geophysical subsurface sensing, microwave and optical waveguides, one can reduce three-dimensional problems into 2.5-dimensional ones because the inhomogeneity is two-dimensional and one of the spatial derivatives in Maxwell's equation can be simplified by using the Fourier transform. Most previous 2.5-D algorithms [e.g., 1, 2] were based on the FDTD method and used non-PML absorbing boundary condition. Very recently, the pseudospectral time-domain (PSTD) method [3, 4] has been proposed for multidimensional problems. The key point of PSTD method is that it uses the fast Fourier transform (FFT) other than finite differences to calculate spatial derivatives and uses Berenger's perfectly matched layers [5] to remove wraparound effect caused by the periodicity assumed in the FFT. In this work we use the PSTD method to solve 2.5-dimensional problems. The theoretical analysis and numerical examples are presented.

II. Formulation

In a Cartesian coordinate system, the three-dimensional Maxwell's equations can be written using complex stretched coordinate variables $e_\eta = a_\eta + i\frac{\omega_\eta}{\omega}$ $(\eta = x, y, z)$ [4, 6] as :

$$a_\eta \mu \frac{\partial \mathbf{H}^{(\eta)}}{\partial t} + \mu\omega_\eta \mathbf{H}^{(\eta)} = -\frac{\partial}{\partial \eta}(\hat{\eta} \times \mathbf{E}) - \mathbf{M}^{(\eta)}, \tag{1}$$

$$a_\eta \epsilon \frac{\partial \mathbf{E}^{(\eta)}}{\partial t} + (a_\eta \sigma + \omega_\eta \epsilon)\mathbf{E}^{(\eta)} + \omega_\eta \sigma \int_{-\infty}^{t} \mathbf{E}^{(\eta)} dt = \frac{\partial}{\partial \eta}(\hat{\eta} \times \mathbf{H}) - \mathbf{J}^{(\eta)}, \tag{2}$$

where $\mathbf{E} = \sum_{\eta = x,y,z} \mathbf{E}^{(\eta)}$ and similarly for other field components. Equations (1) and (2) consist of a total of 12 scalar equations, since both $\mathbf{E}^{(\eta)}$ and $\mathbf{H}^{(\eta)}$ have two scalar components perpendicular to $\hat{\eta}$.

The FFT algorithm is used to represent the spatial derivative in equation (1) to yield

$$a_\eta \mu \frac{\partial \mathbf{H}^{(\eta)}}{\partial t} + \mu\omega_\eta \mathbf{H}^{(\eta)} = \mathcal{F}_\eta^{-1}\{ik_\eta \mathcal{F}_\eta[\hat{\eta} \times \mathbf{E}]\} - \mathbf{M}^{(\eta)}, \tag{3}$$

where $\mathcal{F}_\eta$ and $\mathcal{F}_\eta^{-1}$ denote forward and inverse Fourier transforms in the η direction which are calculated by FFT's. A similar result can be obtained for equation (2).

Now consider an isotropic medium with two dimensional inhomogeneity in the xy plane and invariant in the z direction. Since the source at z=0 may be electric or magnetic and

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may be an even or odd function of z, the electromagnetic fields and their spatial derivatives may be even or odd functions of z. The total field can be written as $\mathbf{E} = \mathbf{E}_e + \mathbf{E}_o$, where e and o denote the even and odd parts, respectively. One can use the cosine and sine transforms for these two parts, for example,

$$\tilde{\mathbf{H}}_e(x, y, k_z, t) = \int_0^\infty \mathbf{H}_e(x, y, z, t)\cos(k_z z)dz \tag{3}$$

$$\tilde{\mathbf{H}}_o(x, y, k_z, t) = \int_0^\infty \mathbf{H}_o(x, y, z, t)\sin(k_z z)dz \tag{4}$$

to simplify Maxwell's equations. It can be shown that sources (J_{xo}, J_{yo}, J_{ze}, M_{xe}, M_{ye}, M_{zo}) will excite fields (E_{xo}, E_{yo}, E_{ze}, H_{xe}, H_{ye}, H_{zo}), and sources (J_{xe}, J_{ye}, J_{zo}, M_{xo}, M_{yo}, M_{ze}) will excite fields (E_{xe}, E_{ye}, E_{zo}, H_{xo}, H_{yo}, H_{ze}). As an example, if the source is a horizontal electric dipole directed along the z direction, then only $J_z \neq 0$ and J_z is even with respect to the z coordinate. Using the sine and cosine transforms and omitting the subscripts e and o, one can rewrite (2) as:

$$a_y\epsilon\frac{\partial \tilde{E}_x^{(y)}}{\partial t} + (a_y\sigma + \omega_y\epsilon)\tilde{E}_x^{(y0} + \omega_y\sigma\int_{-\infty}^{t}\tilde{E}_x^{(y)}dt = \frac{\partial \tilde{H}_z}{\partial y}, \tag{5}$$

$$\epsilon\frac{\partial \tilde{E}_x^{(z)}}{\partial t} + \sigma\tilde{E}_x^{(z)} = k_z\tilde{H}_y \tag{6}$$

$$a_x\epsilon\frac{\partial \tilde{E}_y^{(x)}}{\partial t} + (a_x\sigma + \omega_x\epsilon)\tilde{E}_y^{(x)} + \omega_x\sigma\int_{-\infty}^{t}\tilde{E}_y^{(x)}dt = -\frac{\partial \tilde{H}_z}{\partial x}, \tag{7}$$

$$\epsilon\frac{\partial \tilde{E}_y^{(z)}}{\partial t} + \sigma\tilde{E}_y^{(z)} = -k_z\tilde{H}_x \tag{8}$$

$$a_x\epsilon\frac{\partial \tilde{E}_z^{(x)}}{\partial t} + (a_x\sigma + \omega_x\epsilon)\tilde{E}_z^{(x)} + \omega_x\sigma\int_{-\infty}^{t}\tilde{E}_z^{(x)}dt = \frac{\partial \tilde{H}_y}{\partial x} - \tilde{J}_z^{(x)}, \tag{9}$$

$$a_y\epsilon\frac{\partial \tilde{E}_z^{(y)}}{\partial t} + (a_y\sigma + \omega_y\epsilon)\tilde{E}_z^{(y)} + \omega_y\sigma\int_{-\infty}^{t}\tilde{E}_z^{(y)}dt = -\frac{\partial \tilde{H}_x}{\partial y} - \tilde{J}_z^{(y)}. \tag{10}$$

Similar equations can be written from (1).

In numerical implementation of the 2.5-D PSTD method, we use FFT to calculate the spatial derivatives and central difference for time integration. The stability condition of 2.5-D PSTD method for a plane wave in a homogeneous nonconductive medium can be derived as in [4]. It is shown that $k_{max}c\Delta t \leq 2$ in the PSTD method [4]. According to the

Nyquist sampling theorem, the FFT algorithm provides exact representation for $k_x \leq \frac{\pi}{\Delta x}$ and $k_y \leq \frac{\pi}{\Delta y}$. So from the equation $k^2 = k_x^2 + k_y^2 + k_z^2$ we can get the stability condition:

$$\Delta t \leq \frac{\Delta x}{c\sqrt{\frac{\pi^2}{2} + (\frac{k_z \Delta x}{2})^2}} \tag{11}$$

In the above, k_z is the Fourier integration variable. For the determination of waveguide dispersion curves, it can be taken as an input parameter.

III. Numerical Results

In order to validate the numerical implementation, we compare the results of the 2.5-D PSTD and FDTD in Figure 1. The medium is homogeneous nonconductive. The electric dipole source is located at (x, y, z)=(0.2, 0.2, 0) m for Figure 1(a) and at (x, y, z)=(1.6, 1.6, 0) m for Figure 1(b). The field E_z is computed at (x, y, z)=(0.6, 0.6, 0.3), (0.6, 0.6, 0.4) m for Figure 1(a) with 16 cells/λ_{min} and at (x, y, z)=(4.8, 4.8, 3), (4.8, 4.8, 4) m for Figure 1(b) with 2 cells/λ_{min}. Figure 1(c) and (d) show the 2.5-D PSTD and FDTD methods for wave propagation in a conductive homogeneous medium. The source and receivers are the same as that in Figure 1(b). From Figure 1, it can be seen that the 2.5-D PSTD algorithm gives accurate results even if the grid density is 2 cells/λ_{min}.

For the application, we compute the open dielectric waveguide which is shown in Figure 2. The result is matched well with [7].

IV. Conclusions

A 2.5-dimensional PSTD algorithm has been developed which can be used to solve Maxwell's equations efficiently. The code has been validated by the analytical results and conventional FDTD method. It is also shown that the 2.5D PSTD algorithm is more efficient than FDTD method. Particularly, 2.5D PSTD method can be used to model very large object because it needs a grid density of only 2 cells/λ_{min}. The dispersion curve of a open dielectric waveguide is obtained by using the 2.5-D PSTD algorithm. Further research work will model various waveguide structures.

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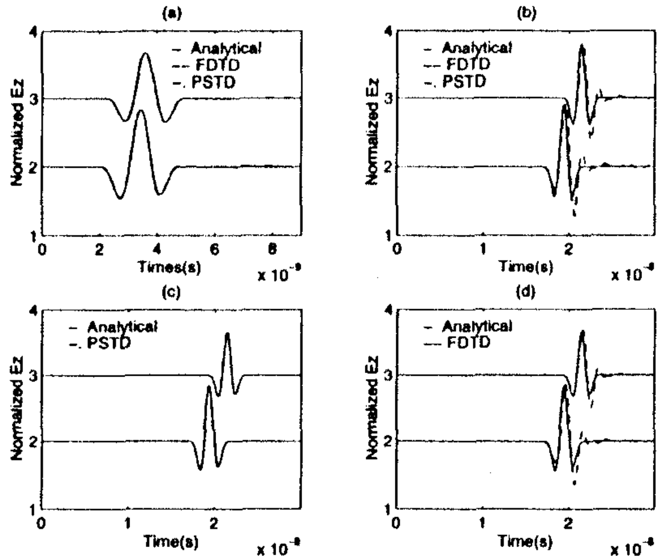

Figure 1. Numerical results of 2.5D PSTD and FDTD algorithms with the analytical solutions. (a) $\Delta x = \lambda_{min}/16$. (b) $\Delta x = \lambda_{min}/2$. (c) and (d) $\Delta x = \lambda_{min}/2$ for conductive homogeneous medium.

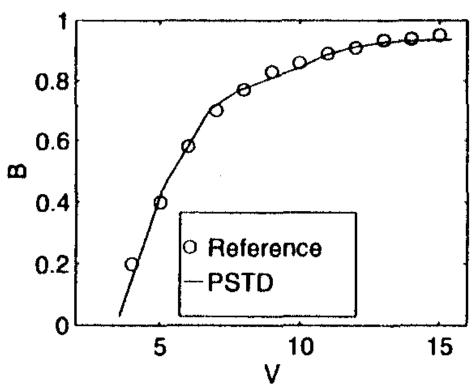

Figure 2. Open dielectric waveguide a=b, ϵ_1=13.1ϵ_0, ϵ_2=ϵ_0. Normalized dispersion curve of the E^y_{11} mode where B=$\frac{[(\frac{\beta}{k_0})^2-1]}{(\epsilon_{r1}-1)}$ and V=$k_0 a\sqrt{(\epsilon_{r1}-1)}$.